Capital Utilization, Obsolescence and Technological Progress

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Abstract

In a standard endogenous R&D growth model with expanding variety of intermediate inputs I incorporate endogenous depreciation rate for the intermediate inputs. The depreciation rate depends negatively on the utilization rate of the intermediate inputs and positively on their durability level, resulting into smaller economic growth relatively to the standard models of expanding variety inputs. The reason is that higher durability for intermediate inputs implies a lower demand for the intermediate inputs which in turn reduces the motivation for innovation. The utilization rate on the other hand, even if it increases the depreciation rate, is responsible for higher demand for the intermediate inputs and therefore it increases the motivation for innovation. The two forces (durability and utilization) have an asymmetric effect on economic growth.

JEL: E22, E23, O3, O40.

Key Words: Capital Utilization; Depreciation; Endogenous Growth; Innovation; Obsolescence.

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1. Introduction

Physical capital is probably the most important and stable engine of growth as established by many empirical studies starting with the influential work of Levine and Renelt (1992). However, the use of physical capital in the production process results in its depreciation-obsolescence over time. The cost of depreciation has been measured by McGrattan and Schmitz (1999) and they find it to be sizable[1]. In addition, the main variable which determines the depreciation rate is that of capital utilization. Firms in practice underuse or overuse physical capital as exemplified in the literature of business cycles where firms have varying capacity utilization depending on the phase of the cycle that the economy performs.[2] As such, firms facing competitive conditions try to invest in durability of the physical capital they use in order to reduce its obsolescence during the production process. At the same time physical capital in a more realistic way is not homogenous but a differentiated input, since economies through R&D activities produce new patents for differentiated types of physical capital.[3] Therefore technological progress is something closely related to the invention of intermediate inputs. In this paper, we are interested in checking how the decisions of firms regarding both utilization and durability rates affect technological progress and consequently economic growth.

We proceed to analyze the endogenous depreciation rate which is endogenized by durability and utilization rates. Gylfason and Zoega (2007) in an extended Solow model of exogenous technological progress have endogenized depreciation rate through durability.[4] The sense of durability has to do mainly with the strength of physical capital during the production process. However, the pressure on physical capital during high rates of production, the utilization rate, creates higher depreciation of physical capital

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[1] They have found that the maintenance activities are 6% of gross national product of Canada. Other empirical attempts to measure the depreciation of physical capital are those of Epstein and Denny (1980), Nadiri and Prucha (1996) and Fraumeni (1997). All these findings suggest that the depreciation rate is of considerable size.

[2] Capacity utilization is mainly capital utilization since physical capital faces installation and other sunk costs constraints in comparison to labour. Empirical estimation of capital-capacity utilization can be found in Cooley et al. (1995) and Dergiades and Tsoulfidis (2007). Moreover, Imbs (1999) and King and Rebelo (1999) have shown empirically varying capital utilization across countries.

[3] This differentiated type of physical capital is what the literature of endogenous R&D growth calls as intermediate inputs.

[4] They solve the Solow model in the case of steady state consumption maximization. They find that higher technological progress leads to lower growth when the depreciation rate is endogenous because the durability level comes at a cost of foregone consumption.

and that make firms target a higher durability level for their machines. An important paper of Chatterjee (2005) analyses the effect of capital utilization on the convergence speed by using a generic production function which under different parameterizations can capture both exogenous and endogenous growth models. He finds that the existence of utilization reduces the speed of convergence and steady steady-state equilibrium because full utilization through higher depreciation acts as a disincentive to investment and growth. However, both paper of Chatterjee (2005) and Gylfason and Zoega (2007) rely on either exogenous growth models or by ignoring endogenous technological progress. For example, Gylfason and Zoega (2007) examine the effect of exogenous technological progress on durability and therefore on the savings behaviour. In the present model, I use an R&D growth model with expanding variety of intermediate inputs with purpose to investigate how the simultaneous presence of endogenous durability and utilization rates affect economic growth through their effect on endogenous technological progress which occurs through the expanding variety of intermediate inputs.

