Green consumers and environmental policy*

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Abstract

The present paper examines how improvements in consumers' environmental awareness influence the choice between output and emission taxes, within a framework of imperfect competition and endogenous choice of abatement level. We first show that in the absence of policy intervention, there exists a level of environmental awareness beyond which welfare is decreasing as market imperfections become more prominent relative to environmental concerns. We also confirm that both output and emission taxes are welfare superior to the free-market case. What is surprising however, is that the welfare performance of an optimally chosen emissions tax is monotonically decreasing in consumers' environmental sensitivity, while the opposite is true for an output tax up to a certain level. At low levels of consumers' environmental awareness an emissions tax is welfare superior, but eventually, there is a level of environmental awareness beyond which an output tax welfare dominates an emissions tax. Therefore, an emissions tax is better suited to societies that have not yet developed high levels of environmental awareness, while societies characterized by high levels of environmental awareness should prefer an output tax.

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1 Introduction

Over the past few decades there has been clear evidence of a continuous increase in consumers' environmental awareness. Based on the emergence of "green consumerism", a substantial literature has been developed examining mainly its effect on private sector's voluntary environmental actions. However, this important behavioral change could also have substantial effects on the design of environmental policies. The present paper addresses this issue by examining the effect that the level of consumer's awareness has on the design of optimal environmental tax policy. We define green consumer's utility function assuming that her choices depend on the product's level of pollution: at higher pollution levels the green consumer decreases her consumption. This assumption is based on increasing evidence that consumers are worried about the level of pollution, they feel well-informed and they believe they can play a role in protecting the environment (see for example Eurobarometer (2014) and (2017)). On the emissions generation side, we assume that firms can engage in end-of-pipe abatement through fixed investments, such as pollution filters and scrapers. Since fixed investments go in pair with imperfect competition, we focus on cases where firms have market power.

Within this framework, we first examine the welfare effect of increasing environmental awareness in the absence of government intervention. We find that increases in consumers' environmental awareness are not monotonically related to social welfare: beyond a certain point, further increases in environmental sensitivity may be welfare decreasing. This apparently strange result is due to the fact that consumers "punish" polluting firms by reducing their consumption in a market where consumption is already too low, due to imperfect competition. Thus, any benefits from further internalizing the externality must be weighted against costs from reducing consumption. This suggests that there is an optimal amount of environmental awareness, which is below the actual marginal environmental damage, a result similar to the adjustment of the Pigouvian tax under imperfect competition.

We then turn to examine the effect of environmental awareness on the optimal choice of environmental taxation: we compare its effect on emissions and output tax. For a given level of investment on abatement, whether the tax is per unit of output or pollution is of little importance, since the relation between the two is fixed. When, however, the choice of such investment is endogenous, the two types of taxes create different incentives for environmental investment and may affect consumption, pollution, and finally total welfare, in very different ways.[1] When consumers' behavior is not affected by environmental considerations, simple intuition suggests that an optimally chosen emissions tax cannot be welfare-inferior to an optimally chosen tax on output, since it aims directly at the desired target, and therefore produces greater incentives for investment in abatement; as a result, the emissions tax provides greater output and cleaner environment. However, in the presence of environmentally aware consumers the

[1] See Crémer and Thisse (1999) and Constantatos and Sartzetakis (1999) on how the taxation per unit of output affects product specification, which is based on fixed costs.

relative performance of the two taxes becomes more complex. We show that increases in consumers' environmental awareness affect negatively the welfare performance of an emissions tax, while on the contrary, they increase welfare when an output tax/subsidy is used. As a result, while for low levels of environmental sensitivity an emissions tax produces higher welfare relative to an output tax as expected, at high such levels the output tax welfare dominates. Intuitively, improvements in consumers' environmental awareness apart from the direct decrease in consumption yield also reductions in the output tax rate which indirectly increase consumption. After a certain level of environmental awareness, the indirect effect on consumption dominates, thus leading to a) greater consumption value, and b) greater incentives for investment in abatement. The above results can be better understood if we consider environmental awareness as an emission reduction instrument, which, despite its voluntary nature, works much more like an emission rather than an output tax. In this respect, it is clear that in the presence of two distortions, environmental awareness complements efficiently the output tax, which can even turn into a subsidy to correct the market distortion when high levels of environmental awareness address effectively the environmental distortion.

