Tricks with the BoD model
and an application to the e-Government Development Index

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ABSTRACT: In this paper we examine the implications of the ideal and the anti-ideal decision-making units on the Benefit-of-the-Doubt (BoD) model. In particular, we verify that, in the presence of an ideal (anti-ideal) decision-making unit, the efficiency scores of the BoD (inverted BoD) model can be obtained without solving the relevant linear programming problems. In this case, it suffices to choose the largest (smallest) of the element-wise divided component indicators of the evaluated decision-making unit by those of the ideal (anti-ideal) decision-making unit. We then illustrate one practical use of this simple enumeration algorithm by providing empirical results for a BoD-based e-Government Development Index within a traffic-light reporting system.

KEYWORDS: BoD model, Ideal and anti-ideal DMU, e-Government Development Index, traffic-light reporting system

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1. Introduction

The Benefit-of-the-Doubt (BoD) model is one of four statistical models recommended by OECD (2008) for constructing composite indicators. It has been used in a large number of applications including but not limiting to the Human Development Index (Despotis, 2005), the OECD Better Life Index (Mizobuchi, 2014), the Quality of Life Indicator (Morais and Camanho, 2011), the Digital Access Indicator (Gaaloul and Khalfallah, 2014), the Technology Achievement Index (Cherchye *et al.,* 2008) and the Environmental Performance Index (e.g., Zanella *et al.,* 2013). Besides these, it has been used in several multi-criteria decision making problems such as the supplier selection, the inventory classification, the quality perception assessment, preference voting for product/project ranking as well as for measuring effectiveness and spatial efficiency, for target setting, and for dealing with min-max strategy games (see Karagiannis (2020) and the references therein).

The BoD is essentially a Data Envelopment Analysis (DEA) model --in particular, an input oriented constant-returns-to-scale (CRS) DEA model with a single constant input (see Färe and Karagiannis (2014)-- and as such it can be estimated by linear programming. However, in two previous studies, it has been noticed (without proof) that under certain circumstances such an estimation may not be necessary. Specifically, Korhonen and Luptacik (2004), in an attempt to obtain a composite eco-efficiency indicator by means of the BoD model and using technical and ecological efficiency scores as component indicators, claimed that “the indicator of the eco-efficiency is simply the better value of efficiency scores obtained from the Models I [technical efficiency] and II [ecological efficiency]” (p. 443).[1] They did not however explicitly refer to the circumstances under which such a simple enumeration algorithm can be applied.[2] Charles and Diaz (2017), on the other hand, were more explicit on that and they argued that "if there is an *outlier* DMU, that is, a DMU that performs better than the others in every dimension … the resulting index would focus *only* on the dimension in which a particular DMU is the closest to the top performer DMU, and disregard completely its performance in all the other dimensions" (p. 757).[3] The outlier DMU that Charles and Diaz (2017) referred to is also known in the DEA literature as the *virtual superefficient DMU* (Bazargan and Vasigh, 2003), the *ideal DMU* (IDMU) (Wang and Luo, 2006), the *perfect object* (Vaninsky, 2011) or the *positive ideal item* (Chen, 2012), and contrast the anti-ideal DMU (ADMU), which is defined as the DMU that perform worse than the other DMUs in every dimension.[4] The IDMU has been used in DEA for several purposes, the most relevant of which to our purposes is that of Vaninsky (2011), who proposed quite simple enumeration algorithms for obtaining conventional DEA efficiency scores when an IDMU is or it is purposely included among the evaluated DMUs.[5]

The first objective of this paper is to show how Vaninsky's (2011) result for the conventional DEA model can be adapted to the BoD model in order to provide a formal proof for that it is not necessary to solve the relevant linear program to estimate the BoD efficiency scores when an IDMU is among the evaluated DMUs. In this case, it suffices to choose the largest of the element-wise divided component indicators of the evaluated DMU by those of the IDMU. Such a simple enumeration algorithm also exists for the BoD model with common (across DMUs) weights and in this case, the efficiency scores will be given by the un-weighted arithmetic average of the element-wise divided component indicators of the evaluated DMU by those of the IDMU. We also develop an analogous enumeration algorithm for the inverted BoD model, which is an output-oriented CRS DEA model with a single constant output and the component indicators treated as inputs (Färe and Karagiannis, 2014), in relation to the ADMU. In this case, it is suffice to choose the smallest of the element-wise divided component indicators of the evaluated DMU by those of the ADMU to obtain the inverted BoD efficiency scores.

The second objective of this paper is to use these results for re-estimating the United Nations (UN) e-Government Development Index (e-GDI) within a traffic-light reporting system, according to which all evaluated DMUs are classified into three groups based on their performance relative to a predetermined target, namely, (a) the *green group*, the DMUs of which outperform the chosen target in terms of all component indicators, (b) the *yellow group*, the DMUs of which meet the chosen target in term of some, but not all, of the component indicators, and (c) the *red group*, the DMUs of which perform below the chosen target in terms of all the component indicators. The chosen target implicitly introduces the notion of the IDMU to the red group and that of the ADMU to the green group. This in turn implies that the e-GDI for the countries in the green and the red groups can be computed by using the simple enumeration algorithms proposed in this paper. For the countries in the yellow group, we follow Ahn and Neumann (2014) suggestion and use the absolute value of the sum of their negative deviations from the target.

The rest of the paper is organized as follows: In the next section we present the main results regarding the implications of the IDMU and the ADMU on the BoD and the inverted BoD model, respectively. In the third section, we consider the e-GDI as a case study and we compare and contrast three variants of it based on equal weights (EW), the BoD model, and the traffic-lights-reporting-system BoD model. Concluding remarks follow in the last section.

2. The BoD (inverted BoD) model with an IDMU (ADMU)

The BoD is a special case of Charnes *et al.* (1978) input-oriented CRS DEA model when there is a single constant input that takes the value of one for all evaluated DMUs. To verify this notice that the multiplier and the envelopment forms of the input-oriented CRS DEA model are given as:

$$
\begin{array}{llll}
\max\limits_{u_j^k, v_i^k} & \sum\limits_{j=1}^{J} u_j^k y_j^k & & \\
s.t. & \sum\limits_{n=1}^{N} v_n^k x_n^h - \sum\limits_{j=1}^{J} u_j^k y_j^h \geq 0 & h = 1, \ldots, K \\
& \sum\limits_{i=1}^{I} v_n^k x_n^k = 1 & \\
& u_j^k \geq 0 & j = 1, \ldots, J \\
& v_n^k \geq 0 & n = 1, \ldots, N
\end{array}
\qquad
\begin{array}{lll}
\min\limits_{\theta^k, \lambda_h^k} & \theta^k & \\
s.t. & \sum\limits_{h=1}^{K} \lambda_h^k x_n^h \leq \theta^k x_n^k & n = 1, \ldots, N \\
& \sum\limits_{h=1}^{K} \lambda_h^k y_j^h \geq y_j^k & j = 1, \ldots, J \\
& \lambda_h^k \geq 0 & h = 1, \ldots, K \\
& \theta^k \ free &
\end{array}
\qquad (1)
$$

where x_n^k is the quantity of the nth input used by the kth DMU, y_j^k is the quantity of the jth output produced by the kth DMU, v_n^k is the relative weight (multiplier) of the nth input for the kth DMU, u_j^k is the relative weight (multiplier) of the jth output for the kth DMU, λ_h^k is the intensity variable of the kth DMU with respect to the hth DMU, and θ^k

is the efficeicny socre of the kth DMU. In the case of the BoD model, there is only one input, i.e. n=1, and in addition, $x^k = 1$ for all DMUs. This implies that $v^k = 1$ for all DMUs and $\sum_{n=1}^{N} v_n^k x_n^k = v^k = 1$ for each DMU. On the other hand, the first inequality constraint in the envelopment form of (1) may be written $\sum_{h=1}^{K} \lambda_h^k \leq \theta^k$ since $x^k = 1$. By substituting these into (1) yields the multiplier and envelopment forms of the BoD model:

$$
\begin{array}{llll}
\max\limits_{u_j^k} & \sum\limits_{k=1}^{K} u_j^k y_j^k & \qquad \min\limits_{\mu_h^k} & \sum\limits_{h=1}^{K} \lambda_h^k \\
s.t. & \sum\limits_{j=1}^{J} u_j^k y_j^h \leq 1 \quad h = 1, \dots, K & \qquad s.t. & \sum\limits_{h=1}^{K} \lambda_h^k y_j^h \geq y_j^k \quad j = 1, \dots, J \\
 & u_j^k \geq 0 \qquad j = 1, \dots, J & & \lambda_h^k \geq 0 \qquad h = 1, \dots, K
\end{array} \tag{2}
$$

where the component indicators are treated as outputs. Furthermore, given Lovell and Pastor (1999) propositions 2 and 3 and Caporaletti *et al*. (1999) Appendix A, one can verify that the BoD model is reciprocal to (a) the output-oriented CRS DEA model with a single constant input, (b) the output-oriented variable-returns-to-scale (VRS) DEA model with a single constant input, and (c) the output-oriented VRS DEA model without inputs (see also Karagiannis, 2020).

