Constructing Composite Indicators with Shannon Entropy: The Case of Human Development Index

Karagiannis, Roxani[1] and Giannis Karagiannis[2]

1 Research Fellow, Center for Planning and Economic Research, 11 Amerikis str., 10672 Athens, Greece, rkarag@kepe.gr,

[2] Professor, Department of Economics, University of Macedonia, 156 Egnatia Str. Thessaloniki, 54006 GREECE; karagian@uom.gr

ABSTRACT: In this paper we propose a weighting scheme for constructing composite indicators based on Shannon entropy. Its main advantages are that it delivers a set of common weights across decision-making units that allow for complete comparison and ranking and is easy to implement. It can also be extended to provide group specific weights. We explore the potential of the suggested weighing method by using it to re-estimate the Human Development Index.

KEYWORDS: Composite Indicators; Weighting; Shannon Entropy; HDI

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1. Introduction

Composite indicators are synthetic indices or aggregates of all component indicators describing a multi-dimensional and often complex issue. Based on an underlying model, these component indicators are compiled into a single index, which is much easier to interpret than a number (quite large in some cases) of separate indicators. The ability of composite indicators to provide simple comparisons of decision making units (DMUs), being either countries, firms, producers, etc., makes them useful tools in policy analysis and public communication. In addition, composite indicators have been proven useful in benchmarking performance. For all these reasons, there are now more than 150 composite indicators, including the Human Development Index (HDI), the Quality of Life Indicator, the Competitiveness Index, the Technology Achievement Index, the Health System Performance Index, the Environmental Performance, and several subjective Well-being Indices.

OECD's (2008) Handbook on Constructing Composite Indicators identifies seven steps in the construction of composite indicators: *first*, development of the theoretical framework that provides the basis for choosing the set of appropriate indicators describing a particular issue. *Second*, selection of the necessary data for the indicators included in the analysis on the basis of their relevance, their relation to each other, measurability, coverage and analytical soundness. *Third*, imputation (when necessary) of missing data for the component indicators. *Fourth*, use of multivariate analysis to confirm the relevance of the component indicators and their relation to each other. *Fifth*, normalization of the component indicators to transform their values into a common scale. *Sixth*, use of a consistent with the theoretical framework scheme of weighting and aggregating the component indicators and *seventh*, robustness test and sensitivity analysis of the estimated values of the composite indicator.

The aim of this paper is on the sixth step in the construction of composite indicators and in particular, on suggesting an alternative weighting method. The proposed weighting method is based on information theory and more precisely, on Shannon (1948) entropy. In the next sections we illustrate how using this method one can derive a set of common (but not necessarily equal) weights for the component indicators, which are not determined *a priori* but rather endogenously on the basis of the relative variability of the component indicators' across DMUs. The resulting set of common weights allow for a complete comparison and ranking of all DMUs. Even though there are other statistical weighting methods that deliver common (across DMUs) weights (see next section for a review), the merits of the proposed method are that it is easy to implement and it is computationally less demanding compared to existing alternatives. These make the proposed weighting method very practical and attractive. For the purposes of this paper, we apply the proposed weighting method to construction of HDI using data for 2012.

The remainder of this paper proceeds as follows. In the next section, we provide a brief literature review of alternative weighting methods. In the third section, we describe the method and the materials used in this paper and in the fourth section, we present the empirical results regarding the HDI for 2012 as well as comparisons with previous results based on other weighting methods. Concluding remarks follow in the last section.

2. Literature Review

According to OECD (2008, p. 31) Handbook on Constructing Composite Indicators, the weighting methods are divided into two categories: statistical and participatory. Three methods fall in the category of statistical methods: namely, (a) factor analysis – principal components; (b) the non-parametric Benefit-of-the Doubt (BoD) model that is a special variant of the general Data Envelopment Analysis model; and (c) the unobservable component model. On the other hand, three methods are included in the category of participatory methods: namely, (a) budget allocation process based on either experts or public opinion; (b) analytical hierarchy process; and (c) conjoint analysis. All these methods are described in more details on pp. 89-99 of OECD (2008) Handbook.

Besides their analytical and operational differences these weighting methods result in different types of weights: *First*, all but the BoD model delivers a set of common (but not necessarily equal) weights for the component indicators. The BoD model in contrast may result in a set of weights that are either DMU-specific or common across DMUs. The former reflects the underlying assumption of the model that each evaluated DMU is allowed to choose, under certain regulatory conditions, its own set of weights in order to show itself in the best possible light relative to other DMUs in the sample. It is thus able to exaggerate its own advantages and at the same time to downplay its own weaknesses in order to obtain the maximal possible evaluation score. Hence, the variable-weights scheme expresses in the best possible way the interest of the evaluated DMUs, which may assign extremely low or high weights to certain indicator(s). The variable-weights scheme is the most affirmative in its resulting outcomes: if afterwards someone is still weak relative to other DMUs in the sample this cannot be put down to the choice of input and output weights.

On the other hand, it has been argued (e.g., Kao and Hung, 2005; Wang, Luo and Lin, 2011) that comparison and ranking of DMUs are meaningful only when they are conducted on common grounds and thus the use of common but not necessarily equal weights, which though are not determined *a priori*, may be favored. Another advantage of common weights is that it can be applied to assess performance for DMUs not being in the sample (Kao and Hung, 2007). For these reasons, several variants of BoD model have been introduced: these include (a) the compromise solution approach (Kao and Hung, 2005), (b) the goal programming approach (Despotis 2005a, b; see also Bernini *et al.*, 2013 and Sayed *et al.*, 2018), (c) the average cross efficiency approach (Karagiannis and Paleologou, 2014), and (d) the meta-goal programming approach (Sayed *et al.*, 2015). Perhaps with the exception of the third approach, all the others are computationally demanding and require rather complex optimization techniques.

Second, all but the budget allocation process, based on either experts or public opinion, delivers a set of *a posteriori* weights. These are endogenous and thus are derived during the evaluation process by means of an optimization procedure. They may be variable or common across the evaluated DMUs and may or may not reflect experts' and stakeholders' opinions. In contrast, the budget allocation process results in a set of *a priori* weights, which are set prior to the evaluation process. They are common to all evaluated DMUs and in several cases, assign equal weights to all indicators included in the analysis.

However, derivation of *a posteriori* weights may not necessarily be based on an optimization procedure as with the existing weighting methods. It may rely on information theory methods and thus be purely data-driven. By being practical and relatively easy to implement, such methods may be proved a powerful tool for the construction of composite indicators, the usefulness of which have not been explored yet in the literature. In the next section, we provide such an alternative based on the notion of entropy.

3. Method and Materials

The purpose of this paper is to propose another weighting scheme that falls in the statistical method category and results in a common (across DMUs) set of weights for component indicators. The proposed weighting scheme is based on Shannon (1948) entropy, which has a central role in information theory and provides an objective weighting that fully exploits the information of the data itself. In particular, the entropy method gives more weight to component indicators with larger variation across DMUs because they provide higher discrimination and are thus more valuable in the decision-making process. Accordingly, component indicators with relatively smaller variation across DMUs are weighted less as they are less important in the decision-making process.

Suppose that a set of R component indicators, capturing different aspects of performance, are arranged in the following matrix format for K DMUs:

$$\begin{bmatrix} y_1^1 & \cdots & y_1^k & \cdots & y_1^K \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ y_r^1 & \cdots & y_r^k & \cdots & y_r^K \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ y_R^1 & \cdots & y_R^k & \cdots & y_R^K \end{bmatrix}$$

where subscripts (*r=1,…,R*) refers to indicators and superscripts (*k=1,…,K*) to DMUs. This is our data for deriving a set of aggregation weights that will be used to estimate the value of a composite indicator for each DMU. Using Shannon entropy, the set of aggregation weights is obtained in four steps: *first*, set $\bar{y}_r^k = \hat{y}_r^k / \sum_{k=1}^{K} \hat{y}_r^k$ for each component indicator, where is $\hat{y}_r^k$ is the normalized value of component indicators.[1]

[1] See below in this section for the alternative normalization processes that may be used for this purpose.

Second, compute the value of entropy as $e_r = -e_0 \sum_{k=1}^{K} \bar{y}_r^k \, ln\bar{y}_r^k$ for each component indicator, where the entropy constant $e_0 = 1/lnK$. *Third*, set $d_r = 1 - e_r$ for each component indicator. *Fourth*, compute the degree of importance of the r[th] component indicator as $m_r = d_r / \sum_{r=1}^{R} d_r = (1 - e_r)/(K - \sum_{r=1}^{R} e_r)$. These degrees of importance are used then as aggregation weights in the construction of the composite indicator and the value of the composite indicator is given as $y^k = \sum_{r=1}^{R} m_r \hat{y}_r^k$.

From the above one can verify that the entropy value of each component indicator is inversely related to its dispersion (variation) across DMUs and thus, more (less) weight is given to component indicators with larger (smaller) variation across DMUs. In particular, the larger (smaller) the variation of a component indicator, the larger (smaller) its entropy value and the smaller (larger) the information provided by this component indicator. Consequently, the more (less) important this component indicator becomes in the decision-making process and as a result, the smaller (larger) is its degree of importance and thus its weight. If a component indicator has no variation across DMUs and thus provides no discrimination at all, its entropy value takes the maximum value of one and consequently, its degree of importance becomes zero. That is, if all DMUs score about equally with respect to a given component indicator, then such a component indicator will be judged unimportant in the decision-making process and ranking. It is then reasonable to assign a small weight to it when considering overall performance.

The main advantages of Shannon (1948) entropy in constructing composite indicators are that *first*, it results in a set of common (across DMUs) weights that allow for complete comparison and ranking of all DMUs and *second*, it is easy to implement. Compared to the equal-weighting (EW) scheme, which assigns the same weight to all component indicators, the proposed weighting scheme provides higher discriminatory power. If for instance we consider two component indicators, one with the same value across DMUs and another with some variation, the EW scheme assigns weights equal to 0.5 to each one while the proposed weighting scheme assigns a weight equal to zero to the former component indicator and a weight equal to one to the latter. On the other hand, compared to the BoD model, the proposed weighting scheme is computationally less demanding, delivers common (across DMUs) weights and for this reason, it provides higher discriminatory power.