The main results of the paper are the following three: i) in comparison to the benchmark expanding variety of intermediate inputs R&D models the growth rate is lower when there is endogenous depreciation rate through durability and utilization, ii) similar to Gylfason and Zoega (2007) higher durability reduces economic growth and iii) the higher utilization rate increases economic growth which is compatible with the empirical finding of Mayshar and Halevy (1997) and the theoretical predictions of Chatterjee (2005). However, the mechanism which explains the above results in the current paper is the following: if the durability level which is incorporated in each intermediate input is high, then there is a lower demand for intermediate inputs which reduces the value of the R&D firms and therefore the motivation of research in finding new intermediate inputs. The utilization rate mitigates the negative effect of durability because the intermediate inputs depreciate faster which leads to higher demand for intermediate inputs. Therefore, economic growth which is the result of the expanding variety of intermediate inputs may not necessarily be higher if there is higher demand for more durable intermediate inputs as there may a lower demand for intermediate inputs.

The above analysis indicates an asymmetric impact of durability and utilization on economic growth.[5]

The paper is organised as follows: section 2 describes the assumptions of the theoretical model and provides the steps for its solution; section 3 provides the decentralized competitive equilibrium with the necessary results described in the various propositions. The fourth section concludes the paper.

2. Theoretical Model - Model Set up

I develop an endogenous growth model *ala* Romer (1987, 1990) with expanding variety of intermediate inputs by incorporating endogenous depreciation rate for the intermediate inputs which are used for the production of the final output. To endogenize the depreciation rate of the intermediate inputs I follow Gylfason and Zoega (2007), but extending their framework as the depreciation rate does not only depend on the durability level of the intermediate inputs but also on the utilization rate of the intermediate inputs. The intermediate inputs are a form of capital and the effect of capital utilization on its depreciation rate has been studied in Chatterjee (2005). I assume that the higher the durability level of the intermediate inputs, the higher will be their impact in the production of the final good (in other words durability acts as a positive externality) and the lower their depreciation rate. However, higher durability comes at a cost. On the other hand, the utilization rate of the intermediate inputs is at its maximum if and only if there is no effect of the utilization rate on the depreciation rate of the intermediate inputs.[6] The economy has three sectors: a competitive final output sector in which labour and intermediate inputs are used as necessary inputs, a competitive R&D sector which discovers new intermediate inputs and a monopolistic competitive sector which produces the invented intermediate inputs

[5] Indicative empirical papers, which have analyzed the existence of asymmetric effects on economic growth due to different reasons, are those of Alimi (2016) and Karadam (2018).

[6] However, in the model the utilization rate increases the production of the final good but it increases also the depreciation rate which is a cost for the firms because they have to buy in advance a higher quantity of the intermediate inputs. In the literature however, the utilization rate plays the role of excess capacity in order firms to respond in shocks and to create entry barriers to competitors.

2.1 Production

There are a number of competitive final goods firms. The good of firm i is produced with labour $\left(L_i\right)$ and specialized inputs $\left(x_{ij}\right)$. However, final output firms for various reasons as it has been proposed in the literature underutilize their physical capital stock and have excess capacity which in the current model implies that only a fraction of the $u_{ij} \in \left(0,1\right)$ of the intermediate inputs is used: $u_{ij}x_{ij}$. Each intermediate input is denoted by j. An important assumption in the model is that the level of durability of each intermediate input affects positively the production of the final output, in other words an intermediate input is more productive if it is more durable because it carries a better technology. The durability in Gylfason and Zoega (2007) is defined as $d \in \left(0,1\right)$. Therefore, by following the standard aggregate production technology (Spence, 1976; Dixit and Stiglitz, 1977; Ethier 1982; Gancia and Zilibotti, 2005) and incorporating the previous assumptions the technology for the production of the final goods of firm i is:

$$Y_{it} = L_{ij}^{1-\alpha}\left[\int_0^{N_t} d_{ij}^{\phi}\left(u_{ij}x_{ij}\right)^{\alpha} dj\right], \qquad \alpha \in \left(0,1\right) \text{ and } \phi < 1 \tag{1}$$

where N_t is the number of intermediate-input varieties discovered until time t, while α and $1-\alpha$ represent the elasticity of of final good with respect to intermediate inputs and labour respectively. The level of durability and utilization initially is assumed to be specific to each intemediate input. The term d_{ij}^{ϕ} shows the positive role of the durability of each intermediate input on its own productivity.[7] Since $d \in \left(0,1\right)$, the contribution of durability is higher if ϕ approaches zero. Moreover, $\phi < 1$ implies that the durability has diminishing returns in the production of the final good. In the way we model the production function, technological progress which in this strand of literature arises from the increase of the number of the intermediate inputs N_t is also affected from the durability level of each of the intermediate inputs d. Moreover, another assumption which differs from literature is the depreciation rate of each intermediate input which is represented by the following function:

[7] In a symmetric equilibrium as it is shown below, the durability is the same across the different intermediate inputs and it plays obviously as a positive externality the role of total factor productivity in the production function. All the variables vary across time but in the Balanced Growth Equilibrium we will show that x_{ij}, u_{ij} and d_{ij} do not vary across time.

$$\delta_{x_{ij}} = \left(1 - d_{ij}\right)^{\beta} u_{ij}^{\lambda}, \qquad \beta > 1 \text{ and } \lambda > 1 \tag{2}$$

From equation (2) it is clear that in the model we assume a depreciation rate for the intermediate inputs which is also endogenous on the level of durability and utilization. If the durability of the intermediate inputs is high their depreciation rate is low and on the contrary if the utilization rate is high their depreciation rate is high.[8] Firms in the intermediate sector are monopolistically competitive. The assumption of $\beta > 1$ captures the idea of diminishing returns of durability as in Gylfason and Zoega (2007) and it results in high decrease of depreciation rate due to an increase in the durability. Finally, the assumption $\lambda > 1$ implies that the increase of the utilization rate leads to a high increase of the depreciation rate.[9] Therefore the firms decide how much labour and intermediate inputs to hire, but also how much should be both the level of the durability that is incorporated in each intermediate input and the utilization rate of each intermediate input. Two extra assumptions are made at this point: first of all, the cost of durability for final output firms is one to one with the level of durability and secondly, the firms know and pay in advance for the depreciation of the intermediate inputs in order to produce a certain amount of output. The price of the final output is normalized to one. Therefore, the maximization problem of the final output firms is:

$$\max_{L_i, x_{ij}, u_{ij}, d_{ij}} \pi_i^Y = L_i^{1-\alpha}\left[\int_0^{N_t} d_{ij}^{\phi}\left(u_{ij}x_{ij}\right)^{\alpha} dj\right] - wL_i - \int_0^{N_t} P_j x_{ij}\left(1 + d_{ij} + \delta_{x_{ij}}\right) dj \tag{3}$$

P_j is the price of each intermediate input j and the first order conditions of the above problem by using $\delta_{x_{ij}}$ from equation (2) are:

$$\frac{\partial \pi_i^Y}{\partial L_i} = 0 \Rightarrow w = \left(1 - \alpha\right) L_i^{-\alpha}\left[\int_0^{N_t} d_{ij}^{\phi}\left(u_{ij}x_{ij}\right)^{\alpha} dj\right] \tag{4}$$

$$\frac{\partial \pi_i^Y}{\partial x_{ij}} = 0 \Rightarrow \alpha L_i^{1-\alpha} d_{ij}^{\phi}\left(x_{ij}\right)^{\alpha-1}\left(u_{ij}\right)^{\alpha} = P_j\left(1 + d_{ij} + \left(1 - d_{ij}\right)^{\beta} u_{ij}^{\lambda}\right) \tag{5}$$

$$\frac{\partial \pi_i^Y}{\partial d_{ij}} = 0 \Rightarrow \phi L_i^{1-\alpha} d_{ij}^{\phi-1}\left(x_{ij}u_{ij}\right)^{\alpha} = P_j x_{ij}\left(1 - \beta\left(1 - d_{ij}\right)^{\beta-1} u_{ij}^{\lambda}\right) \tag{6}$$

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[8] From the previous analysis is obvious that both durability and utilization have both positive and negative impact, therefore it is expected that both of them will have an interior solution.

[9] The assumption of $\lambda > 1$ is used also in Chatterjee (2005). Empirical support of $\lambda > 1$ is in the papers of Finn (1995), Burnside and Eichenbaum (1996), Basu and Kimball (1997) and Dalgaard (2003).

$$\frac{\partial \pi_i^Y}{\partial u_{ij}} = 0 \Rightarrow \alpha L_i^{1-\alpha} d_{ij}^{\phi} \left(x_{ij}\right)^{\alpha-1} = P_j \lambda \left(1 - d_{ij}\right)^{\beta} u_{ij}^{\lambda-\alpha} \tag{7}$$