The discussion of negative externalities in most textbooks leads to the Pigouvian tax levied either on emission or output, assuming, explicitly or implicitly, that output and emission taxes are equivalent. This equivalence is based on the assumption that the amount of emission produced per unit of output is immutable, ignoring the realistic possibilities of engaging in abatement. Recognizing that a given level of output may yield different levels of emissions, breaks up the equivalence of output and emission taxes. A significant literature has been developed examining the optimal choice of environmental tax instrument in different settings. Schmutzler and Goulder (1997) compare emissions and output taxes in a partial equilibrium framework and in the presence of imperfect monitoring of emissions, while Fullerton et al. (2001) and Cremer and Gahvari (2002) in a general equilibrium framework. Goulder et al. (1997) examine the interactions with pre-existing distortionary taxes. More recently, within a Cournot framework, Aoyama and Silva (2016) compare the effectiveness of output and emission taxation in promoting the adoption of advanced abatement technology. Although the latter paper is closer to our analysis, it does not address the effect of consumers' environmental awareness.

The increasing importance of green consumerism has raised the question of the appropriate adjustments to the traditional environmental tax and subsidy policies and furthermore initiated a discussion regarding the effectiveness of information campaigns and advertising, aiming at increase environmental awareness, as an additional policy instrument (see for example Petrakis et al. (2005), Nyborg et al. (2006), Brouhle and Khanna (2007), Sartzetakis et al. (2012) and more recently Podhorsky (2020) in the form of certification standard).[2] The literature has approached the emergence of green consumers using

[2] In a slightly different setting Marsiglio and Tolotti (2020)consider the case in which individual behavior is determined by social effects and intrinsic motivation and find that a subsidy

different frameworks. Most of the models assume that green consumers differentiate products based on their environmental attributes inducing some firms to produce a "greener" variety of the product. This differentiation has been examined mainly within a framework of vertical differentiation (Bansal and Gangopadhyay (2003), Garcia-Gallego and Georgantzis (2009), Bansal (2008), Deltas et al. (2013) and Doni and Ricchiuti (2013)), and less within a framework of horizontal differentiation (Conrad (2005)). Alternatively, Gil-Molto and Varvarigos (2013) examine the case in which environmental consciousness leads consumers to devote resources to reduce pollution (participation in carbon offsetting schemes, donations to NGOs, etc). Although some of these papers confirm our first result, showing that increasing consumers' responsibility is not always welfare improving, no previous work compares emission and output taxes in the presence of environmentally aware consumers.

The present paper contributes to the above two streams of the literature by examining how environmental tax policies perform when consumers' choices depend on the level of their environmental awareness. The paper's main result is that an emissions tax welfare dominates an output tax at low levels of environmental awareness while the opposite is true for high levels of environmental awareness. The paper also sheds light upon the different specifications of the environmentally aware consumer's utility function used in the literature.

The rest of the paper is structured as follows. Section 2 presents the model with an extensive discussion on the construction of green consumer's utility function and social welfare and also lays out the structure of the game. Section 3 presents the firms' choice of output and abatement, while Section 4 presents the benchmark free market equilibrium. Section 5 derives the optimal output and emissions taxes. Section 6 compares the optimal values of output, abatement, environmental damage and welfare under the two tax regimes. Section 7 discusses robustness of the results, and Section 8 concludes the paper.

2 The model

2.1 Production and pollution

Consider a monopolist producing Q units of product X. Marginal production cost is constant, and for simplicity normalized to zero. The production of each unit of X generates δ units of some harmful pollutant. The monopolist can remove a certain amount (or the entirety) of its pollutant's emissions using end of pipe technology.[3] Examples of end of pipe abatement include scrubbers to remove SO_x from flue gases and setting basins and centrefuges to reduce the sediment content of pulp and paper mill effluents. Given the importance of the abatement specification, we take a moment to explain the key modelling assumptions. In particular consider scrubbers that remove SO_2 from combustion,

designed to promote a green technology could have exactly the opposite effect under certain conditions.