[Figure 1]

A graphical comparison of these three weighting schemes is given in Figure 1 where we consider the case of two component indicators y_1 and y_2. Points labeled with capital letters indicate different DMUs and their coordinates correspond to the values of the two component indicators. Notice that it was purposely assumed that DMUs A, D, E and F score equally with respect to y_1. In this two dimensional example, the weights can be represented as a straight line and the further this line lies on the right, the higher the value of the composite indicator. The line corresponding to the EW scheme has slope equal to 45° and it is labeled as aa' while the line corresponding to the Shannon scheme, which is presumed to give more weight to the component indicator with the larger variation (y_2 in our example), is labeled as bb' and it has a smaller slope compared to aa'. For both of these weighting schemes all DMUs are evaluated by means of a common set of weights, which are also equal to each other for the EW scheme but not necessarily equal for the Shannon scheme. According to the EW scheme, DMU B is ranked first as this is how far the aa' line can lie on the right and for the Shannon scheme the same is true for DMU A. On the other hand, the reference line for the BoD model contains several segments, namely $cABCc'$, reflecting its flexibility to provide DMU-specific weights and enveloping all data points as close as possible. Then the value of the composite indicator for each DMU is computed by measuring the distance of points A, B, C, D, E, and F from the relevant reference line along a ray from the origin. Consider for example DMU E: the value of its composite indicator is given by the ratio OE/OE' using the BoD model, by the ratio OE/OE'' using the entropy weighting scheme, and by the ratio OE/OE''' using the EW scheme. As it is expected, the BoD model provides the most optimistic evaluation. On the other hand, comparing the EW and the entropy schemes, one may notice that for DMUs located above the ray from the origin to the point where the EW and the Shannon reference lines intersect, the EW scheme provides more favorable evaluations than the entropy scheme, and *vice versa*.

The sensitivity of composite indicator values to the normalization process used is a well-known issue in the literature; see e.g., Freudenberg (2003), Saisana, Saltelli and Tarantola (2005), OECD (2008). The choice of the normalization procedure affects the resulting values of the component indicators but does not alter the ranking of DMUs implied by them. As a result, both the values of the composite indicator and the corresponding ranking of DMUs are affected by the normalization used regardless of the weighting method. Moreover, not all normalization procedures are suitable to alternative weighing schemes: for example, the BoD model is not translation invariant to the min-max and the z-standardized normalizations (Filippetti and Peyrache, 2010) while the z-standardized normalization cannot be used with the Shannon scheme as it results in $\sum_{k=1}^{K} \hat{y}_r^k = 0$.

In addition, and this is important in the context of the proposed weighting scheme, the choice of the normalization procedure may also affect the variability of component indicators across DMUs and this in turn has an impact on the aggregation weights derived by Shannon entropy. For example, using the min-max normalization, i.e., $\hat{y}_r^k = (y_r^k - \min_k y_r^k)/(\max_k y_r^k - \min_k y_r^k)$, one can verify that

$$\bar{y}_r^k = \frac{\hat{y}_r^k}{\sum_{k=1}^{K} \hat{y}_r^k} = \frac{\dfrac{y_r^k - \min\limits_k y_r^k}{\max\limits_k y_r^k - \min\limits_k y_r^k}}{\dfrac{\sum_{k=1}^{K}(y_r^k - \min\limits_k y_r^k)}{\max\limits_k y_r^k - \min\limits_k y_r^k}} = \frac{y_r^k - \min\limits_k y_r^k}{\sum_{k=1}^{K} y_r^k - K \min\limits_k y_r^k}$$

In this case, the minimum could potentially have great influence because it would increase the dispersion of $\bar{y}_r^k$, which in turn would affect the resulting weight m_r for this component indicator.[2] This holds true for both within-sample and out-of-sample minimum values. In the former case, the minimum could be affected by the presence of outliers, which should be removed from the sample when possible. On the other hand, using a distance-to-a-reference normalization, such as $\hat{y}_r^k = y_r^k / \max_k y_r^k$, one can verify that

$$\bar{y}_r^k = \frac{\hat{y}_r^k}{\sum_{k=1}^{K} \hat{y}_r^k} = \frac{\dfrac{y_r^k}{\max\limits_k y_r^k}}{\dfrac{\sum_{k=1}^{K} \hat{y}_r^k}{\max\limits_k y_r^k}} = \frac{y_r^k}{\sum_{k=1}^{K} y_r^k}$$

which indicates that the weights m_r would not be affected by the normalization procedure. In other words, proceeding with either y_r^k or $\hat{y}_r^k$ would result in the same m_r.

Last but not least, the proposed weighting scheme can also be extended to account for some degree of heterogeneity across DMUs in that it may be applied to

[2] We would like to thank a referee for raising this point.

homogeneous sub-groups of DMUs. In this case, it delivers common within (but different between) group weights. This extension may provide useful insights and information regarding the composite indicator as long as these sub-groups can be identified at the outset.

Recently, Shannon entropy has been used in the construction of composite indicators but in a completely different context than the one employed in this paper. In particular, Nissi and Sarra (2018), inspired by the work of Soleimani-damaneth and Zarepisheh (2009) and Xie *et al*. (2014) within a DEA framework, use the BoD model to estimate alterative values of a composite indicator using sequentially all possible subsets of component indicators. They then apply Shannon entropy to compute a weighted average of the alternative composite indicators. In contrast, in this paper Shannon entropy is applied directly for the construction of a composite indicator from the whole set of component indicators considered by the underlying theoretical framework. To the best of our knowledge this is the first application of Shannon entropy for such a purpose.

4. Empirical Results

We next use the proposed weighting scheme to re-estimate the HDI for 2012, which in UN (2013) publications is obtained using equal weights. It is well known that the HDI is a composite of three component indicators associated with national achievements in education, health, and per capita gross national income. In particular, two education-related variables are considered, namely, the mean years of adults' schooling (*MeanSY*) and the expected years of schooling for children (*ExpSY*). Then the component indicator related to education (*EducSY*) is given by arithmetic means of the two normalized primary education indicators (*MeanSY* and *ExpSY*). The other two component indicators are per capital Gross National Income (*GNIpc*) in 2005 PPP $ and life expectancy at birth (*Lexp*) in years. The relevant data are taken from UNDP's Human Development Report 2013.

For the normalization of the component indicators we follow UNDP and use the min-max transformation, even thought other transformations (such as distance to a reference or standardization) may have been used. In the HDI literature, there are two options for the above minimum and maximum values. The UNDP uses out-of-sample values, which are called "natural zeros" for the minimum values and "aspiration targets" for the maximum values while one can also use within-sample minimum and maximum values. According to UNDP Human Development Report (2013), the minimum and the maximum values of *MeanSY* are taken to be 0 and 18 years and for *ExpSY* are 0 and 15 years. The minimum education of 0 years refers to societies without formal education. The maximum for expected years of schooling is equivalent to a master's degree in most countries. On the other hand, maximum for mean years of schooling is the projected maximum of this indicator for 2025.

The minimum and the maximum values related to material standards of living are respectively 100 and 75,000 (2011 PPP$). The minimum value for GNIpc ($100) is justified by the considerable amount of unmeasured subsistence and nonmarket production in low-income economies, which is not captured in the official data. Since each normalized indicator is considered as a proxy for capabilities in the corresponding dimension, the transformation function from income to capabilities is likely to be concave; that is, each additional dollar of income has a smaller effect on expanding capabilities. Thus, for income, the natural logarithm of the actual, minimum and maximum values is used.

Lastly, the minimum and maximum values of life expectancy at birth are set at 20 and 85 years, respectively. The reasoning for placing the "natural zero" for life expectancy at 20 years is based on historical evidence that no country in 20th century had a life expectancy of less than 20 years. The main advantage of these out-of-sample minimum and maximum values for the proposed weighting scheme is that the lower bound of the normalized values of the component indicators never reach zero and thus there is no problem with the calculation of each component indicator's entropy value.

In contrast, if one uses within-sample minimum and maximum values then the normalized values of the component indicators, namely $\hat{y}_r^k$, lie in the $[0, 1]$ interval. For the proposed weighting scheme it is necessary however to set the lower limit to be equal to an arbitrarily small number in order to be able to compute the entropy value of each component indicator. On the other hand, to examine the sensitivity of our results to the choice of the normalization procedure, we provide estimates of the HDI using a distance-to-a-reference normalization, where the reference is determined by the within-sample maximum value of each component indicator.

Next we present empirical estimates of the HDI based on Shannon entropy and the aforementioned min-max transformation of the component indicators using out-of-sample (labeled as Shannon-1) and within-sample (labeled as Shannon-2) minimum and maximum values as well as the distance to the maximum value transformation (labeled as Shannon-3). The country-level results along with the HDI based on EW and the out-of-sample min-max normalization, country ranks and rank difference are reported in Table A1 in the Appendix while the relevant results are summarized in terms of the estimated kernel densities in Figures 2 and 3. We should mention here that the EW HDI reported by UN is built as a geometric weighted average while our estimates of the HDI using Shannon weighting scheme are built as an arithmetic weighted average. We do not consider this to be an issue in the comparisons that follow as the geometric weighted average can be approximated by the arithmetic weighted average by means of a first-order Taylor approximation around one (see e.g. Färe and Zelenyuk, 2005), which is natural for the normalized component indicators used for constructing the HDI.