If an IDMU, formally defined as a DMU with inputs $x_n^I = \min_k [x_n^k]$ for *n=1,…,N* and outputs $y_j^I = \max_k [y_j^k]$ for *j=1,…,J*, is among the evaluated DMUs or if it is purposely included in the sample, then it is by default the only efficient DMU. This implies that the optimal solution in the envelopment form of (1) includes only one non-zero intensity variable, namely that of the IDMU, $\lambda_I^k \neq 0$, where I refers to the IDMU. Based on this, we can verify that:

PROPOSITION 1 *(Vaninsky, 2011):* In the case of the input-oriented CRS DEA model with an IDMU, $\lambda_I^k = \min_n \left[\frac{x_n^k}{x_n^I}\right]$ and $\theta_*^k = \min_n \left[\frac{x_n^k}{x_n^I}\right] \times \max_j \left[\frac{y_j^k}{y_j^I}\right]$, where "*" denotes its optimal value.

To adapt the above proposition to the BoD model, notice that $\min_n [x_n^k / x_n^I] = 1$ as $x^k = x^I = 1$. Then, the efficiency score of the kth DMU or equivalently its value of the composite indicator is given as:

$$\theta_*^k = \left(\max_j \left[\frac{y_j^k}{y_j^I}\right]\right) \tag{3}$$

Thus, we have the following:

COROLLARY 1: In a BoD model with an IDMU, the efficiency score or equivalently the value of the composite indicator of the k[th] DMU is given by the maximum of the ratios of its component indicators to those of the IDMU.

That is, divide element-wise the component indicators of each DMU by those of the IDMU and from the resultant vector choose the element with the largest numerical value. This provides a simple enumeration algorithm that can be used to obtain θ^k without need of solving the linear programming problem in (2). If at the outset we have normalized the component indicators to lie within the [0,1] range, which implies that for the IDMU $\hat{y}_j^I = 1$ for $j = 1, \ldots, J$, then (3) becomes:

$$\theta_*^k = \left(\max_j [\hat{y}_j^k]\right) \tag{4}$$

This is equivalent to assigning a multiplier value of one to the normalized indicator with the largest value and completely disregard the performance of the evaluated DMU in all the other dimensions.

The above result can be extended to a special variant of the BoD model with common multipliers considered by Toloo and Tavana (2017). They have shown that in this case the values of the composite indicator are obtained without solving an optimization problem as they are equal to the ratio of the sum of the component (normalized) indicators to the maximum of these sums across DMUs. Note that if $\hat{y}_j^I = 1$, then $\sum_{j=1}^J \hat{y}_j^I = J$. Therefore, one can verify that if an IDMU is among the evaluated DMUs then the efficiency score of the k[th] DMU or equivalently, its value of the composite indicator is equal to:

$$\theta^k = \left(\frac{\sum_{j=1}^J \hat{y}_j^k}{\sum_{j=1}^J \hat{y}_j^I}\right) = \left(\frac{\sum_{j=1}^J \hat{y}_j^k}{J}\right) \tag{5}$$

with $u_j^k = 1/J$. Thus, we have the following:

COROLLARY 2: In a common weights BoD model with an IDMU, the efficiency score or equivalently the value of the composite indicator of the k[th] DMU is given by the simple (unweighted) arithmetic average of the normalized component indicators.

Next we consider the implications of the ADMU, defined formally as a DMU with inputs $x_n^A = \max_k [x_n^k]$ for *n=1,…,N* and outputs $y_j^A = \min_k [y_j^k]$ for *j=1,…,J*, on the inverted BoD model. The inverted BoD is a special case of the inverted input-oriented CRS DEA model when there is a single constant input that takes the value of one for all evaluated DMUs. Following Yamada *et al*. (1994) and Etani *et al*. (2002), the inverted of a conventional DEA model is obtained by changing the optimization indication (i.e. from max to min or *vice versa*) and the sign of the inequality constraints. By doing so in (1) we can obtain the multiplier and the envelopment forms of the inverted input-oriented CRS DEA model as follows:[6]

$$
\begin{aligned}
&\min_{\omega_j^k,\xi_i^k} \sum_{j=1}^{J} \omega_j^k y_j^k \\
&s.t. -\sum_{n=1}^{N} \xi_n^k x_n^h + \sum_{j=1}^{J} \omega_j^k y_j^h \geq 0 \quad h = 1,\dots,K \\
&\quad \sum_{n=1}^{N} \xi_n^k x_n^k = 1 \\
&\quad \omega_j^k \geq 0 \qquad j = 1,\dots,J \\
&\quad \xi_n^k \geq 0 \qquad n = 1,\dots,N
\end{aligned}
\qquad
\begin{aligned}
&\max_{\theta^k,\mu_h^k} \ \pi^k \\
&s.t. \ \sum_{h=1}^{K} \mu_h^k x_n^h \geq \pi^k x_n^k \quad n = 1,\dots,N \\
&\quad \sum_{h=1}^{K} \mu_h^k y_j^h \leq y_j^k \qquad j = 1,\dots,J \\
&\quad \mu_h^k \geq 0 \qquad h = 1,\dots,K \\
&\quad \pi^k \ free
\end{aligned}
\tag{6}
$$

where ξ_n^k is the relative weight (multiplier) of the n[th] input for the k[th] DMU, ω_j^k is the relative weight (multiplier) of the j[th] output for the k[th] DMU, μ_h^k is the intensity variable of the k[th] DMU with respect to the h[th] DMU, and π^k is the efficeicny socre of the k[th] DMU.

Assuming that there is only one input with $x^k = 1$ for *k=1,…,K*, $\xi^k = 1$ for all DMUs and $\sum_{n=1}^{N} \xi_n^k x_n^h = \xi^k = 1$ for all DMUs. On the other hand, the first inequality constraint in the envelopment form of (6) may be written as $\sum_{h=1}^{K} \mu_h^k \geq \pi^k$ since $x^k = 1$. By substituting these into (6) yields the multiplier and envelopment forms of the inverted BoD model:

$$
\begin{array}{llll}
\min\limits_{\omega_j^k} & \sum\limits_{k=1}^{K} \omega_j^k y_j^k & & \\
s.t. & \sum\limits_{j=1}^{J} \omega_j^k y_j^h \geq 1 & h = 1, \dots, K \\
 & \omega_j^k \geq 0 & j = 1, \dots, J
\end{array}
\qquad
\begin{array}{llll}
\max\limits_{\mu_h^k} & \sum\limits_{h=1}^{K} \mu_h^k & & \\
s.t. & \sum\limits_{h=1}^{K} \mu_h^k y_j^h \leq y_j^k & j = 1, \dots, J \\
 & \mu_h^k \geq 0 & h = 1, \dots, K
\end{array}
\tag{7}
$$

Thus, the inverted BoD model may alternatively be seen as an inverted input-oriented CRS DEA model with a single constant input or as an output-oriented CRS DEA model with a single constant output. From the latter and by using Lovell and Pastor (1999) propositions 2 and 3 and Caporaletti *et al*. (1999) Appendix A, one can verify that the inverted BoD is reciprocal to (a) the input-oriented CRS DEA model with a single constant input, (b) the input-oriented VRS DEA model with a single constant output, and (c) the input-oriented VRS DEA model without outputs (see also Karagiannis, 2020).

In order to examine the implications of the ADMU in (7), we proceed in two steps. First, we develop a result analogous to proposition 1 above for the inverted input-oriented CRS DEA model in (6) and then, we adapt this result to the case of the inverted BoD model. If an ADMU is among the evaluated DMUs or it is purposely included in the sample, then it is by default the only one with an "inefficiency" score of 1. This implies that there is only one non-zero intensity variable in the optimal solution of the envelopment form in (6), namely that of the ADMU, $\lambda_A^k \neq 0$, where A refers to the ADMU. Then, following a reasoning similar to proposition 1, one can verify that:[7]

PROPOSITION 2: In the case of an inverted input-oriented CRS DEA model with an ADMU, $\mu_A^k = \min_j \left[\frac{y_j^k}{y_j^A}\right]$ and $\pi_*^k = \min_j \left[\frac{y_j^k}{y_j^A}\right] \max_n \left[\frac{x_n^k}{x_n^A}\right]$.

To adapt this proposition to the inverted BoD model, treat it as an input-oriented CRS DEA model with a single constant input and notice that $\max_n [x_n^k / x_n^A] = 1$ as $x^k = x^A = 1$. Then, the efficiency score of the k[th] DMU or equivalently its value of the composite indicator is given as:

$$
\pi_*^k = \left(\min_j \left[\frac{y_j^k}{y_j^A}\right] \right) \tag{8}
$$

Thus, we have the following:

COROLLARY 3: In an inverted BoD model with an ADMU, the "inefficiency" score or equivalently the value of the composite indicator of the k^{th} DMU is given by the minimum of the ratios of its component indicators to those of the ADMU.

That is, divide element-wise the component indicators of each DMU by those of the ADMU and from the resulting vector choose the element with the smallest numerical value.[8] This provides a simple enumeration algorithm that can be used to obtain π^k without the need to solve the linear programming problems in (7).

3. A study case: the e-GDI

The aim of the e-GDI, which is published biannually by UN since 2001, is to measure the readiness and the capacity of national institutions in using Information and Communication Technologies (ICT) for the delivery of public services (UN, 2018), or alternatively, to assess the preparedness of nations in making the transformation to electronic governance (Ayanso *et al.,* 2011). The e-GDI is a regularly cited indicator in popular press as well as in official governmental websites indicating its leading role in measuring the relative diffusion of e-government services within a country and identifying patterns in e-government development and performance.