[3] End of pipe abatement is only one of the options available to abate pollution (others include changes in the production process, in the use of raw material and energy commodities).

which are still used widely globally.[4] Fixed costs[5] are the main part of scrabbers' total cost, with variable costs[6] being of very small importance for most of the techniques.[7] Furthermore, a firm that either chooses higher quality among the existing technologies or invests in end of pipe abatement R&D, increases its ability to remove higher level of emissions. The amount of emissions removed, can be considered independent of output, although scrubbers remove a certain percentage of generated emissions. This is so because coal-fired power plants (base load plants) produce at capacity throughout the year and thus, scrubbers installed on them remove an almost fixed amount of emissions (abatement at capacity emissions).[8]

Let v indicate the monopolist's choice of abatement, which could alternatively be considered as the choice of the level of R&D in improving end of pipe technology. As stated above, increased spending on abatement facilitates the removal of higher level of emissions. Thus, firm's net emissions are $e = \delta Q - v \geq 0$. Without loss of generality we set $\delta = 1$, therefore $v \in [0, Q]$ to ensure non negative emissions. Based on the above discussion, abatement costs are mainly fixed and thus, independent of the level of production. We assume that abatement cost is quadratic in the amount of abated units of pollutant,

$$C = kv^2, \ k \geq 1. \tag{1}$$

The total environmental damage generated from pollution is,

$$D = ze^2, \tag{2}$$

where z is a parameter transforming units of emission into environmental damage, that is, reductions in social welfare.[9]

2.2 Individual Preferences

We assume a finite number $n \geq 1$ of identical consumers. The pure consumption-utility of the representative consumer from good X is,

$$U = \alpha q - \frac{1}{2}q^2 + M \ , \tag{3}$$

[4] Schmalensee and Stavins (2013) report that in 2010, 60% of power plants in USA use scrabbers, while their use in China is more widespread according to many sources, as for example Xu et al. (2009).

[5] Capital and fixed operating costs (administration, maintenance, etc.).

[6] Variable operating and maintenance costs (cost of sorbents/reagents, cost of disposal of by-products, power and water cost, etc.).

[7] See for examle cost data presented in Cropper (2017), Table 13.4 and in Miller (2015), Ch 4.5, p.233-40.

[8] Since in order to serve the total demand, including peaks, other type of plants–with different technology, use of fuel and abatement technology–have to operate, the choice of total output can be considered independent of the choice of end of pipe technology in the coal-fired power plant.

[9] The parameter z results from a complete environmental evaluation study integrating social preferences and technical aspects, determining the "objective" damage caused by emissions.

where $\alpha > 0$, $q \geq 0$ is the individual consumption of X, and M represents the amount of the numeraire-good consumed.[10] Assuming a sufficiently large consumer income as to avoid corner solutions at the choice of good X, allows us to dispense with M hereafter.

The utility function in (3) corresponds to a consumer who cares only about her individual consumption, yet in this work we want to allow consumers to take into account, even partially, the environmental damage product X generates. In modelling environmentally conscious consumers, we first assume that each individual consumer knows the total amount of net emissions, $Q - v$, that the good X generates. Furthermore, she can develop a good, certainly not perfect, understanding of the consequences of the pollution generated by total net emissions, an understanding that reduces her willingness to purchase good X. She also realizes that, although her own consumption affects total consumption, the change is insignificant when n is large. Accordingly, we assume that she ignores the effect of her own consumption on total net emissions when making her choices.[11] That is, the environmentally conscious consumer realizes that neither v, nor Q, are under her control and for that reason she focuses on total emissions, taking e as exogenous.