[Figure 2]
[Figure 3]

The application of Shannon (1948) entropy implies unequal weights for the component indicators, which are 0.397 for *EducS*Y, 0.428 for *GNIpc* and 0.175 for *Lexp* when the UN out-of-sample minimum and maximum values are used and 0.287 for *EducS*Y, 0.311 for *GNIpc* and 0.402 for *Lexp* when the within-sample minimum and maximum values are used (see Table 1). In the former case, *EducSY* and *GNIpc* are weighted more compare to the EW case and the opposite is true for *Lexp* while in the later case, *GNIpc* and *Lexp* are weighted more compared to the EW case and the opposite is true for *EducS*Y. The common weights derived by Shannon (1948) entropy do not change much, as it can be seen from Figure 1, the shape of the HDI kernel density compared to the EW scheme but there are some differences in ranking (see Table A1). In fact, the EW and the common weights schemes with the out-of-sample min-max normalization seem very similar in terms of kernel densities and exhibit high correlation and Spearman's rank correlation coefficients (see Tables 2 and 3). On the other hand, the within-sample min-max normalization seems to induce a downward shift in the HDI kernel density implying lower values of the resulting composite indicator but without affecting the ranking of countries significantly (see Table 3).

[Table 1]
[Table 2]

[Table 3]

To examine further the changes in ranking, we compute the average shift in countries' ranks, which is given as (Saisana, Saltelli and Tarantola, 2005): $R = (\frac{1}{K})\sum_{k=1}^{K}(rank_A(y^k) - rank_B(y^k))$. The relevant results are reported in Table 4. From there we can see that the average shift in countries' rank between the EW scheme and Shannon-1 is almost three positions (2.81) and that between the EW scheme and Shannon-2 a little more than five positions (5.27). Moreover, from the results reported in Table A1, we can see that there is only one difference in top 10 and bottom 10 countries between the EW scheme and Shannon-1 while this difference increases to two when we compare EW and Shannon-2. In the former case, there are relatively large changes in rank position (i.e., more than ten) for five countries while this number increases to twenty one in the latter case.

[Table 4]

In Table 1 we also report the resulting weights when components indicators are normalized using a distance to within-sample maximum value. In this case, *EducS*Y is weighted the most and then *GNIpc* and *Lexp*, with the weights being 0.700, 0.155 and 0.145, respectively. Even though these weights are quite different from those obtained with the within-sample min-max normalization, they do not seem to affect much the shape of the HDI kernel density. However, we may notice (see Figure 3) an upward shift in the left-hand side and at the same time a downward shift in the right-hand side of the distribution. Nevertheless, the two estimates of the HDI exhibit relatively high correlation (0.948) and rank correction (0.943) coefficients (see Table 3) despite the fact that the average change in countries' rank is fourteen. Thus, at least in the present application, the choice of the normalization procedure affects the values of the estimated composite indicator but it has a smaller effect on ranking.

We also estimate the HDI assuming different weights among regions and income groups. The country-level results reported in Table A2 in the Appendix are based on two within-sample normalizations of the component indicators, namely the min-max and the distance to maximum value. Using these data, we apply the four steps procedure described in the previous section to obtain the weights reported in Table 1, which are labeled as Shannon-2 Regional, Shannon-2 by Income Level, Shannon-3 Regional and Shannon-3 by Income Level. We consider seven regions, namely, East Asia and Pacific (27 countries), Europe and Central Asia (50 countries), South Asia (8 countries), Latin America and Caribbean (33 countries), North America (2 countries), Middle East and North Africa (21 countries), and Sub-Saharan Africa (46 countries). In terms of income classes, we consider four groups of countries: low, lower-middle, upper-middle and high-income economies. According to World Bank Country and Lending Groups (2017), as low-income economies are classified those with a GNI per capita of $1,005 or less in 2016 (16 countries); as lower-middle-income economies those with $1,006 and $3,955 (48 countries); as upper middle-income economies those with $3,956 and $12,235 (55 countries); and as high-income economies those with $12,236 or more (68 countries).

From the results reported in Table 1 we see that there is significant between-groups variation in the resulting weights, regardless of whether these groups are defined in terms of geographical regions or income classes. Despite these variations in aggregation weights, the differences in the distribution of the HDI estimated values are however less pronounced as it can be seen be comparing Figures 3 and 4. The use of group-specific weights, i.e., weights that are common within a regional or an income class but different across groups or classes, does alter the estimated kernel density compared to the case of common weights across all countries but not as much as to reduce the high correlation between the estimated composite indicators (see Table 2) or to induce significant differences in ranking (see Table 3). This seems to be true regardless of the choice of the normalization procedure.

[Figure 4]
[Figure 5]

Lastly, we provide some comparative results with previous studies using the BoD model with common weights (see Sayed *et al*., 2015, 2018). The estimated weights are obtained by the goal-programming approach introduced by Despotis (2005a, b) and the meta-goal programming approach suggested by Sayed *et al*. (2015). The relevant results, taken from Sayed *et al.* (2015, 2018), are plotted against our estimates, namely Shaanon-2 and Shannon-3, in Figure 5. From there we can see the rather close relation of Shannon-2 and the HDI based on goal programming BoD, in the terms of both correlation and ranking (see Tables 2 and 3). In contrast, the meta-goal programming approach results in a shift of the HDI kernel density to the left implying a lower mean value. However, there are no significant differences in ranking (see Table 3).

5. Concluding Remarks

In this paper we propose a weighting scheme for constructing composite indicators based on Shannon entropy. Its main advantages are that it delivers a set of common weights across decision-making units that allow for complete comparison and ranking and that it is easy to implement. It can also be extended to provide group specific weights that are common within each group but vary across groups. This represents another advantage of the proposed method. In an empirical application, we explore the potential of the proposed weighing method by using it to re-estimate the HDI. As with other weighting methods, the proposed weighting scheme is sensitive to the choice of the normalization procedure used for the component indicators as the min-max normalization affects the dispersion of $\bar{y}_r^k$ and thus the resulting weights while the distance-to-a-reference normalization does not. In addition, the z-standardized normalization cannot be used with the proposed weighting scheme as $\sum_{k=1}^{K} \hat{y}_r^k = 0$. However, as it provides a higher discrimination in performance terms it should be preferred to the EW scheme that is currently employed in the construction of the HDI.

Table 1. Shannon-based weights of HDI component indicators

	EducSY	GNIpc	Lexp
Shannon -1 (all countries)	0.397	0.428	0.175
Shannon -2 (all countries)	0.287	0.311	0.402
Shannon -3 (all countries)	0.700	0.155	0.145
Shannon -2 Regional			
East Asia and Pacific	0.426	0.374	0.200
Europe and Central Asia	0.172	0.508	0.319
South Asia	0.278	0.218	0.504
Latin America and Caribbean	0.366	0.444	0.189
North America	0.631	0.090	0.279
Middle East and North Africa	0.447	0.383	0.170
Sub-Saharan Africa	0.201	0.295	0.504
Shannon -2 by Income Level			
Low income economies	0.202	0.202	0.595
Lower-middle income economies	0.358	0.085	0.557
Upper-middle income economies	0.295	0.081	0.624
High-income economies	0.335	0.122	0.543
Shannon -3 Regional			
East Asia and Pacific	0.772	0.153	0.075
Europe and Central Asia	0.462	0.351	0.187
South Asia	0.724	0.095	0.181
Latin America and Caribbean	0.724	0.195	0.081
North America	0.866	0.040	0.094
Middle East and North Africa	0.759	0.179	0.061
Sub-Saharan Africa	0.737	0.152	0.111
Shannon -3 by Income Level			
Low income economies	0.852	0.033	0.115
Lower-middle income economies	0.828	0.027	0.145
Upper-middle income economies	0.742	0.041	0.217
High-income economies	0.711	0.078	0.211

Table 2. Pearson's Correlation Coefficients

	EW	Shannon-1	Shannon-2	Shannon-3	Shannon-2 Reg	Shannon-3 Reg	Shannon-2 Inc	Shannon-3 Inc	MGP-BoD	GP-BoD
EW	1.000									
Shannon-1	0.997	1.000								
Shannon-2	0.990	0.983	1.000							
Shannon-3	0.973	0.970	0.948	1.000						
Shannon-2 Reg	0.953	0.946	0.963	0.907	1.000					
Shannon-3 Reg	0.933	0.930	0.907	0.956	0.941	1.000				
Shannon-2 Inc	0.938	0.921	0.959	0.895	0.984	0.922	1.000			
Shannon-3 Inc	0.938	0.934	0.914	0.961	0.939	0.994	0.932	1.000		
MGP-BoD	0.973	0.978	0.966	0.960	0.932	0.920	0.907	0.921	1.000	
GP-BoD	0.995	0.990	0.999	0.958	0.962	0.917	0.954	0.923	0.972	1.000

Table 3. Spearman’s Rank Correlation Coefficient

	Spearman's rho	Prob > |t|	Decision
EW vs Shannon-1	0.997	0.000	Rejected
EW vs Shannon-2	0.992	0.000	Rejected
EW vs Shannon-3	0.966	0.000	Rejected
Shannon-1 vs Shannon-2	0.986	0.000	Rejected
Shannon-1 vs Shannon-3	0.963	0.000	Rejected
Shannon-2 vs Shannon-3	0.943	0.000	Rejected
Shannon-2 vs Shannon-2 Reg	0.961	0.000	Rejected
Shannon-2 vs Shannon-2 Inc	0.958	0.000	Rejected
Shannon-3 vs Shannon-3 Reg	0.955	0.000	Rejected
Shannon-3 vs Shannon-3 Inc	0.962	0.000	Rejected
Shannon-2 Reg vs Shannon-3 Reg	0.941	0.000	Rejected
Shannon-2 Inc vs Shannon-3 Inc	0.934	0.000	Rejected
Shannon-2 vs MGP-BoD	0.980	0.000	Rejected
Shannon-2 vs GP-BoD	0.999	0.000	Rejected

Table 4. Average shifts in countries' ranks

	Shannon-1	Shannon-2	Shannon-3
EW	2.81	5.27	10.34
Shannon-1		6.35	10.49
Shannon-2			14.00