The UN's e-GDI is based on three component indicators reflecting different aspects of e-government. The first is the *Online Service Index* (OSI), which is obtained by using an Online Service Questionnaire survey. The total number of positive answers to a number of binary questions is normalized by using the min-max normalization, i.e., $(y_j^k - \min_k y_j^k)/(\max_k y_j^k - \min_k y_j^k)$, to lie within the [0,1] range, based on within sample minimum and maximum values. The second component indicator is the *Telecommunication Infrastructure Index* (TII), which measures the status of development of telecommunication infrastructure within each country. The TII is based on five sub-indicators, namely (a) the number of estimated Internet users, (b) the number of main fixed telephone lines, (c) the number of mobile subscribers, (d) the number of wireless broadband subscriptions, and (e) the number of fixed broadband subscriptions, all expressed per 100 inhabitants. These five sub-indicators are converted into z-scores and then their simple arithmetic average is normalized using the min-max normalization to lie within the [0,1] range. The third component indicator measures the capabilities of the countries citizens to effectively handle ICT via the *Human Capital Index* (HCI). The HCI is the weighted average of the z-scores of four sub-indicators, namely (a) adult literacy rate, (b) gross enrolment ratio, (c) expected years of schooling and (d) mean years of schooling, with the literacy rate receiving a weight of 1/3 and the other three sub-indicators a weight of 2/9. The resulting weighted average is then normalized to lie within the [0,1] range using the min-max normalization.

3.1. Comparison of the EW and BoD-based EGDI

The UN's e-GDI is obtained by taking the simple arithmetic average of the normalized values of the OSI, the TII and the HCI and is thus a composite indicator with equal weights. The choice of this weighting scheme has been criticized in the literature. For example, Whitmore (2012) argued that it lacks an empirical and/or a theoretical basis and for this reason, he used Factor Analysis to select a subset of the (relevant) sub-indicators proposed by UN and to obtain the corresponding aggregation weights. Ayanso *et al.* (2011), on the other hand, relied on Principal Components Analysis to obtain the aggregation weights for the TII and the HCI as well as for the e-GDI.

As an alternative to Whitmore (2012) and Ayanso *et al*. (2011), we first re-estimate the e-GDI using the BoD model. By means of the BoD model, each country is given the benefit-of-the-doubt to select those aggregation weights which would make it appear in the best possible light. Thus, the weights obtained by solving model (2) are allowed to vary across countries and component indicators, and this gives rise to the most optimistic estimates of the e-GDI. Our empirical results are based on the normalized values of the OSI, the TII and the HCI, taken from the UN (2016) and reproduced in the Appendix Table A1. The average of the estimated weights for the OSI, the TII and the HCI are respectively 0.177, 0.078 and 0.759, far different from the equal weights used by the UN. Almost 12% of the evaluated countries assigned a weight equal to one to OSI, a much higher percentage (around 69%) assigned a weight equal to one to HCI and only one country (Monaco) assigned a weight equal to one to TII. The rest of the evaluated countries (around 19%) assigned non-zero weights to combinations of two or three of the component indicators.

The results regarding the EW- and the BoD-based e-GDI are given in Table 1 and in Figure 1. As it was expected, the BoD model provided on average more optimistic estimates than the EW scheme: 0.666 vs. 0.492. In addition, the EW scheme resulted in a much lower minimum value as well as a slightly higher standard deviation (see Table 1). On the other hand, there is almost no difference between the average and the median values in the EW estimates while there is a second decimal point difference between the average and median value in the BoD based estimates. In addition, there are more countries in the [0.9, 1] range for the BoD rather than the EW estimates (25 vs. 2). These imply that the distribution of the BoD estimates of the e-GDI is skewed to the right, as it can be seen from Figure 1, while that of the EW has twin peaks around the values of 0.3 and 0.5 but it is still more evenly distributed. Another difference between the BoD-based and the EW e-GDI is that the former resulted in a number of countries, namely UK, Australia, Korea, Singapore and Monaco, attaining the highest possible value of the e-GDI. The first four of these countries are also ranked at the top of the EW e-GDI, while Monaco is ranked in the 31st position. Recall that Monaco is the only country that attain a BoD score of one by relying solely on its TII component indicator.

We have also found significant differences in countries' rankings between the EW and the BoD-based e-GDI (see Table A1). In particular, 189 out of the 193 countries changed rank position when the aggregation rule shifts from the EW to the BoD despite the fact that the rank correlation between the two is as high as 0.938. From these, 88 countries exhibited a rank change from 1 to 10 positions, while 19 countries exhibited changes of over 30 positions. The average rank shift, calculated as $R = (\frac{1}{K}) \sum_{h=1}^{K} \left| rank_{EW}^{h} - rank_{BoD}^{h} \right|$ (see Saisana *et al.,* 2005), is around 14 positions. In general, the majority of cases with rank increases (88.4%) is related to countries with relatively less balanced performance (identified as those assigning in the BoD model a weight equal to one into only one of the component indicators). On the other hand, most of the countries (70.6%) with relatively balanced performance (i.e., those assigning in the BoD model a positive weight into two or all component indicators), deteriorated their rank position in the BoD-based e-GDI compared to the EW-based e-GDI. However, these changes appear on average to be relatively smaller (around 10 positions) compared to those (around 14 positions) of countries with less balanced performance. Nevertheless, both models seem to agree on the best and the worst performers: 7 out of the best 10 countries and 8 out of the 10 worst ones in the

EW-based e-GDI are also ranked in the top-10 and bottom-10 positions in the BoD-based e-GDI.

At the lower panel of Table 1 we report aggregate estimates of the e-GDI by continent and income class, which are based on simple arithmetic averages.[9] For both weighting schemes, Europe stood first and Americas second while Africa had the lower e-GDI. There is only a difference between the EW and the Bod-based e-GDI regarding the third and fourth position, i.e., whether Oceania or Asia is ranked third or fourth. On the other hand, for both weighting schemes, high and upper middle income countries were above while lower middle and low income countries were below the average e-GDI. However, for the above average countries, the difference are more pronounced when using the EW scheme.

As we have mentioned, the purpose of the e-GDI was to become a useful tool for "countries and areas where the potential of ICT has not yet been fully explored and where capacity development support might be helpful" (UN, 2018, p. 8) by providing guidelines to improve the efficiency with which national institutions use ICT for delivering public services (Rorissa *et al.,* 2011; Ayanso *et al.,* 2011). This objective is however difficult to pursue with either the EW scheme, which allows countries to compensate for their weaknesses instead of bringing them forth, or the BoD model, which it is based on country-specific benchmarks (peers), which might not be directly comparable with each other. To deal with these issues, we present next empirical results from a model (i.e., the traffic-light-based BoD model) that is based on a pre-determined benchmark.

3.2. A traffic-lights-reporting-system-based e-GDI.

The aim of the traffic-light scheme is to conduct *a priori* clustering of countries based on the values of the component indicators corresponding to a benchmark unit. Then, countries are classified into three groups, i.e., green, yellow and red. If the values of all component indicators fall short of those of the benchmark unit the country is classified in the red group while if the values of all component indicators exceed those of the benchmark unit the country is classified in the green group. The rest of the countries are included in the yellow group. Thus, countries in the green group meet (i.e. outperform) the targets set by the benchmark, countries in the yellow group meet some but not all benchmark targets, and countries in the red group fail to meet any target.

The framework of traffic-lights reporting system provides a nice basis for applying our theoretical results as the benchmark unit acts, at the same time, as an IDMU for the red group and as an ADMU for the green group. As a result, we can use Corollaries 1 and 3 to obtain the estimated values of the composite indicator for these two groups without the need of solving (2) and (7). From these we can provide a complete ranking of countries in these two groups but not an intra-group ranking. For the yellow group, we follow Ahn and Neumann (2014) suggestion and provide a ranking of countries using their percentage negative deviations from the benchmark unit, given as:[10]

$$d^k = \left| \sum_{j=1}^{J} \frac{y_j^k - y_j^{APU}}{y_j^{APU}} \right| \tag{9}$$

d^k can take values higher that 100%, e.g. a country may lag behind the target by 70% and 40% for two different component indicators, resulting to a d_k equal to 110%.

For the purpose of this paper, we consider the Average Production Unit (APU) as the benchmark unit.[11] The APU has long served the performance evaluation literature as being the unit reflecting structural efficiency (Førsund and Hjalmarsson, 1979), with the difference of its efficiency score from aggregate (group) efficiency representing the extent of reallocation efficiency (Karagiannis, 2015).[12] In the BoD model, the APU reflects the "world average" in terms of achievements with respect to component indicators and as such it was chosen for benchmarking. One could of course have used other benchmarking targets reflecting policy objectives or social goals but in the absence of convincing figures for the case of the e-GDI we have chosen the "world average".

The traffic-lights clustering of countries based on the APU is depicted in Figure 2. The green group contains 69 countries which include the 86% of European countries and roughly ¼ of the North and South American countries (i.e., US, Canada, Brazil, Argentina and Chile) along with other developed and high-income countries such as Australia, Japan and Russia as well as some countries of the Arabic peninsula (see also Table 2). Only two countries from Africa are classified in the green group, namely, Mauritius and South Africa. The yellow group contains 63 countries. The geographical region with the highest representation in this group is the North and South America, with 62.9% of its countries and Oceania with 57,1% of its countries.