Equation (4) is the inverse demand for labour by firm *i*. Equation (5) is the inverse demand for the intermediate input *j*. Equation (6) describes the equalization of the marginal benefit of durability (left hand side of eq. (6)) with the marginal cost of durability (right hand side of eq. (6)). Finally, eq. (7) describes the equalization of the marginal benefit of utilization (left hand side of eq. (7)) with the marginal cost of utilization (right hand side of eq. (7)). By solving eq. (5) with respect to x_{ij} we get the demand for the intermediate input:

$$x_{ij} = \left(\frac{\alpha L_i^{1-\alpha} d_{ij}^{\phi} \left(u_{ij}\right)^{\alpha}}{P_j \left(1 + d_{ij} + \left(1 - d_{ij}\right)^{\beta} u_{ij}^{\lambda}\right)} \right)^{\frac{1}{1-\alpha}} \tag{8}$$

By replacing $L_i^{1-\alpha} d_{ij}^{\phi} \left(x_{ij} u_{ij}\right)^{\alpha}$ from condition (5) into condition (6) we get:

$$d_{ij} = \frac{\phi + \left(1 - d_{ij}\right)^{\beta} u_{ij}^{\lambda} \left(\alpha\beta + \phi\right)}{\alpha - \phi} \tag{9}$$

showing the optimal level of durability for an intermediate input *j* that is required by any firm *i*. By combining condition (5) together with (10) we get the following result:

$$1 + d_{ij} = \left(\lambda - 1\right)\left(1 - d_{ij}\right)^{\beta} u_{ij}^{\lambda} \tag{10}$$

If eq. (9) is solved with respect to $\left(1 - d_{ij}\right)^{\beta} u_{ij}^{\lambda}$ and by replacing it into eq. (10) we get the solution of the optimal level of durability for each intermediate input *j*:

$$d_{ij}^{*} = \frac{\left(\alpha\beta + \phi\lambda\right)}{\lambda\left(\alpha - \phi\right) - \alpha\left(1 + \beta\right)} \tag{11}$$

Equation (11) shows the optimal level of durability that an intermediate input *j* must have for firm *i*, and it is expressed on the exogenous parameters of the model. From equation (9) we can have an expression for the utilization rate u_{ij}^{λ}:

$$u_{ij} = \left[\frac{d_{ij}\left(\alpha - \phi\right) - \phi}{\left(1 - d_{ij}\right)^{\beta} \left(\alpha\beta + \phi\right)} \right]^{\frac{1}{\lambda}} \tag{12}$$

By replacing in eq. (12) the equilibrium solution for durability from eq. (11) we get the optimal level of utilization for each intermediate input:

$$u_{ij}^{*} = \left[\frac{\alpha \left[\lambda \left(\alpha - \phi \right) - \alpha \left(1 + \beta \right) \right]^{\beta - 1}}{\left[\lambda \left(\alpha - 2\phi \right) - \alpha \left(1 + 2\beta \right) \right]^{\beta}} \right]^{\frac{1}{\lambda}} \tag{13}$$

Equation (13) shows the optimal level of utilization that an intermediate input *j* must have for firm *i*, and it is expressed on the exogenous parameters of the model. It can be observed from eq. (11) and (13) that the optimal level of durability and utilization are the same for every input *j* and firm *i* and in the rest of the paper they are referred as d^{*} and u^{*}. Therefore, the total demand for the intermediate input *j* from all the firms is:

$$x_{j} = L \left(\frac{\alpha d^{*\phi} u^{*\alpha}}{P_{j} \left(1 + d^{*} + \left(1 - d^{*} \right)^{\beta} u^{*\lambda} \right)} \right)^{\frac{1}{1-\alpha}} \tag{14}$$

2.2 Intermediate Producers

The intermediate firms are monopolistic competitors since every intermediate input in the production function is not perfect substitutable by any other. Moreover, the cost of producing one unit of any intermediate input j is one as it is standard in the literature if the durability level of the intermediate input is zero. If the durability level of the intermediate input is d_{j} positive and different from zero, then the cost of producing this intermediate input is $x_{j}\left(1 + d_{j}\right)$ which is adjusted both for the quantity and durability of x_{j}. Any representative intermediate firm maximizes the following problem:

$$\max_{P_j} \pi_{j}^{I} = P_{j} x_{j} \left(1 + d_{j} \right) - x_{j} \left(1 + d_{j} \right) \tag{15}$$

s.t. eq. (14) and eq. (11). The solution of the above problem gives us the usual price mark up:

$$P_{j} = \left(1 / \alpha \right) > 1 \text{ for } \alpha \in \left(0,1 \right) \tag{16}$$