Figure 1. Performance Assessment using the EW, Shannon and BoD Weighting Schemes

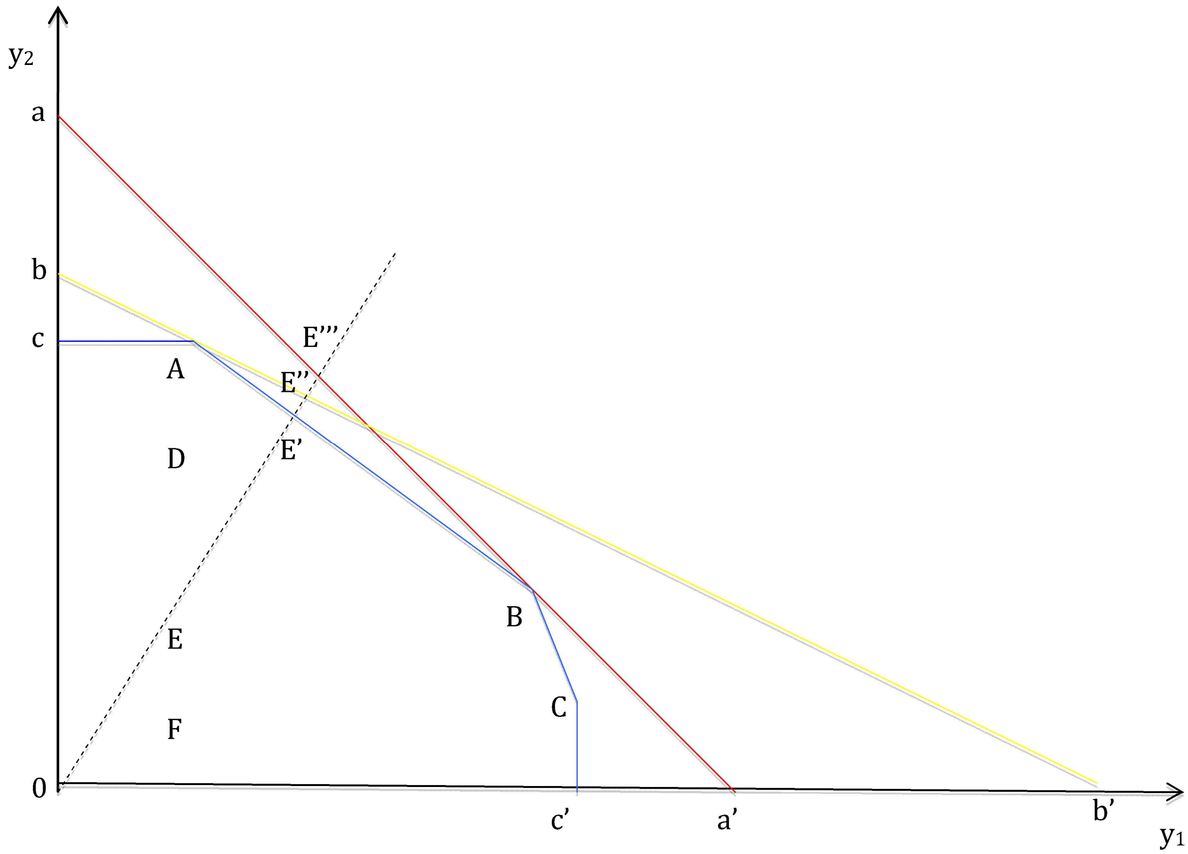


Figure 2. Kernel density for EW, Shannon-1 and Shannon-2 HDI, 2012

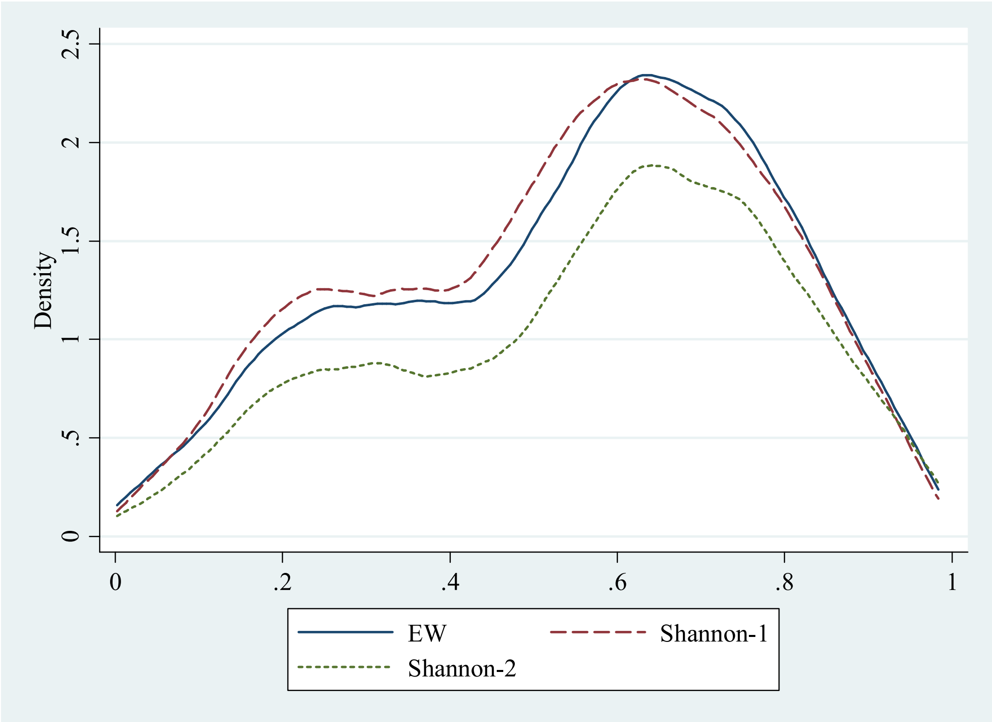


Figure 3. Kernel density for Shannon-2 and Shannon-3 HDI, 2012

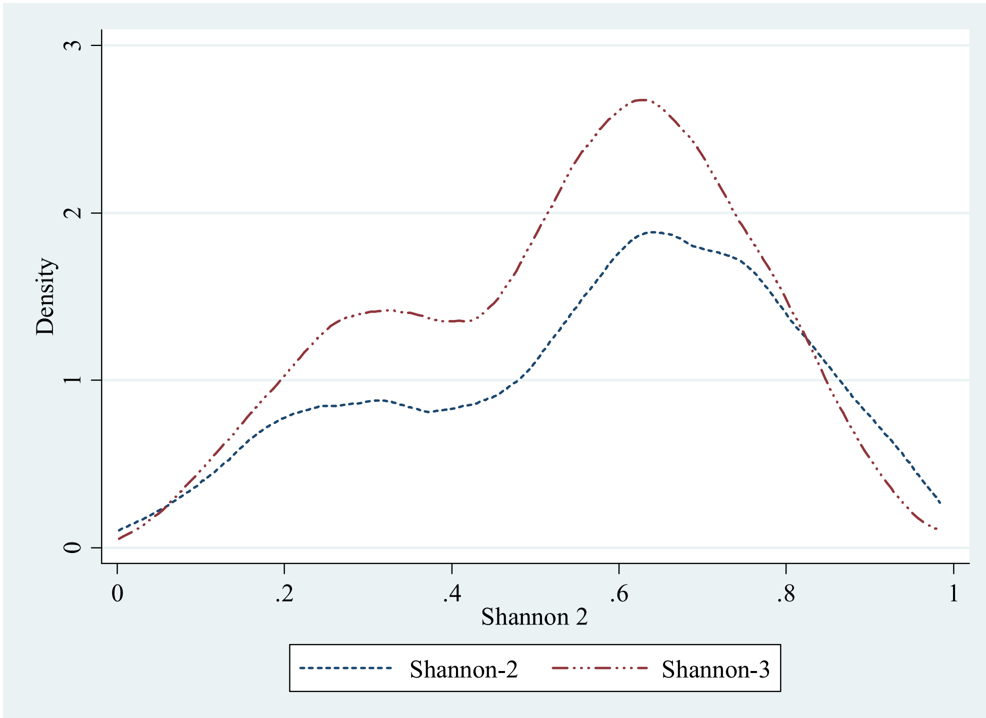


Figure 4. Kernel density for Shannon-2 and Shannon-3 by region and by income level HDI, 2012