However, we find only six European countries in this group. In the red group, there are 61 countries mostly from Africa and sub-Saharan Africa in particular. This group also includes some low- and lower-middle income Oceanic (e.g. Nauru) and Central American (e.g. Haiti and Honduras) countries, along with 11 Asian countries, but no European country. Moreover, from Table 2, we can see that the percentage of countries into the green (red) group increases (decreases) with income while the percentage of countries into the yellow group is higher for the middle income class. There is only one high-income country in the red group (i.e., Equatorial Guinea) while there is no low-income country in the green group. Comparing the green and red group countries with the top- and bottom-ranked countries in the EW- and the BoD-based e-GDI, we see that 92.5% of the 40 top-ranked countries are included in the green group while all the 40 bottom ranked countries are in the red group.

The empirical results regarding this traffic-light-based e-GDI are reported in Table 3 while detailed country results are given in Appendix's Table A1. The top performing countries in the green group include 8 high-income Northern and Western European countries (Belgium, Denmark, Finland, UK, Ireland, Sweden, Netherlands and Norway) along with Australia and New Zealand. Australia is ranked at the top of the green group's countries, followed by Belgium. The average value of e-GDI in this group was 1.243, which means that, on average, the performance of the countries in the green group could worsen up to 24,3% in terms of OSI, TII and HCI but with the countries still being able to meet the predetermined targets. Alternatively, we can say that on average countries in the green group outperform the predetermined targets by at least 24.3%. Regarding the distribution of the e-GDI scores in the green group, we can see from Figure 3 that it has twin peaks around the values of 1.1 and 1.3.

On the other hand, the distribution of the deviation scores for the yellow group is highly skewed to the left (see Figure 3), implying that most of these countries fail to meet the targets by small percentages. In particular, 30 countries fail to meet the target in only one of the component indicators while the remaining 33 fail at two indicators.[13] The problem is largely concentrated into the OSI and the TII for which 39 and 42 out of the 63 countries in the yellow group have achievements below the average, while only 15 countries perform worse than the average for the HCI. China and Romania are the two best-performing countries in the yellow group, with the former only marginally failing to achieve the TII target by 1.02% while the latter is below the OSI target by 1.25%. Only 13 countries have a sum of negative deviations below 10%. The worst-performing countries are the Marshall Islands and the Democratic Republic of Korea.

In contrast, the distribution of the e-GDI in the red group is highly skewed to the right, implying that for most of these countries the maximum among the values of the OSI, the TII and the HCI indicators is close to the "world average". Only 13 countries obtain a score below 0.5. These countries are all but one (Haiti) located in Africa and they are identified as the worst among the poor performing countries, indicating where supranational support to develop the capacity for e-government services is most urgently needed. The remaining countries in the red group had scores higher than 0.5, with Algeria, Cape Verde and Rwanda being at the top. The average effectiveness of the countries in the red group was 0.683 implying that on average these countries need to improve their performance by 46,4% in terms of the OSI, the TII and the HCI.

4. Concluding Remarks

In this paper we deal with BoD type models with an IDMU or an ADMU that turn to have analytical solutions. As a result, the values of the composite indicator can be obtained without need for solving the relevant linear programs, reducing considerably the computation burden. In particular, they are obtained by using the maximum of the ratios of each DMU's component indicators to those of the IDMU. We have also shown that a similar result holds for the inverted BoD model when an ADMU is among the evaluated DMUs. Then, the values of the composite indicator are obtained by using the minimum of the ratios of each DMU's component indicators to those of the ADMU. In addition, we illustrate the applicability of these results in the case of the traffic-light reporting system, where the chosen benchmark unit (the APU in our case) acts as an IDMU for the red group (poor performers) and as an ADMU for the green group (good performers).

We have used these results to re-estimate the e-GDI using the BoD and the traffic-light-based BoD models instead of the EW scheme used by the UN. Our empirical results indicate that all countries with relatively balanced performance are classified in the green group while countries with less balanced performance are found in the red group. This means that countries in the red group should pay more attention to the component indicators at which they are relatively worse in order to gradually transform their operating mix to a more balanced one. For the majority of the countries in the red group, these indicators are OSI and TII. This provides an indication on where should international agencies such as the UN put their effort in supporting the future developments undertaken from the countries in the red group. Nevertheless, possible transfer of policy measures used in the green group should be done with particular care, as most of these are European highly developed countries whereas the majority of the countries in the red group are from Africa and are mostly low income countries.

Table 1: Frequency distribution and descriptive statistics of the UN and the BoD-based e-GDI.

	EW	BoD
(0,0.1)	4	2
[0.1,0.2)	12	4
[0.2,0.3)	28	4
[0.3,0.4)	28	15
[0.4-0.5)	27	16
[0.5,0.6)	30	22
[0.6,0.7)	24	32
[0.7,0.8)	22	45
[0.8,0.9)	16	28
[0.9,1)	2	20
1	0	5
maximum	0.919	1.000
minimum	0.027	0.066
average	0.492	0.666
median	0.497	0.700
st. dev.	0.215	0.207
by continent		
Africa	0.288	0.456
Americas	0.525	0.710
Asia	0.513	0.688
Europe	0.724	0.863
Oceania	0.415	0.691
by income class		
High Income	0.735	0.868
Upper Middle Income	0.509	0.716
Lower Middle Income	0.386	0.572
Low Income	0.230	0.401

Table 2: Traffic-light country classification by continent and income class.

	% of countries classified as		
	green	yellow	red
by continent			
Africa	3.70%	18.50%	77.80%
Americas	25.70%	62.90%	11.40%
Asia	40.40%	36.20%	23.40%
Europe	86.00%	14.00%	0.00%
Oceania	14.30%	57.10%	28.60%
by income class			
High Income	85.50%	12.70%	1.80%
Upper Middle Income	32.10%	57.20%	10.70%
Lower Middle Income	8.70%	37.00%	54.30%
Low Income	0.00%	19.40%	80.60%

Table 3: Frequency distributions and descriptive statistics of the e-GDI and the absolute negative deviations for the green, the red and the yellow group.

green group		yellow group		red group	
(1,1.1)	16	(0% , 40%)	38	(0,0.2)	2
[1.1,1.2)	10	[40% , 80%)	10	[0.2,0.4)	5
[1.2,1.3)	16	[80% , 120%)	7	[0.4,0.6)	14
[1.3,1.4)	17	[120% , 160%)	6	[0.6,0.8)	19
[1.4,1.6)	10	[160% , 200%)	2	[0.8-0.1)	21
Maximum	1.555		185.524		0.997
Minimum	1.017		1.021		0.157
Average	1.243		50.695		0.683
Median	1.263		34.112		0.752
st. dev.	0.144		47.039		0.209

Figure 1: Kernel density distributions of the EW and the BoD-based e-GDI.

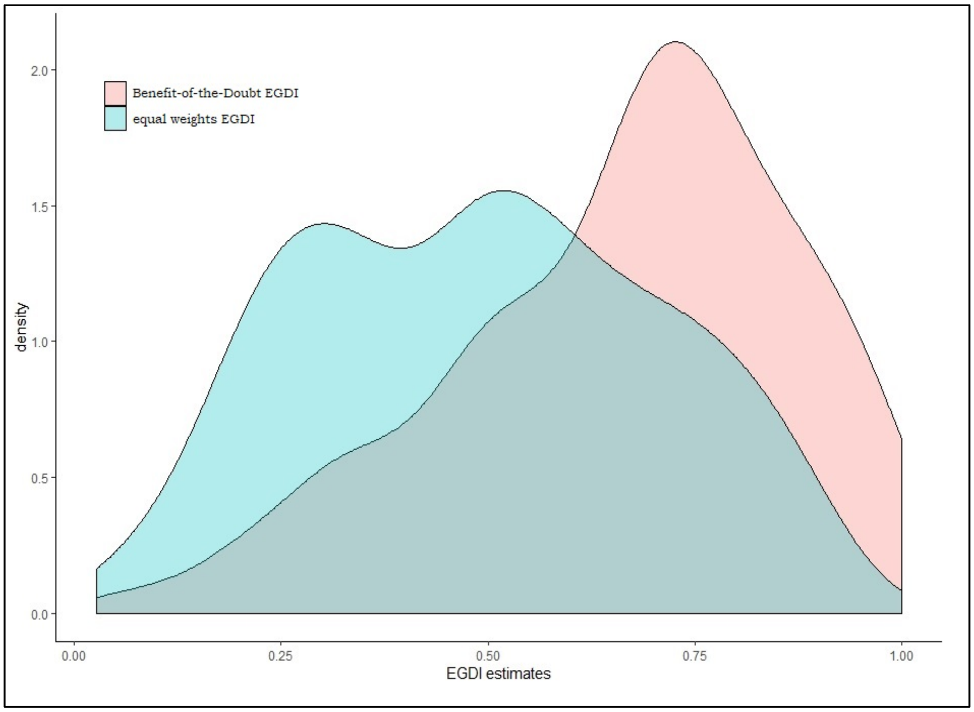


Figure 2: Country groups based on the traffic-lights reporting system.