2.3 R&D Sector

The firms which produce the patents for the production of the intermediate inputs work in a perfect competition environment. The sunk cost for innovation is $\eta > 0$ units of final output and we assume that the invention of a new intermediate togeher with the different levels of durability that can be incoroporated into it are invented with certainty. The patents are sold to the intermediate firms which are necessary for the production of the intermediate goods with their required level of durability. The price of selling the patent of the intermediate good V_N is the present value of the perpetual profits of the intermediate firms:

$$V_N = \int_t^{\infty} \pi_\tau^I e^{-\int_t^\tau r(s)ds} d\tau, \qquad \tau > t \tag{17}$$

By replacing the price value of selling an intermediate input from eq. (16) into the total demand of an intermediate input j we have:

$$x_j = L\alpha^{\frac{2}{1-\alpha}} \left(\frac{d^{*\phi} u^{*\alpha}}{\left(1 + d^* + \left(1 - d^*\right)^\beta u^{*\lambda}\right)} \right)^{\frac{1}{1-\alpha}} \tag{18}$$

By replacing into eq. (17) the profit function of the intermediate producers from eq. (15), together with eq. (18) and eq. (16) we have:

$$V_N = L\alpha^{\frac{2}{1-\alpha}} \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{d^{*\phi} u^{*\alpha}}{\left(1 + d^* + \left(1 - d^*\right)^\beta u^{*\lambda}\right)} \right)^{\frac{1}{1-\alpha}} \int_t^{\infty} e^{-r(\tau - t)} d\tau \tag{19}$$

By solving for the integral in eq. (19) under the assumption of a constant interest rate over time, we get in equilibrium the price of the patent to be equal with:

$$V_N = \frac{L\alpha^{\frac{2}{1-\alpha}}}{r} \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{d^{*\phi} u^{*\alpha}}{\left(1 + d^* + \left(1 - d^*\right)^\beta u^{*\lambda}\right)} \right)^{\frac{1}{1-\alpha}} \tag{20}$$

From eq. (20) we can observe that the price of a patent is constant as well in equilibrium since L, r, d^* and u^* are constant as well. Moreover, the higher the price of patents V_N the higher will be the motivation of performing R&D. In equilibrium, due to the perfect competition conditions that exist in the R&D sector, the profits of an R&D firm should be zero which implies that the price of the patent V_N is equal to the sunk cost of invention η which result into the following equilibrium value for the interest rate:

$$r = \frac{L\alpha^{\frac{2}{1-\alpha}}}{\eta}\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{d^{*\phi}u^{*\alpha}}{\left(1+d^{*}+\left(1-d^{*}\right)^{\beta}u^{*\lambda}\right)}\right)^{\frac{1}{1-\alpha}} \tag{21}$$

In the above eq. (21), there is the equilibrium value of the real return rate of the households' asset holdings which is used for transforming the variables into present value.

2.4 Households

It is assumed that the households live forever, are homogeneous and face the following instantaneous utility function and no population growth:

$$U\left(C_t\right) = \frac{C_t^{1-\theta}-1}{1-\theta}, \qquad \theta > 1 \tag{22}$$

where θ is the inverse of the intertemporal elasticity of substitution of consumption.[10] The asset holdings of the households (i.e. owners of intermediate firms) equal the aggregate value of intermediate firms:

$$A_t = N_t V_N \tag{23}$$

Moreover, the equation of asset accumulation for households is defined as:

$$\dot{A}_t = r_t A_t + wL - C_t \tag{24}$$

The time preference of the households with which they discount their utility function is the discount factor $\rho > 0$. The solution for the above problem of the households is as usual:

$$\frac{\dot{C}}{C} = \frac{r-\rho}{\theta} \tag{25}$$

Equation (25) shows the growth rate of consumption over time in the balanced growth path equilibrium (an equilibrium where all the variables grow at constant growth rates).

[10] Blundell et al. (1994) and Attanasio and Browning (1995) find θ to be close to 1 at country level while Evans (2004) and Percoco (2008) after allowing for market demand preferences show that θ is close to 1.5.

3. Decentralized Competitive Equilibrium and Comparative Statics

Proposition 1: The growth rate of: consumption, the number of patents and output is the same.