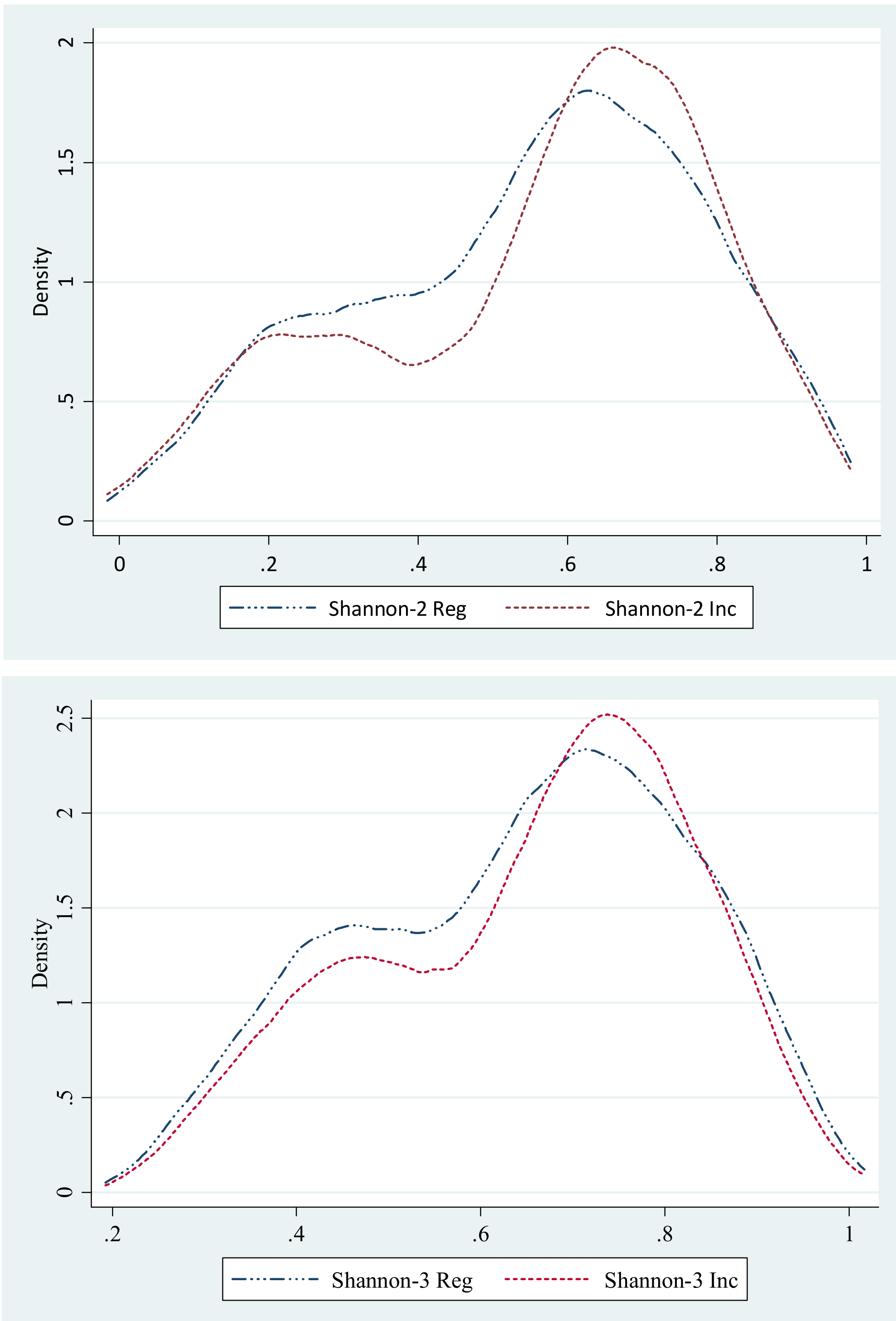
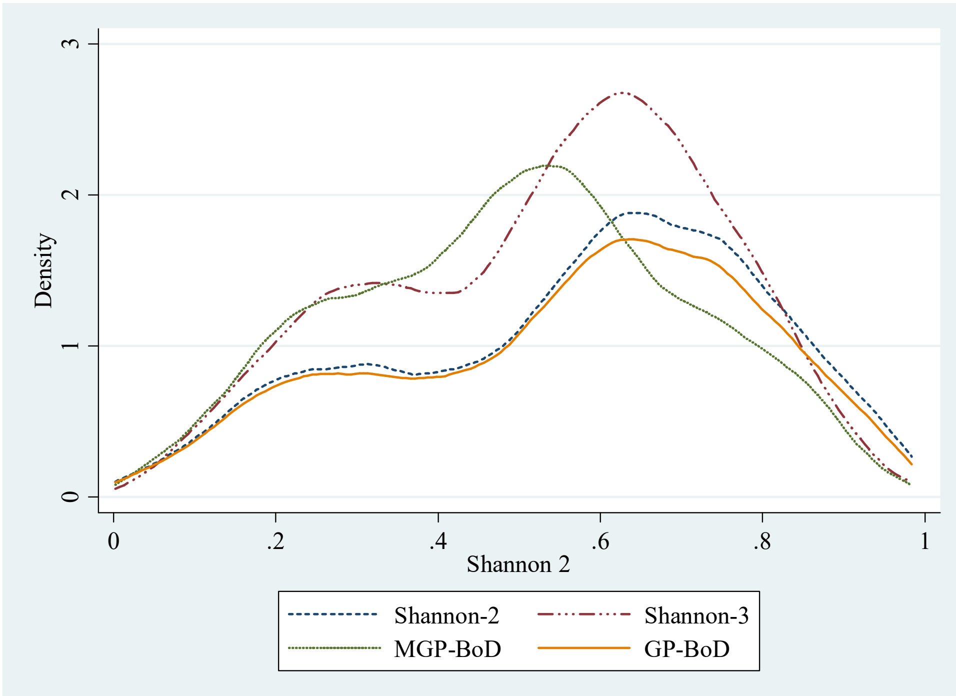


Figure 5. Kernel density for Shannon-2, Shannon-3, MGP-BoD and GP-BoD HDI, 2012



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Appendix

Table A1. HDI, country ranks and rank differences, 2012

	Countries	EW	Country Rank	Shannon -1	Country Rank	Shannon -2	Country Rank	Shannon - 3	Country Rank	Rank difference (2)-(4)	Rank difference (2)-(6)	Rank difference (4)-(6)	Rank difference (6)-(8)
1	Afghanistan	0.374	175	0.358	176	0.131	182	0.405	177	-1	-7	-6	5
2	Albania	0.749	70	0.699	80	0.680	67	0.732	69	-10	3	13	-2
3	Algeria	0.713	93	0.673	94	0.623	95	0.690	97	-1	-2	-1	-2
4	Andorra	0.846	33	0.808	34	0.809	28	0.764	55	-1	5	6	-27
5	Angola	0.508	148	0.510	140	0.284	158	0.510	151	8	-10	-18	7
6	Antigua and Barbuda	0.760	67	0.726	68	0.663	70	0.727	72	-1	-3	-2	-2
7	Argentina	0.811	45	0.777	46	0.737	43	0.794	44	-1	2	3	-1
8	Armenia	0.729	87	0.684	88	0.642	84	0.747	63	-1	3	4	21
9	Australia	0.938	2	0.919	2	0.914	1	0.949	2	0	1	1	-1
10	Austria	0.895	18	0.857	21	0.851	18	0.840	24	-3	0	3	-6
11	Azerbaijan	0.734	82	0.699	81	0.624	93	0.748	62	1	-11	-12	31
12	Bahamas	0.794	49	0.765	50	0.725	48	0.718	76	-1	1	2	-28
13	Bahrain	0.796	48	0.761	52	0.715	53	0.750	61	-4	-5	-1	-8
14	Bangladesh	0.515	146	0.472	149	0.411	140	0.492	155	-3	6	9	-15
15	Barbados	0.825	38	0.789	41	0.756	40	0.801	38	-3	-2	1	2
16	Belarus	0.793	50	0.767	48	0.680	66	0.816	33	2	-16	-18	33
17	Belgium	0.897	17	0.862	17	0.846	19	0.859	18	0	-2	-2	1
18	Belize	0.702	96	0.652	103	0.631	92	0.682	104	-7	4	11	-12
19	Benin	0.436	166	0.417	166	0.249	164	0.448	166	0	2	2	-2
20	Bhutan	0.538	140	0.551	135	0.464	131	0.515	147	5	9	4	-16
21	Bolivia, Plurinational	0.675	108	0.642	108	0.539	115	0.712	82	0	-7	-7	33

	State of												
22	Bosnia and Herzegovina	0.735	81	0.689	86	0.658	73	0.710	84	-5	8	13	-11
23	Botswana	0.634	119	0.652	104	0.422	136	0.665	109	15	-17	-32	27
24	Brazil	0.730	85	0.695	83	0.645	83	0.695	95	2	2	0	-12
25	Brunei Darussalam	0.855	30	0.832	29	0.802	31	0.774	51	1	-1	-2	-20
26	Bulgaria	0.782	57	0.746	58	0.688	61	0.783	49	-1	-4	-3	12
27	Burkina Faso	0.343	183	0.351	179	0.186	176	0.350	186	4	7	3	-10
28	Burundi	0.355	178	0.353	178	0.143	178	0.446	168	0	0	0	10
29	Cambodia	0.543	138	0.507	143	0.391	141	0.554	135	-5	-3	2	6
30	Cameroon	0.495	150	0.482	147	0.266	163	0.544	138	3	-13	-16	25
31	Canada	0.911	11	0.873	10	0.866	13	0.875	13	1	-2	-3	0
32	Cape Verde	0.586	132	0.564	130	0.536	116	0.558	133	2	16	14	-17
33	Central African Republic	0.352	180	0.327	184	0.106	186	0.388	179	-4	-6	-2	7
34	Chad	0.340	184	0.346	180	0.127	184	0.355	185	4	0	-4	-1
35	Chile	0.819	40	0.774	47	0.764	36	0.785	48	-7	4	11	-12
36	China	0.699	101	0.656	101	0.611	100	0.655	114	0	1	1	-14
37	Colombia	0.719	91	0.681	91	0.634	89	0.685	102	0	2	2	-13
38	Comoros	0.429	169	0.409	168	0.287	157	0.455	165	1	12	11	-8
39	Congo	0.534	142	0.510	141	0.341	149	0.544	137	1	-7	-8	12
40	Congo, Democratic Republic of the	0.304	186	0.292	187	0.072	187	0.406	176	-1	-1	0	11
41	Costa Rica	0.773	62	0.725	69	0.722	49	0.729	71	-7	13	20	-22
42	Cτte d'Ivoire	0.432	168	0.403	169	0.233	169	0.424	171	-1	-1	0	-2
43	Croatia	0.805	47	0.764	51	0.732	45	0.773	52	-4	2	6	-7
44	Cuba	0.780	59	0.733	63	0.729	46	0.811	35	-4	13	17	11
45	Cyprus	0.848	31	0.809	33	0.798	33	0.798	42	-2	-2	0	-9