Following the above specification of environmentally conscious consumers, the representative consumer's behavior is dictated by the following utility function, which is a decreasing function of the monopolist's net emissions:

$$\widetilde{U}(q; e, \phi) = \begin{cases} (\alpha - \phi e)\, q - \frac{1}{2}q^2 + M & \text{if } e \geq 0 \\ U & \text{if } e \leq 0 \end{cases}, \tag{4}$$

where $\phi \geq 0$ is a taste parameter expressing consumer's aversion to pollution. In order to rule out some uninteresting cases, we assume that α is large enough as to keep the term $(\alpha - \phi e)$ positive. Consumers with different values of ϕ differ with respect to the intensity of their aversion towards total emissions. Since we are using a *representative-consumer* model, we assume that all consumers within a given society and at a given point of time have the same preferences, i.e., the same value of ϕ. Different societies, or the same society at different time points may be characterized by different values of ϕ.

When $\phi = 0$, consumers do not react to pollution and thus, $\widetilde{U} = U$. When $\phi > 0$, $\widetilde{U}$ allows for partial internalization of the environmental externality: socially responsible consumers adopt a consumption pattern other than the one dictated by strict consumption-utility maximization. This provides an element

[10] This is a simplified version of the more general utility function specified along two dimensions of firm heterogeneity, vertical product differentiation and substitutability: $U = \sum_{i=1,n} \alpha q_i - \frac{1}{2}\left(\sum_{i=1,n} q_i^2 + 2\gamma \sum_{i \neq j} q_i q_j\right) + M$, where $\gamma \in [0, 1)$ measures the degree of substitutability between products (see Hackner (2000) and Garella and Petrakis (2008)). In the present paper, we assume $\gamma = 0$, which implies no substitutability among product types and allows us to concentrate on a single monopoly.

[11] As it will become apparent after deriving environmentally conscious consumer's demand in (5), assuming that she takes into account the impact of her consumption on total emissions, would simply change slightly the slope of her demand function without any qualitative results on the model.

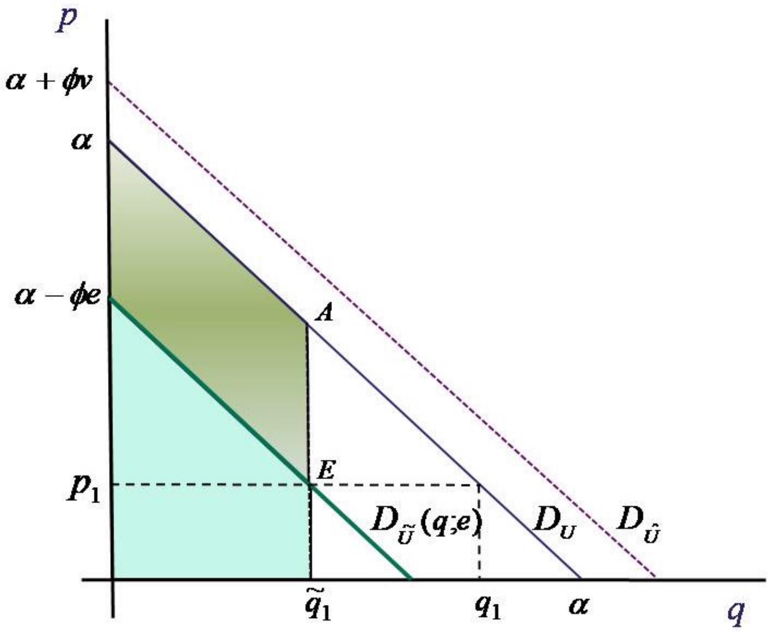


Figure 1: Individual consumer's Demand curve

of vertical differentiation to the model, since a reduction of a product's overall emissions corresponds to an increase in its quality.[12] In order to avoid more than 100% internalization of the environmental damage, we assume that $\phi \leq z$.[13]

For any given $e \geq 0$, the representative consumer's inverse demand function is obtained by maximizing (4) with respect to q, assuming e is treated as exogenous by the individual:[14]

$$p = \begin{cases} (\alpha - \phi e) - q & \text{if } e \geq 0 \\ a - q & \text{if } e \leq 0. \end{cases} \tag{5}$$

In Figure 1, the line $D_{\widetilde{U}}(q;e)$ (green line) illustrates an individual's inverse demand represented in (5) for $e > 0$, while $D_U(q)$ (blue line) represents the demand deriving from (3), corresponding to either $\phi = 0$, or $e \leq 0$. For any

[12] A reduction in a product's total emissions mitigates the lowering of consumers' willingness-to-pay due to the good's environmental impact.