Proof: The proof is on the Appendix.

Proposition 2: The common growth rate of the economy in the Balanced Growth Path Equilibrium equals to:

$$g^* = \frac{\dot{N}}{N} = \frac{\dot{Y}}{Y} = \frac{\dot{C}}{C} = \frac{1}{\theta}\left[\frac{L\alpha^{\frac{2}{1-\alpha}}}{\eta}\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{d^{*\phi}u^{*\alpha}}{\left(1+d^* + \left(1-d^*\right)^{\beta} u^{*\lambda}\right)}\right)^{\frac{1}{1-\alpha}} - \rho\right] \quad (26)$$

By also substituting in eq. (26) the endogenous value for u^* from eq. (13) and the endogenous value for d^* from eq. (11).

Proof: By substituting r from eq. (21) into eq. (25) and using the result of Proposition 1.

Proposition 3: As long as $r > \rho$ then $g^* > 0$ which together with $\theta > 1$ satisfies also the TVC condition.

Proof: By assuming $r > \rho$ then $g^* > 0$ and together with $\theta > 1$ satisfies the standard condition for satisfying TVC $g^* < r$.

Proposition 4: First of all, for having $d^* \in \left(0,1\right)$ the following two conditions should hold: $\lambda\left(\alpha-\phi\right)-\alpha\left(1+\beta\right)>0$ and $\alpha\beta+\phi\lambda<\lambda\left(\alpha-\phi\right)-\alpha\left(1+\beta\right)$. Secondly, for having $u^*>0$ the condition $\lambda\left(\alpha-2\phi\right)-\alpha\left(1+2\beta\right)>0$ is necessary and for $u^*<1$ it is necessary the calibration of eq. (13).

Proof: The proof is on the Appendix.

Proposition 5: In order $\delta_x \in (0,1)$, $u \in (0,1)$, $d \in (0,1)$ and $g^* > 0$ together with the assumptions of the model regarding the parameter values $\alpha \in (0,1)$, $\beta > 1$, $\lambda > 1$ and $\phi < 1$, we impose the following parameter values: $\alpha = 0.7$, $\beta = 1.1$, $\lambda = 10$ and $\phi = 0.09$.

Proof: The proof is on the Appendix.

Proposition 6: In comparison to the standard expanding variety models the new term that appears in the equilibrium equation of the growth rate is:

$\left(\frac{d^{*\phi} u^{*\alpha}}{\left(1 + d^* + (1-d^*)^\beta u^{*\lambda}\right)}\right)^{\frac{1}{1-\alpha}}$. The growth rate now in comparison to the standard expanding variety models is lower.

Proof: By using the parameter values from Proposition 5 into

$\left(\frac{d^{*\phi} u^{*\alpha}}{\left(1 + d^* + (1-d^*)^\beta u^{*\lambda}\right)}\right)^{\frac{1}{1-\alpha}}$ and using eqs. (11) and (13) the main result of the proposition can be proven.

Intuitively, if it is assumed $d^* = 1$ and $u^* = 1$, the new term in the equation of the common growth rate of the economy is less than one independently of the choice of the parameter values. This is because the higher durability level will reduce the demand for the quantity for each intermediate input which in turn will reduce the value of the patents and therefore the motivation of producing new intermediate inputs. Therefore, there exists an asymmetrical effect of durability and utilization on economic growth.

Proposition 7: The effect of the durability on the common growth rate of the economy g^* is negative and on the contrary the effect of the utilization rate on the common growth rate of the economy g^* is positive.

Proof: The above results hold for the parameter values used in Proposition 5.

Intuitively, as it has been mentioned above the high durability reduces the motivation of research but if the utilization rate is higher than the durability then there is higher depreciation for the intermediate inputs which results into higher demand for intermediate inputs and therefore the value of the patents is high and this is a motivation of research and therefore growth. Propositions 6 and 7 formally indicate an asymmetric effect of durability and utilization on economic growth.

Proposition 8: The following results hold regarding the effect of the main parameters on the equilibrium level of durability:

$$\frac{\partial d^*}{\partial \alpha} < 0, \frac{\partial d^*}{\partial \beta} > 0, \frac{\partial d^*}{\partial \phi} > 0 \text{ and } \frac{\partial d^*}{\partial \lambda} < 0.$$

Proof: By differentiating eq. (11) with respect to the different parameters.