46	Czech Republic	0.873	28	0.836	28	0.805	30	0.867	15	0	-2	-2	15
47	Denmark	0.901	15	0.871	13	0.845	21	0.877	11	2	-6	-8	10
48	Djibouti	0.445	164	0.420	164	0.268	162	0.408	175	0	2	2	-13
49	Dominica	0.745	72	0.701	78	0.685	63	0.690	99	-6	9	15	-36
50	Dominican Republic	0.702	96	0.662	99	0.615	97	0.659	111	-3	-1	2	-14
51	Ecuador	0.724	89	0.681	89	0.651	78	0.696	91	0	11	11	-13
52	Egypt	0.662	112	0.620	114	0.578	107	0.628	122	-2	5	7	-15
53	El Salvador	0.680	107	0.636	111	0.582	105	0.654	115	-4	2	6	-10
54	Equatorial Guinea	0.554	136	0.591	121	0.353	147	0.508	152	15	-11	-26	-5
55	Eritrea	0.351	181	0.317	185	0.213	172	0.364	184	-4	9	13	-12
56	Estonia	0.846	33	0.815	32	0.761	37	0.860	17	1	-4	-5	20
57	Ethiopia	0.396	173	0.382	173	0.247	166	0.410	173	0	7	7	-7
58	Fiji	0.702	96	0.668	97	0.584	104	0.762	58	-1	-8	-7	46
59	Finland	0.892	21	0.858	19	0.844	24	0.852	20	2	-3	-5	4
60	France	0.893	20	0.853	23	0.854	17	0.847	21	-3	3	6	-4
61	FYROM	0.740	78	0.698	82	0.659	72	0.709	86	-4	6	10	-14
62	Gabon	0.683	106	0.671	95	0.528	119	0.666	108	11	-13	-24	11
63	Gambia	0.439	165	0.422	163	0.273	160	0.432	170	2	5	3	-10
64	Georgia	0.745	72	0.704	77	0.656	74	0.797	43	-5	-2	3	31
65	Germany	0.920	5	0.885	6	0.872	12	0.895	6	-1	-7	-6	6
66	Ghana	0.558	135	0.521	139	0.413	139	0.600	127	-4	-4	0	12
67	Greece	0.860	29	0.819	31	0.809	29	0.829	27	-2	0	2	2
68	Grenada	0.770	63	0.732	64	0.698	58	0.763	56	-1	5	6	2
69	Guatemala	0.581	133	0.553	133	0.500	127	0.536	141	0	6	6	-14
70	Guinea	0.355	178	0.356	177	0.178	177	0.386	180	1	1	0	-3
71	Guinea-Bissau	0.364	176	0.364	175	0.131	181	0.408	174	1	-5	-6	7
72	Guyana	0.636	118	0.589	122	0.522	123	0.639	120	-4	-5	-1	3

73	Haiti	0.456	161	0.416	167	0.302	152	0.467	163	-6	9	15	-11
74	Honduras	0.632	120	0.584	126	0.546	113	0.612	124	-6	7	13	-11
75	Hong Kong, China (SAR)	0.906	13	0.869	14	0.878	7	0.829	28	-1	6	7	-21
76	Hungary	0.831	37	0.799	37	0.744	42	0.841	23	0	-5	-5	19
77	Iceland	0.906	13	0.873	11	0.872	10	0.881	10	2	3	1	0
78	India	0.554	136	0.525	138	0.426	135	0.530	143	-2	1	3	-8
79	Indonesia	0.629	122	0.594	120	0.522	124	0.616	123	2	-2	-4	1
80	Iran, Islamic Republic of	0.742	76	0.707	75	0.651	79	0.714	79	1	-3	-4	0
81	Iraq	0.590	131	0.549	136	0.481	130	0.557	134	-5	1	6	-4
82	Ireland	0.916	7	0.885	7	0.872	11	0.910	5	0	-4	-4	6
83	Israel	0.900	16	0.857	20	0.860	15	0.873	14	-4	1	5	1
84	Italy	0.881	25	0.839	27	0.844	23	0.834	25	-2	2	4	-2
85	Jamaica	0.730	85	0.687	87	0.635	87	0.733	68	-2	-2	0	19
86	Japan	0.912	10	0.867	15	0.884	4	0.864	16	-5	6	11	-12
87	Jordan	0.700	100	0.654	102	0.608	101	0.696	92	-2	-1	1	9
88	Kazakhstan	0.754	69	0.735	62	0.623	94	0.788	46	7	-25	-32	48
89	Kenya	0.519	145	0.493	144	0.327	150	0.581	130	1	-5	-6	20
90	Kiribati	0.629	121	0.587	123	0.505	126	0.647	118	-2	-5	-3	8
91	Korea, Republic of	0.909	12	0.871	12	0.861	14	0.890	7	0	-2	-2	7
92	Kuwait	0.790	54	0.790	40	0.734	44	0.690	96	14	10	-4	-52
93	Kyrgyzstan	0.622	125	0.585	125	0.500	128	0.690	98	0	-3	-3	30
94	Lao People's Democratic Republic	0.543	138	0.507	142	0.429	134	0.524	145	-4	4	8	-11
95	Latvia	0.814	44	0.782	43	0.720	50	0.824	30	1	-6	-7	20
96	Lebanon	0.745	72	0.711	73	0.651	81	0.709	85	-1	-9	-8	-4
97	Lesotho	0.461	158	0.451	152	0.209	173	0.513	148	6	-15	-21	25

98	Liberia	0.388	174	0.369	174	0.215	171	0.473	161	0	3	3	10
99	Libya	0.769	64	0.742	59	0.696	59	0.740	66	5	5	0	-7
100	Liechtenstein	0.883	24	0.864	16	0.846	20	0.775	50	8	4	-4	-30
101	Lithuania	0.818	41	0.790	39	0.717	52	0.824	29	2	-11	-13	23
102	Luxembourg	0.875	26	0.844	24	0.831	26	0.791	45	2	0	-2	-19
103	Madagascar	0.483	151	0.447	153	0.369	146	0.529	144	-2	5	7	2
104	Malawi	0.418	170	0.396	170	0.216	170	0.481	158	0	0	0	12
105	Malaysia	0.769	64	0.729	66	0.682	65	0.733	67	-2	-1	1	-2
106	Maldives	0.688	104	0.647	106	0.633	91	0.630	121	-2	13	15	-30
107	Mali	0.344	182	0.334	183	0.135	179	0.368	183	-1	3	4	-4
108	Malta	0.847	32	0.805	36	0.795	34	0.803	37	-4	-2	2	-3
109	Mauritania	0.467	155	0.442	155	0.292	155	0.448	167	0	0	0	-12
110	Mauritius	0.737	80	0.705	76	0.651	77	0.688	100	4	3	-1	-23
111	Mexico	0.775	61	0.732	65	0.707	54	0.730	70	-4	7	11	-16
112	Micronesia, Federated States of	0.645	117	0.602	119	0.525	122	0.665	110	-2	-5	-3	12
113	Moldova, Republic of	0.660	113	0.619	115	0.543	114	0.696	94	-2	-1	1	20
114	Mongolia	0.675	108	0.641	110	0.554	110	0.706	88	-2	-2	0	22
115	Montenegro	0.791	52	0.753	56	0.705	56	0.799	41	-4	-4	0	15
116	Morocco	0.591	130	0.558	132	0.514	125	0.540	140	-2	5	7	-15
117	Mozambique 185	0.327	185	0.342	181	0.132	180	0.376	181	4	5	1	-1
118	Myanmar	0.498	149	0.466	150	0.374	144	0.486	157	-1	5	6	-13
119	Namibia	0.608	128	0.586	124	0.450	132	0.591	129	4	-4	-8	3
120	Nepal	0.463	157	0.430	162	0.373	145	0.458	164	-5	12	17	-19
121	Netherlands	0.921	4	0.886	5	0.874	9	0.889	8	-1	-5	-4	1
122	New Zealand	0.919	6	0.902	4	0.888	3	0.957	1	2	3	1	2
123	Nicaragua	0.599	129	0.552	134	0.526	120	0.580	131	-5	9	14	-11

124	Niger	0.304	186	0.293	186	0.129	183	0.309	187	0	3	3	-4
125	Nigeria	0.471	153	0.452	151	0.242	168	0.492	156	2	-15	-17	12
126	Norway	0.955	1	0.925	1	0.912	2	0.931	3	0	-1	-1	-1
127	Oman	0.731	84	0.719	70	0.660	71	0.649	117	14	13	-1	-46
128	Pakistan	0.515	146	0.478	148	0.385	143	0.479	159	-2	3	5	-16
129	Palau	0.791	52	0.759	53	0.687	62	0.817	32	-1	-10	-9	30
130	Palestine, State of	0.670	110	0.625	112	0.578	106	0.688	101	-2	4	6	5
131	Panama	0.780	59	0.739	60	0.706	55	0.744	64	-1	4	5	-9
132	Papua New Guinea	0.466	156	0.437	158	0.326	151	0.421	172	-2	5	7	-21
133	Paraguay	0.669	111	0.623	113	0.574	109	0.658	113	-2	2	4	-4
134	Peru	0.741	77	0.700	79	0.654	75	0.717	78	-2	2	4	-3
135	Philippines	0.654	114	0.613	117	0.532	117	0.674	107	-3	-3	0	10
136	Poland	0.821	39	0.786	42	0.747	41	0.799	40	-3	-2	1	1
137	Portugal	0.816	43	0.781	44	0.773	35	0.760	59	-1	8	9	-24
138	Qatar	0.834	36	0.826	30	0.800	32	0.700	89	6	4	-2	-57
139	Romania	0.786	56	0.747	57	0.695	60	0.787	47	-1	-4	-3	13
140	Russian Federation	0.788	55	0.766	49	0.666	69	0.812	34	6	-14	-20	35
141	Rwanda	0.434	167	0.418	165	0.242	167	0.473	160	2	0	-2	7
142	Saint Kitts and Nevis	0.745	72	0.709	74	0.653	76	0.706	87	-2	-4	-2	-11
143	Saint Lucia	0.725	88	0.681	90	0.642	85	0.696	93	-2	3	5	-8
144	Saint Vincent and the Grenadines	0.733	83	0.695	84	0.634	88	0.713	80	-1	-5	-4	8
145	Samoa	0.702	96	0.659	100	0.606	102	0.741	65	-4	-6	-2	37
146	Sao Tome and Principe	0.525	144	0.491	145	0.389	142	0.531	142	-1	2	3	0
147	Saudi Arabia	0.782	57	0.757	54	0.701	57	0.724	74	3	0	-3	-17