[13] Overinternalization of the externality could be due to either misinformation, or consumers hating emissions *per se,* independently of the damage they create. This assumption is by no means crucial, but it is reasonable and in some instances makes proofs somewhat easier.

[14] Striclty speaking, since $e = \delta Q - v = \delta \left(\sum_{j \neq i} q_j + q_i \right) - v$, the consumption of individual i *does* affect emissions. Nevertheless, we assume that n is sufficiently large for this effect to be ignored by i. Except for a slight modification of the equilibrium expressions, no qualitative result is affected by this assumption.

given level of environmental sensitivity ϕ the green consumer's demand $D_{\widetilde{U}}$ has been drawn for a given level of $e > 0$, its position depending negatively on the level of e. For any given ϕ, the $D_U(q)$ line constitutes the highest possible position of the demand function since, in the absence of warm glow effects (see below), if in some way total emissions disappear, there is no longer point to show environmental responsibility.

Rather than *emissions-haters*, environmental vertical-differentiation models commonly assume consumers to be *abatement-lovers,* enjoying consumption utility according to the function $\widehat{U} = (\alpha + \phi v)\, q - \frac{1}{2}q^2$.[15] The dashed purple line denoted by $D_{\widehat{U}}$ on Figure 1 illustrates the abatement-lover's demand. It is clear that with both abatement and net emissions being positive, the demand function of the abatement-lover lies above that of the environmentally insensitive consumer (deriving from utility U in (3)) while that of the emissions-hater green consumer (deriving from utility specification $\widetilde{U}$ in (4)) lies below that of the not-environmentally conscious consumer.

From the specification in (5) it follows that for $\phi > 0$, and $e > 0$, at any price p_1, the environmentally sensitive consumer chooses $\widetilde{q}_1$ instead of q_1–as indicated by the demand function deriving from $\widetilde{U}$ –but still values her consumption according to the demand curve D_U, deriving from U. In terms of Figure 1, the total consumption value of the $\widetilde{q}_1$ units consumed corresponds to the area $0\alpha A\widetilde{q}_1$, instead of the area $0(\alpha - \phi e)E\ \widetilde{q}_1$ under $D_{\widetilde{U}}$. We term the former as "*social responsibility*" (SR) approach and the latter "*hedonic*" approach. In the SR approach, environmental awareness is viewed as a self-imposed sacrifice of $q_1 - \widetilde{q}_1$ valuable units, whereas according to the hedonic approach the quantity reduction is the result of a change in preferences, due to an acquired genuine distaste for the good.

We use the term "*hedonic*" because consumer's involvement creates psychological rewards, whether positive or negative, that must be taken into account by the social planner. For instance, in many vertical differentiation environmental models, consumers value product-types with superior environmental performance as higher qualities and their consumption yields higher utility.[16] This is equivalent to assuming that through a *warm-glow* effect, greener consumption creates happiness on its own, over and above any positive impact it may have on the environment. Consequently, the social planner should favor the consumption of greener goods even if their impact on the environment were completely illusionary. In our context, following the hedonic approach would imply negative rewards: since the consumer is motivated by her aversion to total

[15] Social consciousness is commonly interpreted as appreciation of a particular characteristic of the good that is environmentally friendly, for instance, abatement. The equivalent of (4) assuming *abatement loving* consumers is: $\widehat{U} = (\alpha + \phi v)\, q - \frac{1}{2}q^2$, where v (abatement) corresponds to the quality-measurement parameter. The use of $\widehat{U}$ is appropriate when green behavior is motivated by private benefits from the specific characteristic (e.g., health benefits from avoiding environment-damaging pesticides). While in our context we consider the use of (4) as more appropriate, note that it makes little difference whether one uses $\widetilde{U}$ or $\widehat{U}$, see Section7.