The interpretation of the above results is the following: a high α means that the production function of the final good is affected a lot by the quantity of the intermediate input and therefore the durability level should be less since the quantity of the intermediate input is more important for production in comparison to its durability level. A high β means that the depreciation rate will be reduced a lot in increases of durability, and therefore durability in equilibrium should be high. For a given level of ϕ, durability should be higher since $d \in (0,1)$. Finally, since high λ implies that the increase of the utilization rate leads to a higher increase of the depreciation rate, this is a requirement for having higher durability.

Proposition 9: The following results hold regarding the effect of the main parameters on the equilibrium level of utilization:

$$\frac{\partial u^*}{\partial \alpha} > 0, \frac{\partial u^*}{\partial \beta} > 0, \frac{\partial u^*}{\partial \phi} > 0 \text{ and } \frac{\partial u^*}{\partial \lambda} < 0.$$

Proof: By differentiating eq. (13) with respect to the different parameters.

A high α means that the production function of the final good is affected a lot by the quantity of the intermediate input and therefore the utilization rate of the intermediate inputs should be high. A high β means that the depreciation rate will be reduced a lot in increases of durability, and therefore there is space also for increases in the utilization rate. . Similar is the explanation for the effect of ϕ. Since high ϕ results in higher durability which in turn reduces the depreciation rate, this gives space for higher utilization as well. Finally, since high λ implies that the increase of the utilization rate leads to a high increase of the depreciation rate, the utilization rate should be chosen at a lower rate by firms.

Proposition 10: The following results hold regarding the effect of the main parameters on the equilibrium level of economic growth g^*:

$$\frac{\partial g^*}{\partial \beta} < 0, \frac{\partial g^*}{\partial \phi} > 0, \frac{\partial g^*}{\partial \lambda} < 0 \text{ and } \frac{\partial g^*}{\partial \alpha} < 0.$$

Proof: By substituting in eq. (26) the endogenous value for u^* from eq. (13) and the endogenous value for d^* from eq. (11) and differentiating with respect to the different parameters.

A high β increases durability more than the increase of the utilization rate which leads to a lower demand for intermediate inputs. This in turn, reduces the value of the R&D firms and therefore the innovation is lower even if the durability per intermediate input is higher. A high ϕ results in both higher durability and higher utilization which increases the growth rate of the economy, which in turn reduces the depreciation rate, giving space for higher utilization as well. Similarly, a high λ leads to a reduction both in durability and utilization which shrinks the economy. Finally, a high α even if it reduces durability and increases utilization which are both positive forces for economic growth, at the same time it reduces directly the mark up of the monopolistic profits of the intermediate firms which in turn reduces the demand for R&D products by intermediate firms and therefore resulting in lower economic growth.

4. Concluding Remarks

The current paper seeks to shed light on how the existence of endogenous depreciation rate through durability and utilization affects economic growth and technological progress in an expanding variety of inputs R&D growth model. The literature has endogenized separately the depreciation rate on durability and utilization. In the current paper the depreciation rate is determined by durability and utilization simultaneously and we explore how that affects the growth rate and technological progress of the economy.

The main result of the paper is that higher durability level for the intermediate inputs reduces the demand for them which reduces the profits and motivation for innovation firms. Therefore, the quantity of the intermediate inputs is lower (which is the main determinant of technological progress in this type of R&D model). On the other hand, the utilization rate increases the need for hiring more intermediate inputs which mitigate the negative effects of durability. Therefore, the main message of the paper is that even if durability is a signal of technological progress, at the same time it shrinks the profitability of the R&D sector and therefore the incentives for innovation and for further technological progress. Moreover, since the utilization rate is affected even more by the business cycles of the economy in comparison to its negative impact on depreciation rate, it is wise for policy makers to subsidize the R&D firms in order for them to weather the lower demand for intermediate inputs due to negative durability effects. The coexistence of durability and utilization has an asymmetric impact on economic growth and in the end their simultaneous presence reduces economic growth. The reason is that the negative consequences of durability on the R&D's profitability are higher than the positive impact of utilization, since the firms do not take into account this asymmetric impact.

For future extensions in the current paper it would be important to explore empirically the validity of the main findings of the paper vis-à-vis the effects of durability and utilization on technological progress. The data for utilization at country level can be constructed by comparing the maximum output, which a country can produce at a specific time period with its available resources, with actual output. Furthermore, it is also possible to create an index for durability by using data from OECD for infrastructure maintenance.