148	Senegal	0.470	154	0.438	157	0.295	153	0.468	162	-3	1	4	-9
149	Serbia	0.769	64	0.727	67	0.682	64	0.764	54	-3	0	3	10
150	Seychelles	0.806	46	0.777	45	0.717	51	0.766	53	1	-5	-6	-2
151	Sierra Leone	0.359	177	0.341	182	0.108	185	0.392	178	-5	-8	-3	7
152	Singapore	0.895	18	0.862	18	0.857	16	0.811	36	0	2	2	-20
153	Slovakia	0.840	35	0.807	35	0.759	38	0.832	26	0	-3	-3	12
154	Slovenia	0.892	21	0.856	22	0.837	25	0.883	9	-1	-4	-3	16
155	Solomon Islands	0.530	143	0.491	146	0.418	137	0.507	153	-3	6	9	-16
156	South Africa	0.629	122	0.642	109	0.416	138	0.674	106	13	-16	-29	32
157	Spain	0.885	23	0.844	25	0.844	22	0.845	22	-2	1	3	0
158	Sri Lanka	0.715	92	0.667	98	0.634	90	0.717	77	-6	2	8	13
159	Sudan	0.414	171	0.392	171	0.275	159	0.371	182	0	12	12	-23
160	Suriname	0.684	105	0.646	107	0.576	108	0.654	116	-2	-3	-1	-8
161	Swaziland	0.536	141	0.544	137	0.291	156	0.578	132	4	-15	-19	24
162	Sweden	0.916	7	0.878	8	0.874	8	0.877	12	-1	-1	0	-4
163	Switzerland	0.913	9	0.875	9	0.880	5	0.856	19	0	4	4	-14
164	Syrian Arab Republic	0.648	116	0.604	118	0.586	103	0.605	125	-2	13	15	-22
165	Tajikistan	0.622	125	0.583	127	0.496	129	0.684	103	-2	-4	-2	26
166	Tanzania, United Republic of	0.476	152	0.442	154	0.293	154	0.497	154	-2	-2	0	0
167	Thailand	0.690	103	0.650	105	0.611	99	0.643	119	-2	4	6	-20
168	Timor-Leste	0.576	134	0.561	131	0.431	133	0.550	136	3	1	-2	-3
169	Togo	0.459	159	0.432	160	0.272	161	0.521	146	-1	-2	-1	15
170	Tonga	0.710	95	0.670	96	0.613	98	0.754	60	-1	-3	-2	38
171	Trinidad and Tobago	0.760	67	0.737	61	0.651	80	0.712	83	6	-13	-19	-3
172	Tunisia	0.712	94	0.677	92	0.638	86	0.681	105	2	8	6	-19
173	Turkey	0.722	90	0.692	85	0.646	82	0.659	112	5	8	3	-30
174	Turkmenistan	0.698	102	0.673	93	0.550	112	0.719	75	9	-10	-19	37

175	Uganda	0.456	161	0.436	159	0.248	165	0.512	149	2	-4	-6	16
176	Ukraine	0.740	78	0.713	71	0.618	96	0.799	39	7	-18	-25	57
177	United Arab Emirates	0.818	41	0.794	38	0.757	39	0.726	73	3	2	-1	-34
178	United Kingdom	0.875	26	0.842	26	0.830	27	0.820	31	0	-1	-1	-4
179	United States	0.937	3	0.912	3	0.878	6	0.930	4	0	-3	-3	2
180	Uruguay	0.792	51	0.754	55	0.727	47	0.762	57	-4	4	8	-10
181	Uzbekistan	0.654	114	0.615	116	0.531	118	0.698	90	-2	-4	-2	28
182	Vanuatu	0.626	124	0.582	128	0.525	121	0.601	126	-4	3	7	-5
183	Venezuela, Bolivarian Republic of	0.748	71	0.713	72	0.668	68	0.713	81	-1	3	4	-13
184	Viet Nam	0.617	127	0.572	129	0.554	111	0.596	128	-2	16	18	-17
185	Yemen	0.458	160	0.440	156	0.353	148	0.437	169	4	12	8	-21
186	Zambia	0.448	163	0.430	161	0.198	174	0.511	150	2	-11	-13	24
187	Zimbabwe	0.397	172	0.388	172	0.192	175	0.543	139	0	-3	-3	36

Table A2. HDI with varying regional and income-class weights and country ranks, 2012

	Countries	Shannon-2 Reg	Country Rank	Shannon-2 Inc	Country Rank	Shannon-3 Reg	Country Rank	Shannon-3 Inc	Country Rank
1	Afghanistan	0.113	183	0.098	183	0.397	174	0.362	176
2	Albania	0.655	72	0.735	48	0.763	56	0.737	67
3	Algeria	0.588	97	0.656	90	0.673	104	0.691	95
4	Andorra	0.826	25	0.813	32	0.821	36	0.765	57
5	Angola	0.258	161	0.197	171	0.503	150	0.488	151
6	Antigua and Barbuda	0.653	73	0.663	85	0.721	73	0.727	72
7	Argentina	0.718	45	0.752	44	0.788	52	0.797	44
8	Armenia	0.607	88	0.695	68	0.763	57	0.755	61
9	Australia	0.904	2	0.936	1	0.946	2	0.953	2
10	Austria	0.855	12	0.858	19	0.872	18	0.841	24
11	Azerbaijan	0.611	86	0.639	99	0.768	55	0.749	63
12	Bahamas	0.709	48	0.712	59	0.712	80	0.716	77
13	Bahrain	0.691	52	0.715	57	0.737	67	0.750	62
14	Bangladesh	0.441	132	0.453	133	0.492	153	0.458	156
15	Barbados	0.735	39	0.771	39	0.794	47	0.804	40
16	Belarus	0.672	58	0.680	78	0.823	35	0.816	34
17	Belgium	0.845	15	0.854	21	0.882	14	0.861	18
18	Belize	0.573	101	0.697	67	0.668	106	0.688	100
19	Benin	0.247	163	0.242	164	0.437	167	0.413	167
20	Bhutan	0.470	125	0.473	131	0.508	147	0.504	142
21	Bolivia, Plurinational State of	0.538	109	0.553	124	0.707	85	0.715	79
22	Bosnia and Herzegovina	0.639	76	0.706	64	0.747	64	0.713	80
23	Botswana	0.377	143	0.343	146	0.664	107	0.650	116

24	Brazil	0.619	83	0.669	81	0.686	94	0.693	93
25	Brunei Darussalam	0.777	32	0.785	36	0.758	59	0.771	51
26	Bulgaria	0.675	57	0.707	62	0.801	44	0.784	49
27	Burkina Faso	0.197	173	0.172	175	0.334	186	0.299	186
28	Burundi	0.125	180	0.124	180	0.438	166	0.419	165
29	Cambodia	0.378	142	0.410	141	0.534	137	0.530	137
30	Cameroon	0.238	166	0.236	166	0.539	135	0.521	140
31	Canada	0.844	16	0.878	15	0.860	23	0.878	14
32	Cape Verde	0.573	100	0.578	115	0.542	133	0.524	139
33	Central African Republic	0.091	185	0.081	185	0.377	179	0.346	179
34	Chad	0.119	182	0.088	184	0.341	185	0.306	185
35	Chile	0.723	43	0.791	34	0.776	54	0.791	45
36	China	0.570	103	0.643	98	0.634	116	0.653	113
37	Colombia	0.601	92	0.664	84	0.676	101	0.684	101
38	Comoros	0.301	152	0.317	151	0.442	165	0.415	166
39	Congo	0.330	147	0.321	149	0.536	136	0.515	141
40	Congo, Democratic Republic of the	0.054	187	0.056	187	0.398	173	0.380	172
41	Costa Rica	0.666	63	0.778	37	0.717	77	0.732	71
42	Cτte d'Ivoire	0.725	41	0.748	46	0.805	42	0.776	50
43	Croatia	0.670	60	0.813	31	0.800	45	0.827	29
44	Cuba	0.796	29	0.812	33	0.834	32	0.801	42
45	Cyprus	0.791	31	0.819	28	0.876	16	0.870	15
46	Czech Republic	0.235	168	0.216	168	0.411	170	0.382	171
47	Denmark	0.842	19	0.851	23	0.893	10	0.879	12
48	Djibouti	0.251	162	0.243	163	0.383	177	0.358	177
49	Dominica	0.635	78	0.728	52	0.678	100	0.689	98

50	Dominican Republic	0.581	99	0.644	97	0.648	112	0.656	110
51	Ecuador	0.605	90	0.700	65	0.685	96	0.699	90
52	Egypt	0.522	112	0.624	103	0.605	122	0.628	121
53	El Salvador	0.546	108	0.619	106	0.644	115	0.655	112
54	Equatorial Guinea	0.326	148	0.238	165	0.499	151	0.485	152
55	Eritrea	0.243	165	0.271	157	0.346	184	0.312	184
56	Estonia	0.745	37	0.772	38	0.863	21	0.862	17
57	Ethiopia	0.262	160	0.264	160	0.395	175	0.371	174
58	Fiji	0.589	95	0.618	107	0.757	60	0.770	52
59	Finland	0.842	20	0.853	22	0.877	15	0.854	20
60	France	0.847	14	0.871	16	0.874	17	0.851	21
61	FYROM	0.648	74	0.693	69	0.749	63	0.709	86
62	Gabon	0.514	114	0.490	130	0.661	109	0.659	109
63	Gambia	0.282	157	0.267	159	0.418	169	0.390	170
64	Georgia	0.608	87	0.711	60	0.793	50	0.809	37
65	Germany	0.865	9	0.883	12	0.908	6	0.897	6
66	Ghana	0.416	136	0.451	134	0.592	125	0.587	127
67	Greece	0.794	30	0.832	26	0.852	25	0.834	26
68	Grenada	0.664	65	0.741	47	0.755	61	0.768	54
69	Guatemala	0.448	129	0.542	126	0.520	141	0.531	136
70	Guinea	0.179	174	0.178	173	0.373	180	0.336	180
71	Guinea-Bissau	0.111	184	0.101	182	0.399	172	0.372	173
72	Guyana	0.485	122	0.559	121	0.628	120	0.622	122
73	Haiti	0.265	158	0.334	148	0.453	164	0.437	162
74	Honduras	0.486	121	0.592	111	0.597	123	0.589	126
75	Hong Kong, China (SAR)	0.836	21	0.885	9	0.813	39	0.830	28
76	Hungary	0.729	40	0.755	43	0.848	27	0.844	23