[16] See for instance the case of abatement-loving green consumers, described earlier.

emissions, higher levels of the latter would imply higher reduction of a product's consumption value. Again, even if the presence or impact of such emissions on the environment were completely illusionary, the social planner should restrict the good's consumption in order to make consumers happier.

In this work we use the SR approach, assuming that green behavior of a socially responsible person comes directly as a self-restriction, rather than indirectly through a change in her tastes. In other words, we define social responsibility *as a conscious altering of behavior*, not of preferences. This implies a dichotomy between *decisions-guiding utility* and *derived utility*: consumers act as if they were guided by the utility function $\widetilde{U}$ in (4), but for any given quantity finally consumed, their satisfaction is given by U in (3). In other words, social responsibility leads to reductions in the consumed quantity *without affecting the value of the units finally consumed.*

2.3 Aggregate Demand and First Best

The social planner, as well as the monopolist, base their decisions after observing aggregate demand. The latter is obtained by first deriving individual direct demand from (5), aggregating it over n consumers and then substituting $e = Q - v$ to get, $Q(p; v, \phi, n) = \frac{n(a+\phi v)}{1+\phi n} - \frac{n}{1+\phi n} p$.[17] Inverting this we derive the aggregate inverse demand curve,

$$p(Q; v, \phi, n) = \begin{cases} (\alpha + \phi v) - \frac{1+\phi n}{n} Q & \text{if } Q \geq v \\ \alpha - \frac{1}{n} Q & \text{if } Q \leq v. \end{cases} \tag{6}$$

Note that the lower part of the above is the aggregate demand deriving from U in (3), depicted as D_U on Figure 2. Its presence is due to the fact that warm glow effects are ruled out, and therefore as soon as net emissions become non-positive ($Q \leq v$), there is no longer room for green behavior. Thus, the effective aggregate demand–depicted by the green line on figure 2–is composed of two segments: one pertaining to $D_U(Q; n) = \alpha - \frac{1}{n} Q$, up to $Q = v$, and a second, pertaining to $D_{\widetilde{U}}(Q; n, \phi, v) = (\alpha + \phi v) - \frac{1+\phi n}{n} Q$, for $Q \geq v$. Because we consider that within the current state of technology non-positive net emissions correspond to a rather unrealistic outcome, we restrict all equilibrium outcomes to lie on the $D_{\widetilde{U}}$ segment of the demand function.[18]

Social welfare W represents consumption value minus the sum of environmental damage and abatement cost. Since we have assumed no production costs, for any given quantity Q the consumption value is the area below the demand curve up to Q, and comprises consumer surplus, profits and tax revenue; any redistribution among these components does not affect welfare as long as it

[17] Notice the change in the demand function's arguments. Although e is exogenous to the individual consumer's choises, for both the monopolist and the government e is endogenous, depending on the choice of Q and v.

[18] This is obtained by placing an upper bound on the admissible values of ϕ (see the discussion *i)* after the exrpession (14), and *ii)* after Lemma 1)