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APPENDIX

Proposition 1: The growth rate of: consumption, the number of patents and output is the same.

Proof: From eq. (18) the growth rate of the quantity of any specific intermediate input x_j is zero since there is no population growth. Moreover, by transforming the production function in eq. (1) into aggregate level we have $Y_t = d^{\phi} L^{1-\alpha} N_t \left(x_{ij} u_{ij}\right)^{\alpha}$ which implies that the growth rate of the output is the same as the growth rate of the number of the different intermediate inputs: $\frac{\dot{Y}}{Y} = \frac{\dot{N}}{N}$. Finally, the total output in the economy is used for consumption C, invention of new intermediates $\eta \dot{N}$ and for the production of intermediates with the demanded level of durability $Nx\left(1+d\right)$: $Y_t = C_t + \eta \dot{N}_t + N_t x\left(1+d\right)$. The previous can be written as: $\frac{\dot{N}_t}{N_t} = \frac{1}{\eta}\left[\frac{Y_t}{N_t} - \frac{C_t}{N_t} - x\left(1+d\right)\right]$. From this equation in order the growth rate of the number of patents to be constant in the Balanced Growth Path Equilibrium it is necessary to hold: $\frac{\dot{N}}{N} = \frac{\dot{Y}}{Y} = \frac{\dot{C}}{C}$.

Proposition 4: First of all, for having $d^* \in \left(0,1\right)$ the following two conditions should hold: $\lambda\left(\alpha-\phi\right) - \alpha\left(1+\beta\right) > 0$ and $\alpha\beta + \phi\lambda < \lambda\left(\alpha-\phi\right) - \alpha\left(1+\beta\right)$. Secondly, for having $u^* > 0$ the condition $\lambda\left(\alpha-2\phi\right) - \alpha\left(1+2\beta\right) > 0$ is necessary and for $u^* < 1$ it is necessary the calibration of eq. (13).

Proof: For having $d^* \in \left(0,1\right)$ it is straightforward to set the necessary conditions in eq. (11). For having $u^* > 0$ if $\lambda\left(\alpha-\phi\right) - \alpha\left(1+\beta\right) > 0$ holds for $d^* > 0$, it is necessary to impose need to impose $\lambda(\alpha - 2\phi) - \alpha(1 + 2\beta) > 0$. For $u^* < 1$ it is necessary to set parameter values otherwise the inequality is not straightforward. The conditions $\lambda(\alpha - \phi) - \alpha(1 + \beta) > 0$ and $\alpha\beta + \phi\lambda < \lambda(\alpha - \phi) - \alpha(1 + \beta)$ can hold if λ and α are high and if β and ϕ are low.

Proposition 5: In order $\delta_x \in (0,1)$, $u \in (0,1)$, $d \in (0,1)$ and $g^* > 0$ together with the assumptions of the model regarding the parameter values $\alpha \in (0,1)$, $\beta > 1$, $\lambda > 1$ and $\phi < 1$, we impose the following parameter values: $\alpha = 0.7$, $\beta = 1.1$, $\lambda = 10$ and $\phi = 0.09$.

Proof: In the literature the range of capital's depreciation varies between 0.05 to 0.14. The utilization rate is between 0.7 to 0.9.[11] The parameter α takes this value as it is proposed by Mankiw, Romer and Weil (1992). [12] Therefore, $\beta = 1.1$ and $\lambda = 10$ satisfy $\delta_x \in (0.05, 0.14)$.[13] Finally, if the parameter value ϕ approaches zero then the durability has the highest positive impact on the production of goods since by assumption durability takes values between zero and one as in Gylfason and Zoega (2007).

[11] Cooley et al. (1995) provide estimates for the capital utilization of USA which is around 0.82. Also Epstein and Denny (1980), Nadiri and Prucha (1996) and Fraumeni (1997) find a range of depreciation rate between 0.12 to 0.14.

[12] According to Mankiw, Romer and Weil (1992) the parameter α in the absence of human incorporates its impact as well in a broad sense of capital and it should take a value around 0.7.

[13] If I had set a scaling parameter into eq. (2) for depreciation rate in the form $\delta_{x_{ij}} = A(1-d)^{\beta} u^{\lambda}$, then λ can take a much lower value.