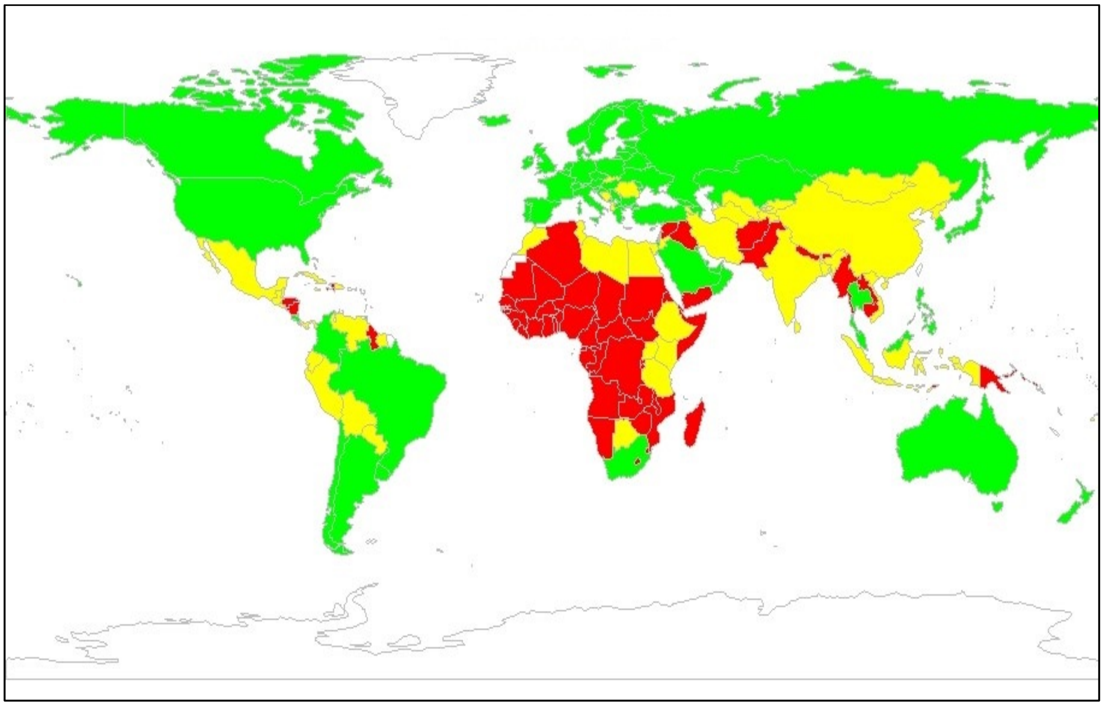
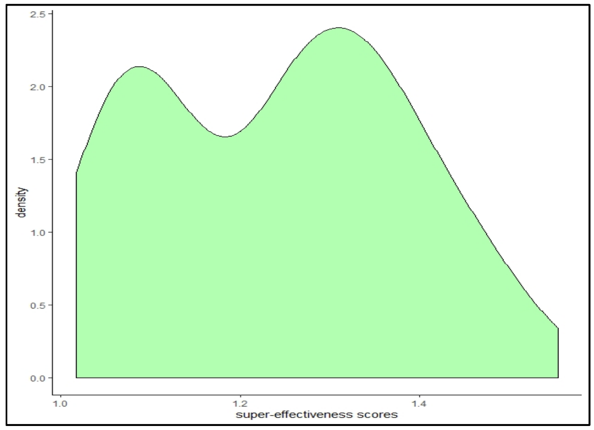
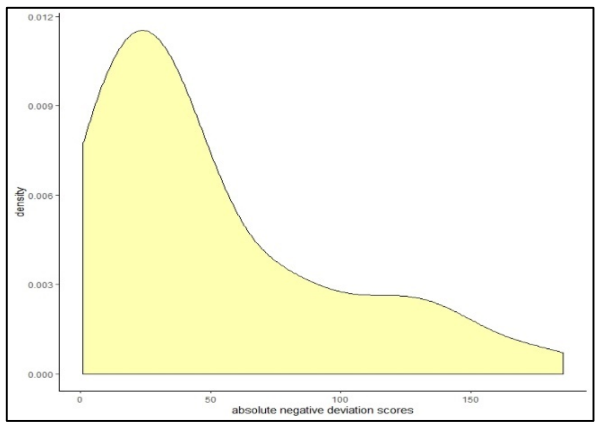


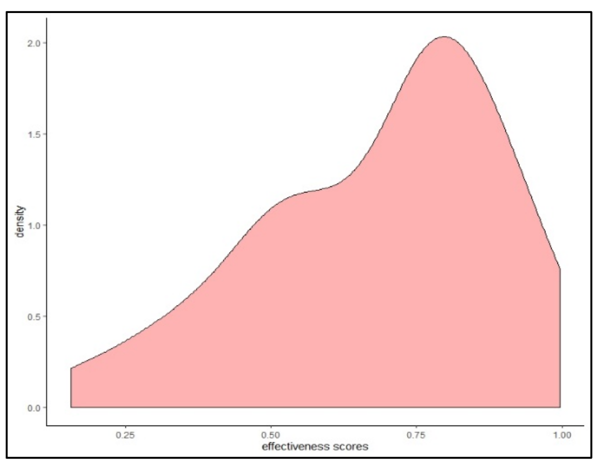
Figure 3: Kernel density distributions of the traffic-light BoD e-GDI and the absolute negative deviations for the green, the red and the yellow group.



e-GDI for the green countries



absolute negative deviation for the yellow countries



e-GDI for the red countries

Table A1: United Nations', BoD and traffic-lights based estimates of the e-GDI: Country scores, ranks and rank differences, 2016.

	Country	Region	Level of Income	OSI	TII	HCI	UN e-GDI	rank	BoD e-GDI	rank	rank difference (UN e-GDI- BoD e-GDI)	traffic light group	traffic light score	traffic light rank
1	Australia	Oceania	High Income	0.978	0.765	1.000	0.914	2	1.000	1	1	GREEN	1.555	1
2	Belgium	Europe	High Income	0.710	0.681	0.971	0.787	19	0.971	8	11	GREEN	1.510	2
3	Denmark	Europe	High Income	0.775	0.825	0.953	0.851	9	0.989	6	3	GREEN	1.482	3
4	Finland	Europe	High Income	0.942	0.759	0.944	0.882	5	0.965	9	-4	GREEN	1.468	4
5	New Zealand	Oceania	High Income	0.942	0.714	0.940	0.865	8	0.957	11	-3	GREEN	1.462	5
6	United Kingdom	Europe	High Income	1.000	0.818	0.940	0.919	1	1.000	1	0	GREEN	1.462	5
7	Ireland	Europe	High Income	0.725	0.660	0.922	0.769	26	0.922	17	9	GREEN	1.433	7
8	Sweden	Europe	High Income	0.877	0.813	0.921	0.870	6	0.976	7	-1	GREEN	1.432	8
9	Netherlands	Europe	High Income	0.928	0.752	0.918	0.866	7	0.947	13	-6	GREEN	1.428	9
10	Norway	Europe	High Income	0.804	0.728	0.903	0.812	18	0.917	20	-2	GREEN	1.404	10
11	Slovenia	Europe	High Income	0.848	0.588	0.895	0.777	21	0.895	26	-5	GREEN	1.392	11
12	Germany	Europe	High Income	0.841	0.734	0.888	0.821	15	0.915	21	-6	GREEN	1.381	12
13	United States of America	Americas	High Income	0.928	0.717	0.882	0.842	12	0.930	16	-4	GREEN	1.370	13
14	Republic of Korea	Asia	High Income	0.942	0.853	0.880	0.892	3	1.000	1	2	GREEN	1.367	14
15	Spain	Europe	High Income	0.913	0.649	0.878	0.814	17	0.918	19	-2	GREEN	1.365	15
16	Estonia	Europe	High Income	0.891	0.733	0.876	0.833	13	0.913	23	-10	GREEN	1.362	16
17	Poland	Europe	High Income	0.703	0.586	0.875	0.721	36	0.875	32	4	GREEN	1.360	17
18	Argentina	Americas	Upper Middle Income	0.710	0.503	0.880	0.698	41	0.880	30	11	GREEN	1.356	18
19	Lithuania	Europe	High Income	0.826	0.626	0.872	0.775	23	0.872	33	-10	GREEN	1.355	19
20	Iceland	Europe	High Income	0.623	0.781	0.894	0.766	27	0.931	15	12	GREEN	1.348	20
21	Israel	Asia	High Income	0.862	0.618	0.862	0.781	20	0.876	31	-11	GREEN	1.340	21
22	Canada	Americas	High Income	0.957	0.672	0.857	0.829	14	0.956	12	2	GREEN	1.333	22
23	Latvia	Europe	High Income	0.609	0.583	0.851	0.681	45	0.851	37	8	GREEN	1.317	23
24	France	Europe	High Income	0.942	0.750	0.845	0.846	10	0.942	14	-4	GREEN	1.313	24
25	Kazakhstan	Asia	Upper Middle Income	0.768	0.567	0.840	0.725	33	0.840	40	-7	GREEN	1.306	25
26	Austria	Europe	High Income	0.913	0.710	0.840	0.821	16	0.913	22	-6	GREEN	1.305	26
27	Switzerland	Europe	High Income	0.601	0.798	0.858	0.753	28	0.911	24	4	GREEN	1.301	27
28	Singapore	Asia	High Income	0.971	0.841	0.836	0.883	4	1.000	1	3	GREEN	1.300	28
29	Hungary	Europe	Upper Middle Income	0.630	0.562	0.832	0.675	46	0.832	43	3	GREEN	1.293	29