77	Iceland	0.857	11	0.895	5	0.896	9	0.885	10
78	India	0.435	133	0.433	136	0.526	140	0.495	146
79	Indonesia	0.492	119	0.556	122	0.596	124	0.616	123
80	Iran, Islamic Republic of	0.627	80	0.668	82	0.700	88	0.712	81
81	Iraq	0.430	135	0.504	129	0.532	138	0.525	138
82	Ireland	0.853	13	0.892	6	0.913	5	0.914	5
83	Israel	0.825	26	0.884	10	0.863	20	0.878	13
84	Italy	0.834	23	0.866	18	0.863	22	0.839	25
85	Jamaica	0.606	89	0.673	80	0.724	72	0.737	68
86	Japan	0.843	18	0.906	4	0.851	26	0.868	16
87	Jordan	0.570	102	0.657	89	0.679	98	0.701	89
88	Kazakhstan	0.616	84	0.607	109	0.796	46	0.787	48
89	Kenya	0.311	149	0.338	147	0.576	129	0.569	130
90	Kiribati	0.487	120	0.539	127	0.631	117	0.633	119
91	Korea, Republic of	0.843	17	0.880	13	0.884	13	0.894	7
92	Kuwait	0.709	47	0.693	71	0.674	102	0.682	103
93	Kyrgyzstan	0.449	128	0.555	123	0.696	90	0.690	96
94	Lao People's Democratic Republic	0.385	141	0.456	132	0.497	152	0.492	147
95	Latvia	0.708	49	0.729	51	0.834	33	0.826	30
96	Lebanon	0.630	79	0.654	92	0.695	91	0.710	84
97	Lesotho	0.174	175	0.166	177	0.508	146	0.488	150
98	Liberia	0.214	171	0.232	167	0.463	160	0.449	159
99	Libya	0.670	59	0.707	63	0.726	70	0.742	65
100	Liechtenstein	0.898	3	0.814	29	0.847	29	0.769	53
101	Lithuania	0.711	46	0.717	56	0.836	31	0.824	31
102	Luxembourg	0.857	10	0.821	27	0.846	30	0.790	46

103	Madagascar	0.389	140	0.422	139	0.517	143	0.503	144
104	Malawi	0.206	172	0.209	169	0.472	158	0.454	157
105	Malaysia	0.659	71	0.690	74	0.717	76	0.735	69
106	Maldives	0.660	69	0.689	75	0.629	119	0.630	120
107	Mali	0.132	179	0.126	179	0.355	182	0.317	182
108	Malta	0.755	34	0.813	30	0.789	51	0.807	38
109	Mauritania	0.299	153	0.278	155	0.435	168	0.406	168
110	Mauritius	0.667	62	0.653	94	0.678	99	0.688	99
111	Mexico	0.669	61	0.726	53	0.720	75	0.734	70
112	Micronesia, Federated States of	0.506	116	0.560	120	0.650	111	0.653	114
113	Moldova, Republic of	0.507	115	0.585	113	0.713	78	0.690	97
114	Mongolia	0.551	106	0.583	114	0.695	92	0.710	85
115	Montenegro	0.682	53	0.735	49	0.811	40	0.803	41
116	Morocco	0.441	131	0.561	119	0.511	145	0.537	135
117	Mozambique 185	0.123	181	0.112	181	0.363	181	0.327	181
118	Myanmar	0.331	146	0.400	143	0.457	162	0.451	158
119	Namibia	0.446	130	0.424	138	0.582	126	0.580	128
120	Nepal	0.411	137	0.430	137	0.459	161	0.423	164
121	Netherlands	0.870	8	0.884	11	0.905	7	0.891	8
122	New Zealand	0.885	5	0.918	2	0.957	1	0.963	1
123	Nicaragua	0.450	126	0.585	112	0.563	132	0.555	132
124	Niger	0.145	178	0.150	178	0.291	187	0.246	187
125	Nigeria	0.222	170	0.207	170	0.484	155	0.460	155
126	Norway	0.909	1	0.918	3	0.939	3	0.932	3
127	Oman	0.627	81	0.637	100	0.630	118	0.645	117
128	Pakistan	0.399	139	0.395	144	0.475	157	0.439	161

129	Palau	0.696	50	0.697	66	0.813	38	0.819	33
130	Palestine, State of	0.538	110	0.633	101	0.670	105	0.679	105
131	Panama	0.676	56	0.722	54	0.735	68	0.747	64
132	Papua New Guinea	0.284	156	0.321	150	0.388	176	0.371	175
133	Paraguay	0.531	111	0.624	105	0.646	113	0.662	107
134	Peru	0.625	82	0.683	76	0.708	84	0.717	76
135	Philippines	0.518	113	0.564	118	0.660	110	0.662	106
136	Poland	0.741	38	0.758	41	0.824	34	0.801	43
137	Portugal	0.770	33	0.790	35	0.805	41	0.764	59
138	Qatar	0.754	35	0.756	42	0.680	97	0.691	94
139	Romania	0.678	55	0.719	55	0.804	43	0.789	47
140	Russian Federation	0.665	64	0.657	87	0.821	37	0.811	35
141	Rwanda	0.235	167	0.245	162	0.463	159	0.445	160
142	Saint Kitts and Nevis	0.635	77	0.657	88	0.698	89	0.707	87
143	Saint Lucia	0.603	91	0.682	77	0.686	95	0.697	91
144	Saint Vincent and the Grenadines	0.616	85	0.653	93	0.705	86	0.712	82
145	Samoa	0.585	98	0.659	86	0.730	69	0.740	66
146	Sao Tome and Principe	0.402	138	0.416	140	0.519	142	0.504	143
147	Saudi Arabia	0.681	54	0.690	73	0.711	81	0.722	74
148	Senegal	0.302	151	0.298	154	0.456	163	0.433	163
149	Serbia	0.663	67	0.715	58	0.786	53	0.768	55
150	Seychelles	0.722	44	0.707	61	0.760	58	0.764	58
151	Sierra Leone	0.089	186	0.073	186	0.382	178	0.349	178
152	Singapore	0.822	27	0.850	24	0.794	48	0.809	36
153	Slovakia	0.753	36	0.767	40	0.848	28	0.834	27
154	Slovenia	0.819	28	0.856	20	0.890	11	0.888	9

155	Solomon Islands	0.366	145	0.450	135	0.477	156	0.472	153
156	South Africa	0.372	144	0.315	152	0.674	103	0.655	111
157	Spain	0.833	24	0.866	17	0.869	19	0.850	22
158	Sri Lanka	0.659	70	0.693	70	0.720	74	0.724	73
159	Sudan	0.303	150	0.270	158	0.352	183	0.316	183
160	Suriname	0.555	105	0.594	110	0.645	114	0.650	115
161	Swaziland	0.247	164	0.186	172	0.576	130	0.557	131
162	Sweden	0.870	7	0.887	8	0.897	8	0.879	11
163	Switzerland	0.881	6	0.891	7	0.887	12	0.858	19
164	Syrian Arab Republic	0.506	117	0.654	91	0.577	128	0.607	124
165	Tajikistan	0.450	127	0.547	125	0.693	93	0.682	102
166	Tanzania, United Republic of	0.293	154	0.304	153	0.486	154	0.469	154
167	Thailand	0.564	104	0.648	95	0.620	121	0.641	118
168	Timor-Leste	0.430	134	0.410	142	0.529	139	0.537	134
169	Togo	0.264	159	0.271	156	0.513	144	0.498	145
170	Tonga	0.597	93	0.666	83	0.744	65	0.764	60
171	Trinidad and Tobago	0.664	66	0.624	104	0.709	83	0.707	88
172	Tunisia	0.593	94	0.676	79	0.662	108	0.681	104
173	Turkey	0.661	68	0.647	96	0.724	71	0.659	108
174	Turkmenistan	0.551	107	0.531	128	0.742	66	0.715	78
175	Uganda	0.232	169	0.250	161	0.504	149	0.491	148
176	Ukraine	0.588	96	0.630	102	0.794	49	0.804	39
177	United Arab Emirates	0.723	42	0.733	50	0.710	82	0.721	75
178	United Kingdom	0.834	22	0.838	25	0.856	24	0.821	32
179	United States	0.890	4	0.878	14	0.928	4	0.931	4
180	Uruguay	0.694	51	0.748	45	0.754	62	0.767	56

181	Uzbekistan	0.496	118	0.570	116	0.712	79	0.693	92
182	Vanuatu	0.481	124	0.570	117	0.577	127	0.602	125
183	Venezuela, Bolivarian Republic of	0.641	75	0.692	72	0.704	87	0.711	83
184	Viet Nam	0.482	123	0.613	108	0.568	131	0.571	129
185	Yemen	0.290	155	0.374	145	0.406	171	0.393	169
186	Zambia	0.167	177	0.171	176	0.507	148	0.490	149
187	Zimbabwe	0.167	176	0.175	174	0.539	134	0.539	133