30	Japan	Asia	High Income	0.877	0.828	0.827	0.844	11	0.962	10	1	GREEN	1.286	30
31	Russian Federation	Europe	High Income	0.732	0.609	0.823	0.722	35	0.823	45	-10	GREEN	1.280	31
32	Montenegro	Europe	Upper Middle Income	0.681	0.522	0.817	0.673	47	0.816	48	-1	GREEN	1.269	32
33	Portugal	Europe	High Income	0.746	0.584	0.813	0.714	38	0.813	49	-11	GREEN	1.264	33
34	Italy	Europe	High Income	0.870	0.647	0.813	0.776	22	0.870	35	-13	GREEN	1.2632	34
35	Chile	Americas	High Income	0.775	0.497	0.812	0.695	42	0.812	50	-8	GREEN	1.2629	35
36	Greece	Europe	High Income	0.580	0.603	0.890	0.691	43	0.890	28	15	GREEN	1.254	36
37	Croatia	Europe	High Income	0.746	0.597	0.805	0.716	37	0.805	52	-15	GREEN	1.251	37
38	Saudi Arabia	Asia	High Income	0.674	0.573	0.800	0.682	44	0.799	54	-10	GREEN	1.243	38
39	Liechtenstein	Europe	High Income	0.667	0.729	0.798	0.731	32	0.850	38	-6	GREEN	1.240	39
40	Bulgaria	Europe	Upper Middle Income	0.565	0.560	0.788	0.638	52	0.788	58	-6	GREEN	1.223	40
41	Uruguay	Americas	High Income	0.775	0.614	0.782	0.724	34	0.791	55	-21	GREEN	1.216	41
42	Serbia	Europe	Upper Middle Income	0.819	0.543	0.777	0.713	39	0.821	46	-7	GREEN	1.208	42
43	Luxembourg	Europe	High Income	0.717	0.819	0.775	0.771	25	0.919	18	7	GREEN	1.205	43
44	Cyprus	Asia	High Income	0.536	0.492	0.778	0.602	64	0.778	62	2	GREEN	1.160	44
45	Costa Rica	Americas	Upper Middle Income	0.638	0.513	0.744	0.631	53	0.744	76	-23	GREEN	1.156	45
46	Qatar	Asia	High Income	0.674	0.604	0.732	0.670	48	0.751	69	-21	GREEN	1.138	46
47	Malta	Europe	High Income	0.797	0.699	0.731	0.742	30	0.829	44	-14	GREEN	1.136	47
48	Kuwait	Asia	High Income	0.652	0.743	0.729	0.708	40	0.834	42	-2	GREEN	1.133	48
49	Georgia	Asia	Lower Middle Income	0.638	0.418	0.776	0.611	61	0.776	63	-2	GREEN	1.127	49
50	Republic of Moldova	Europe	Lower Middle Income	0.594	0.485	0.719	0.599	65	0.719	86	-21	GREEN	1.118	50
51	Bahrain	Asia	High Income	0.826	0.776	0.718	0.773	24	0.903	25	-1	GREEN	1.116	51
52	Azerbaijan	Asia	Upper Middle Income	0.681	0.485	0.716	0.627	56	0.716	89	-33	GREEN	1.113	52
53	Trinidad and Tobago	Americas	High Income	0.529	0.497	0.708	0.578	70	0.708	94	-24	GREEN	1.100	53
54	Mauritius	Africa	Upper Middle Income	0.703	0.460	0.707	0.623	58	0.715	90	-32	GREEN	1.099	54
55	Andorra	Europe	High Income	0.507	0.686	0.698	0.630	55	0.762	66	-11	GREEN	1.085	55
56	Malaysia	Asia	Upper Middle Income	0.717	0.440	0.695	0.618	60	0.723	84	-24	GREEN	1.081	56
57	Thailand	Asia	Upper Middle Income	0.551	0.412	0.694	0.552	77	0.694	101	-24	GREEN	1.079	57
58	Lebanon	Asia	Upper Middle Income	0.515	0.491	0.688	0.565	73	0.688	102	-29	GREEN	1.070	58
59	Ukraine	Europe	Lower Middle Income	0.587	0.397	0.839	0.608	62	0.839	41	21	GREEN	1.0692	59
60	The former Yugoslav Republic of Macedonia	Europe	Upper Middle Income	0.609	0.469	0.688	0.589	69	0.688	103	-34	GREEN	1.0691	60
61	Oman	Asia	High Income	0.594	0.515	0.680	0.596	66	0.680	109	-43	GREEN	1.057	61
62	Brazil	Americas	Upper Middle Income	0.732	0.503	0.679	0.638	51	0.732	81	-30	GREEN	1.055	62
63	Belarus	Europe	Upper Middle Income	0.486	0.630	0.872	0.663	49	0.872	34	15	GREEN	1.0502	63
64	United Arab Emirates	Asia	High Income	0.891	0.688	0.675	0.752	29	0.891	27	2	GREEN	1.0496	64

65	Czech Republic	Europe	High Income	0.478	0.595	0.863	0.645	50	0.863	36	14	GREEN	1.035	65
66	Colombia	Americas	Upper Middle Income	0.790	0.381	0.700	0.624	57	0.790	57	0	GREEN	1.028	66
67	South Africa	Africa	Upper Middle Income	0.558	0.381	0.725	0.555	76	0.725	83	-7	GREEN	1.026	67
68	Philippines	Asia	Lower Middle Income	0.667	0.379	0.684	0.577	71	0.684	106	-35	GREEN	1.022	68
69	Turkey	Asia	Upper Middle Income	0.601	0.378	0.791	0.590	68	0.791	56	12	GREEN	1.017	69
70	China	Asia	Upper Middle Income	0.768	0.367	0.686	0.607	63	0.768	65	-2	YELLOW	1.021%	1
71	Romania	Europe	Upper Middle Income	0.457	0.453	0.774	0.561	75	0.774	64	11	YELLOW	1.247%	2
72	Bosnia and Herzegovina	Europe	Upper Middle Income	0.449	0.405	0.682	0.512	92	0.682	107	-15	YELLOW	2.805%	3
73	Barbados	Americas	High Income	0.442	0.640	0.811	0.631	54	0.819	47	7	YELLOW	4.384%	4
74	Slovakia	Europe	High Income	0.442	0.550	0.782	0.592	67	0.782	60	7	YELLOW	4.384%	5
75	Albania	Europe	Upper Middle Income	0.594	0.353	0.652	0.533	82	0.652	120	-38	YELLOW	4.874%	6
76	Brunei Darussalam	Asia	High Income	0.507	0.351	0.731	0.530	83	0.731	82	1	YELLOW	5.359%	7
77	Tunisia	Africa	Upper Middle Income	0.717	0.348	0.640	0.568	72	0.717	88	-16	YELLOW	6.881%	8
78	Viet Nam	Asia	Lower Middle Income	0.573	0.372	0.599	0.514	89	0.599	131	-42	YELLOW	6.895%	9
79	Ecuador	Americas	Upper Middle Income	0.630	0.344	0.713	0.563	74	0.713	92	-18	YELLOW	7.353%	10
80	Armenia	Asia	Lower Middle Income	0.428	0.392	0.734	0.518	87	0.734	80	7	YELLOW	7.520%	11
81	Bahamas	Americas	High Income	0.428	0.384	0.721	0.511	93	0.721	85	8	YELLOW	7.520%	12
82	Jordan	Asia	Upper Middle Income	0.457	0.346	0.734	0.512	91	0.734	79	12	YELLOW	8.061%	13
83	Venezuela	Americas	Upper Middle Income	0.435	0.354	0.750	0.513	90	0.750	73	17	YELLOW	10.546%	14
84	Seychelles	Africa	Upper Middle Income	0.406	0.462	0.686	0.518	86	0.686	104	-18	YELLOW	12.215%	15
85	Mexico	Americas	Upper Middle Income	0.848	0.311	0.699	0.620	59	0.848	39	20	YELLOW	16.084%	16
86	El Salvador	Americas	Lower Middle Income	0.486	0.327	0.604	0.472	104	0.604	129	-25	YELLOW	18.195%	17
87	Dominican Republic	Americas	Upper Middle Income	0.507	0.299	0.668	0.491	98	0.668	112	-14	YELLOW	19.372%	18
88	Grenada	Americas	Upper Middle Income	0.370	0.399	0.782	0.517	88	0.782	61	27	YELLOW	20.046%	19
89	Fiji	Oceania	Upper Middle Income	0.413	0.333	0.751	0.499	96	0.751	70	26	YELLOW	21.029%	20
90	Kyrgyzstan	Asia	Upper Middle Income	0.428	0.312	0.751	0.497	97	0.751	71	26	YELLOW	23.362%	21
91	Mongolia	Asia	Lower Middle Income	0.515	0.284	0.760	0.519	84	0.760	67	17	YELLOW	23.441%	22
92	Egypt	Africa	Lower Middle Income	0.471	0.303	0.605	0.459	108	0.605	128	-20	YELLOW	24.460%	23
93	Peru	Americas	Upper Middle Income	0.630	0.269	0.715	0.538	81	0.715	91	-10	YELLOW	27.537%	24
94	Panama	Americas	Upper Middle Income	0.333	0.420	0.718	0.490	99	0.718	87	12	YELLOW	27.898%	25
95	Monaco	Europe	High Income	0.319	1.000	0.876	0.732	31	1.000	1	30	YELLOW	31.035%	26
96	Bolivia	Americas	Lower Middle Income	0.493	0.253	0.700	0.482	101	0.700	97	4	YELLOW	31.768%	27
97	Paraguay	Americas	Lower Middle Income	0.601	0.254	0.641	0.499	95	0.641	123	-28	YELLOW	31.810%	28
98	Iran (Islamic Republic of)	Asia	Upper Middle Income	0.333	0.351	0.710	0.465	106	0.710	93	13	YELLOW	33.204%	29
99	Uzbekistan	Asia	Lower Middle Income	0.688	0.246	0.695	0.543	80	0.701	96	-16	YELLOW	33.627%	30
100	Morocco	Africa	Lower Middle Income	0.739	0.343	0.474	0.519	85	0.739	77	8	YELLOW	33.954%	31

101	Sri Lanka	Asia	Lower Middle Income	0.652	0.245	0.737	0.545	79	0.737	78	1	YELLOW	34.112%	32
102	Dominica	Americas	Upper Middle Income	0.304	0.431	0.638	0.458	109	0.638	124	-15	YELLOW	34.926%	33
103	Saint Vincent and the Grenadines	Americas	Upper Middle Income	0.297	0.376	0.675	0.449	115	0.675	110	5	YELLOW	35.729%	34
104	Suriname	Americas	Upper Middle Income	0.297	0.412	0.655	0.455	110	0.655	119	-9	YELLOW	35.729%	35
105	Jamaica	Americas	Upper Middle Income	0.355	0.319	0.686	0.453	112	0.686	105	7	YELLOW	37.138%	36
106	Botswana	Africa	Upper Middle Income	0.283	0.422	0.655	0.453	113	0.655	118	-5	YELLOW	38.866%	37
107	Saint Kitts and Nevis	Americas	High Income	0.283	0.530	0.698	0.503	94	0.698	99	-5	YELLOW	38.866%	38
108	Indonesia	Asia	Lower Middle Income	0.362	0.302	0.680	0.448	116	0.680	108	8	YELLOW	40.350%	39
109	Saint Lucia	Americas	Upper Middle Income	0.275	0.409	0.674	0.453	114	0.674	111	3	YELLOW	40.424%	40
110	San Marino	Europe	High Income	0.239	0.613	0.800	0.551	78	0.800	53	25	YELLOW	48.276%	41
111	Maldives	Asia	Upper Middle Income	0.232	0.437	0.630	0.433	117	0.630	125	-8	YELLOW	51.878%	42
112	Guatemala	Americas	Lower Middle Income	0.667	0.236	0.535	0.479	102	0.667	113	-11	YELLOW	53.364%	43
113	Tonga	Oceania	Upper Middle Income	0.370	0.230	0.810	0.470	105	0.810	51	54	YELLOW	58.012%	44
114	Antigua and Barbuda	Americas	Lower Middle Income	0.181	0.541	0.745	0.489	100	0.745	75	25	YELLOW	60.802%	45
115	Kenya	Africa	Low Income	0.558	0.181	0.517	0.419	119	0.558	139	-20	YELLOW	70.921%	46
116	Libya	Africa	Upper Middle Income	0.109	0.429	0.759	0.432	118	0.759	68	50	YELLOW	76.485%	47
117	Palau	Oceania	Upper Middle Income	0.109	0.368	0.887	0.455	111	0.887	29	82	YELLOW	77.209%	48
118	Belize	Americas	Upper Middle Income	0.319	0.183	0.645	0.383	122	0.645	121	1	YELLOW	81.613%	49
119	India	Asia	Lower Middle Income	0.746	0.143	0.502	0.464	107	0.746	74	33	YELLOW	83.439%	50
120	Samoa	Oceania	Lower Middle Income	0.341	0.158	0.708	0.402	121	0.708	95	26	YELLOW	83.849%	51
121	Uganda	Africa	Low Income	0.500	0.113	0.467	0.360	128	0.500	152	-24	YELLOW	97.007%	52
122	Bangladesh	Asia	Low Income	0.623	0.119	0.397	0.380	124	0.623	126	-2	YELLOW	106.087%	53
123	Turkmenistan	Asia	Upper Middle Income	0.087	0.256	0.658	0.334	140	0.658	117	23	YELLOW	112.220%	54
124	Tanzania (united republic of)	Africa	Low Income	0.573	0.090	0.397	0.353	130	0.573	134	-4	YELLOW	113.967%	55
125	Tajikistan	Asia	Low Income	0.123	0.187	0.700	0.337	139	0.700	98	41	YELLOW	123.064%	56
126	Cuba	Americas	Upper Middle Income	0.196	0.110	0.751	0.352	131	0.751	72	59	YELLOW	127.941%	57
127	Micronesia (Federated States of)	Oceania	Lower Middle Income	0.145	0.120	0.666	0.310	146	0.666	114	32	YELLOW	136.398%	58
128	Kiribati	Oceania	Lower Middle Income	0.210	0.067	0.660	0.312	145	0.660	116	29	YELLOW	136.629%	59
129	Tuvalu	Oceania	Upper Middle Income	0.022	0.198	0.665	0.295	151	0.665	115	36	YELLOW	141.922%	60
130	Ethiopia	Africa	Low Income	0.529	0.050	0.221	0.267	157	0.529	144	13	YELLOW	152.273%	61
131	Marshall Islands	Oceania	Upper Middle Income	0.029	0.085	0.695	0.270	156	0.695	100	56	YELLOW	170.848%	62
132	Democratic People's Republic of Korea	Asia	Low Income	0.022	0.036	0.782	0.280	153	0.782	59	94	YELLOW	185.524%	63
133	Algeria	Africa	Upper Middle Income	0.065	0.193	0.641	0.300	150	0.641	122	28	RED	0.997	1
134	Cape Verde	Africa	Lower Middle Income	0.457	0.363	0.603	0.474	103	0.603	130	-27	RED	0.988[g]	2
135	Rwanda	Africa	Low Income	0.457	0.108	0.452	0.339	138	0.463	163	-25	RED	0.988[g]	2
136	Ghana	Africa	Lower Middle Income	0.449	0.259	0.546	0.418	120	0.546	141	-21	RED	0.972	4

137	Gabon	Africa	Upper Middle Income	0.152	0.307	0.616	0.358	129	0.616	127	2	RED	0.958	5
138	Vanuatu	Oceania	Lower Middle Income	0.167	0.168	0.588	0.308	149	0.588	132	17	RED	0.915	6
139	Swaziland	Africa	Lower Middle Income	0.275	0.160	0.588	0.341	136	0.588	133	3	RED	0.914	7
140	Nigeria	Africa	Lower Middle Income	0.413	0.196	0.378	0.329	143	0.413	167	-24	RED	0.893	8
141	Honduras	Americas	Lower Middle Income	0.312	0.201	0.571	0.361	127	0.571	135	-8	RED	0.888	9
142	Guyana	Americas	Lower Middle Income	0.283	0.243	0.569	0.365	126	0.569	136	-10	RED	0.885	10
143	Zambia	Africa	Lower Middle Income	0.370	0.118	0.564	0.351	132	0.564	137	-5	RED	0.8772	11
144	Zimbabwe	Africa	Low Income	0.261	0.217	0.564	0.347	134	0.564	138	-4	RED	0.8769	12
145	Namibia	Africa	Upper Middle Income	0.283	0.267	0.555	0.368	125	0.555	140	-15	RED	0.863	13
146	Nepal	Asia	Low Income	0.399	0.168	0.471	0.346	135	0.471	162	-27	RED	0.862	14
147	Nicaragua	Americas	Lower Middle Income	0.384	0.211	0.545	0.380	123	0.545	142	-19	RED	0.848	15
148	Congo	Africa	Low Income	0.044	0.171	0.534	0.250	162	0.534	143	19	RED	0.831	16
149	Senegal	Africa	Lower Middle Income	0.377	0.196	0.403	0.325	144	0.403	168	-24	RED	0.815	17
150	Nauru	Oceania	Upper Middle Income	0.094	0.245	0.521	0.287	152	0.521	145	7	RED	0.811	18
151	Sao Tome and Principe	Africa	Lower Middle Income	0.044	0.155	0.519	0.239	168	0.519	146	22	RED	0.807	19
152	Equatorial Guinea	Africa	High Income	0.080	0.124	0.517	0.240	165	0.517	147	18	RED	0.804	20
153	Lesotho	Africa	Lower Middle Income	0.138	0.179	0.515	0.277	154	0.515	148	6	RED	0.800	21
154	Bhutan	Asia	Lower Middle Income	0.319	0.219	0.514	0.351	133	0.514	149	-16	RED	0.799	22
155	Togo	Africa	Low Income	0.319	0.104	0.506	0.310	147	0.506	150	-3	RED	0.786	23
156	Angola	Africa	Upper Middle Income	0.348	0.144	0.502	0.331	142	0.502	151	-9	RED	0.780	24
157	Burundi	Africa	Low Income	0.152	0.033	0.498	0.228	173	0.498	153	20	RED	0.774	25
158	Iraq	Asia	Upper Middle Income	0.355	0.165	0.480	0.333	141	0.480	159	-18	RED	0.768	26
159	Lao People's Democratic Republic	Asia	Lower Middle Income	0.283	0.154	0.491	0.309	148	0.491	154	-6	RED	0.763	27
160	Comoros	Africa	Low Income	0.051	0.107	0.489	0.216	176	0.488	155	21	RED	0.759	28
161	Syrian Arab Republic	Asia	Lower Middle Income	0.326	0.209	0.486	0.340	137	0.486	156	-19	RED	0.756	29
162	Timor-Leste	Asia	Lower Middle Income	0.217	0.073	0.484	0.258	160	0.484	157	3	RED	0.753	30
163	Myanmar	Asia	Low Income	0.159	0.066	0.484	0.236	169	0.484	158	11	RED	0.752	31
164	Cameroon	Africa	Lower Middle Income	0.217	0.131	0.479	0.276	155	0.479	160	-5	RED	0.745	32
165	Cambodia	Asia	Low Income	0.051	0.249	0.479	0.259	158	0.479	161	-3	RED	0.744	33
166	Pakistan	Asia	Lower Middle Income	0.326	0.130	0.319	0.258	159	0.329	176	-17	RED	0.705	34
167	Malawi	Africa	Low Income	0.217	0.049	0.454	0.240	166	0.454	164	2	RED	0.705	35
168	Madagascar	Africa	Low Income	0.225	0.051	0.449	0.242	163	0.449	165	-2	RED	0.698	36
169	Solomon Islands	Oceania	Lower Middle Income	0.167	0.115	0.440	0.241	164	0.440	166	-2	RED	0.684	37
170	Afghanistan	Asia	Low Income	0.304	0.107	0.283	0.231	171	0.304	182	-11	RED	0.658	38
171	Democratic Republic of the Congo	Africa	Low Income	0.087	0.079	0.397	0.188	180	0.397	169	11	RED	0.617	39
172	Mozambique	Africa	Low Income	0.203	0.099	0.389	0.231	172	0.389	170	2	RED	0.605	40

173	Yemen	Asia	Lower Middle Income	0.145	0.147	0.383	0.225	174	0.383	171	3	RED	0.595	41
174	Mali	Africa	Low Income	0.094	0.215	0.236	0.182	182	0.249	185	-3	RED	0.579	42
175	South Sudan	Africa	Low Income	0.123	0.053	0.361	0.179	183	0.361	172	11	RED	0.561	43
176	Liberia	Africa	Low Income	0.239	0.104	0.358	0.234	170	0.358	174	-4	RED	0.557	44
177	Sudan	Africa	Lower Middle Income	0.217	0.186	0.358	0.254	161	0.358	173	-12	RED	0.557	44
178	Guinea-Bissau	Africa	Low Income	0.109	0.083	0.354	0.182	181	0.354	175	6	RED	0.550	46
179	Gambia	Africa	Low Income	0.196	0.196	0.327	0.240	167	0.327	177	-10	RED	0.528	47
180	Papua New Guinea	Oceania	Lower Middle Income	0.167	0.074	0.324	0.188	179	0.324	178	1	RED	0.504	48
181	Benin	Africa	Low Income	0.145	0.147	0.320	0.204	177	0.320	179	-2	RED	0.497	49
182	Haiti	Americas	Low Income	0.167	0.100	0.312	0.193	178	0.312	180	-2	RED	0.486	50
183	Djibouti	Africa	Lower Middle Income	0.022	0.070	0.310	0.134	187	0.310	181	6	RED	0.481	51
184	Mauritania	Africa	Lower Middle Income	0.065	0.154	0.302	0.173	184	0.302	183	1	RED	0.469	52
185	Côte d'Ivoire	Africa	Lower Middle Income	0.188	0.171	0.296	0.219	175	0.296	184	-9	RED	0.461	53
186	Burkina Faso	Africa	Low Income	0.188	0.123	0.168	0.160	185	0.188	191	-6	RED	0.408	54
187	Eritrea	Africa	Low Income	0.022	0.000	0.249	0.090	190	0.249	186	4	RED	0.387	55
188	Sierra Leone	Africa	Low Income	0.116	0.122	0.241	0.159	186	0.241	187	-1	RED	0.374	56
189	Central African Republic	Africa	Low Income	0.000	0.038	0.199	0.079	191	0.199	188	3	RED	0.309	57
190	Chad	Africa	Low Income	0.138	0.048	0.192	0.126	188	0.192	189	-1	RED	0.298	58
191	Guinea	Africa	Low Income	0.087	0.091	0.190	0.123	189	0.190	190	-1	RED	0.296	59
192	Somalia	Africa	Low Income	0.015	0.067	0.000	0.027	193	0.066	193	0	RED	0.179	60
193	Niger	Africa	Low Income	0.073	0.056	0.050	0.059	192	0.072	192	0	RED	0.157	61

Note: (a) columns 2-8 are taken from the UN (2016) e-Government Survey, pp. 147-153; (b) New Zealand and United Kingdom, Cape Verde and Rwanda, and Liberia and Sudan are tied for the 5th position of the "green" group and the 2nd and 44th positions of the "red" group, respectively.

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Footnotes

[1] Throughout the paper when a quotation is used, words or phrases in brackets are our own additions.

[2] A similar claim was made by Prieto and Zofio (2001, p. 49) for the RAM DEA model with a single constant input: “There is yet an extreme case when all provision standards are not reached by any observation ... In this situation, the reference peer ... would be point E [the point with coordinates the highest values of the component indicators] and calculation of the RAM measure would not require optimizing techniques”. However, this DEA formulation goes beyond the purposes of this paper, which focus only on radial DEA models.

[3] This requirement is fulfilled in Korhonen and Luptacik (2004) as one of their evaluated units (i.e., unit #1) had a score of one for both technical and ecological efficiency.

[4] From these, we will use the term IDMU from now on.

[5] Martin *et al*. (2017) and Charles and D’Alessio (2019) included the IDMU in the sample to provide a complete ranking of the evaluated DMUs. Jahanshahloo *et al.* (2010) and Sun *et al.* (2013) used the IDMU and the ADMU to obtain a common set of (input and output) weights that allow for complete ranking of all (efficient and inefficient) DMUs. Wang and Luo (2006), on the other hand, incorporated the IDMU and the ADMU into DEA in a way similar to that of the most preferred solution (MPS) in Value Efficiency Analysis (VEA) to evaluate respectively the best and worst relative efficiencies of DMUs and then combine these two using the relative closeness index adopted from TOPSIS. Wang *et al.* (2011) provided a number of neutral cross efficiency models, which set the secondary goal by means of either (a) minimizing the distance from the IDMU, (b) maximizing the distance from the ADMU, (c) maximizing the distance between the IDMU and the ADMU, or (d) maximizing the index of relative closeness. See also Carillo and Jorge (2018) for a variant of the maximizing the distance between the IDMU and the ADMU cross efficiency model and Shi *et al.* (2017) for an alternative neutral cross efficiency model by minimizing the deviation of a DMU from the ideal virtual frontier and maximizing the deviation from the anti-ideal virtual frontier.

[6] The inverted DEA models evaluate DMUs against a worst-practice frontier and aim in identifying the worst- (instead of the best) performing DMUs.

[7] The proof of proposition 2 is similar to that of proposition 1, which is given in Vaninsky (2011).

[8] This is similar to the result obtained by Mishra and Nathan (2018) using the lexim ordering approach, where the entire weight in the construction of composite indicators is given to the component indicator with the smallest value.

[9] Karagiannis (2017) has shown that the simple arithmetic average is the theoretically consistent scheme to aggregate composite indicators, derived from the BoD model, across DMUs.

[10] Notice that d_k can take values higher that 100%, as a particular country may lag behind the target by, say, 70% for a particular component indicator and another 40% for another component indicator, and in this case d_k is equal to 110%.

[11] The APU is the same with Yang *et al*. (2018) μ virtual DMU, but their approach for dividing the evaluated DMUs into those that perform better than the μ virtual DMU (higher DMUs), those that perform worse than the μ virtual DMU, and the overlapped DMUs is completely different than that of the traffic-light reporting system.

[12] Structural efficiency evaluates industry performance by considering the industry as a single production unit that has at its disposal or coordinates (by means of centralized resource allocation) the inputs of its constituent firms (thus allowing for reallocation of inputs across the latter) while aggregate efficiency measures industry performance when the constituent firms operate independently, i.e., have complete control of their inputs and the observed industry structure (i.e., the allocation of individual inputs) is taken as given. Their ratio is equal to reallocation efficiency (see Karagiannis, 2015).

[13] The fact that the yellow group of countries are ranked based on their negative deviations from the target, as suggested by Ahn and Neumann (2014), might seem unfair for countries such as Monaco, which is ranked 26th out of the 63 countries in the yellow group, having a negative deviation of 31.04% from the average OSI while at the same time outperforming the average TII and HCI by 169.48% and 36.14%, respectively. If we compute the d^k's based on both positive and negative deviations from the target (and without using absolute values), then Monaco is ranked first in the yellow group. Nevertheless, the correlation between the rankings based on negative and on both negative and positive deviations from the target is as high as 0.849, with 7 out of the 10 top countries of the former being ranked in the top-10 of the latter, while all bottom-10 ranked countries are the same.