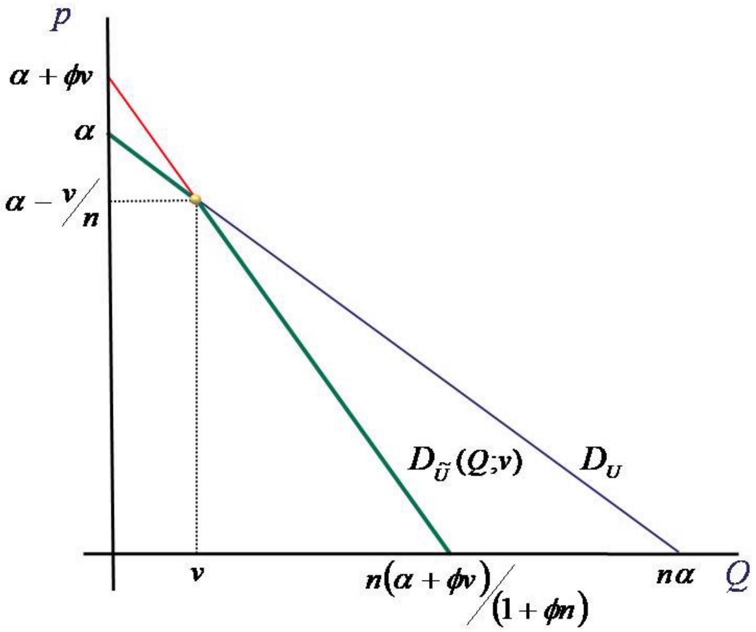


Figure 2: Aggregate Demand curve

leaves total consumption value unaffected (transfer). Hence,

$$W = \sum_{i=1}^{n} u_i - D - C \ ,$$

in which D and C must be substituted by (2) and (1), respectively. In accordance with the SR approach, u_i is substituted by (3),[19] to obtain,

$$W = \sum_{i=1}^{n} U_i - D - C = \sum_{i=1}^{n} \left(\alpha q_i - \frac{1}{2} q_i^2 \right) - z \left(Q - v \right)^2 - kv^2.$$

As it turns out, the parameter α enters multiplicatively in all the expressions and plays no role in any of the results, it is therefore normalized to $\alpha = 1$, without loss of generality. We also assume that $z \geq \alpha = 1$ implying that the environmental damage is of at least equal importance for welfare, as consumption is. However, no analytical results are possible without resorting to some further simplifying assumptions that are not mere normalizations, and for this reason their role is discussed in a special section, where it is shown that they do not qualitatively affect the results. Along with the already stated assumption $\delta = 1$, we also assume $k = 1$, and also set $n = 1$.[20] Using these assumptions aggregate demand is, $p = (\alpha + \phi v) - (1 + \phi) q$, and the social welfare function simplifies to,

$$W = (1 + 2vz) q - \left[\left(\frac{1}{2} + z \right) q^2 + (1 + z) v^2 \right] . \tag{7}$$

Armed with the above we can determine the first best outcome, in which a benevolent regulator is able to determine directly both quantity and abatement level. Direct maximization of (7) obtains (hereafter, the superscript "$*$" denotes first-best values):[21]

$$q^* = \frac{1 + z}{1 + 3z}, \ v^* = \frac{z}{1 + 3z}. \tag{8}$$

Since for all finite values of z, $q^* > v^*$, net emissions at the first-best, $e^* > 0$, are positive. Net emissions e^* and the resulting level of environmental damage D^* are:

$$e^* = \frac{1}{1 + 3z}, \ D^* = \frac{z}{(1 + 3z)^2}. \tag{9}$$

[19] The traditional method of substituting (4) into the social welfare function corresponds to the *hedonic* approach. As shown in section 7 using (4) instead of (3) in W does not significantly alter our results.

[20] With a single consumer it may look like individual and aggregate demand coincide. However, this ignores the fact that we keep assuming that (even if alone) the individual consumer does not realize the impact of her consumption on emissions (as if it were caused by the consumption of other people), whereas the social planner and the monopolist do. By introducing this "myopia", the assumption of $n = 1$ remains a purely simplifying one, without qualitative consequences for the model.

[21] It can be easily verified that $\left(\partial^2 W / \partial q^2 \right) = -2(1 + 2z) < 0$, and $\left(\partial^2 W / \partial v^2 \right) = -2 (1 + 2z) < 0$.

Finally, social welfare at the first best is,

$$W^{*}=\frac{1+z}{2\left(1+3z\right)}=\frac{1}{2}q^{*}. \tag{10}$$

Note that, given the specification of the welfare function in (7), the equilibrium values at the first best are independent of consumer's environmental consciousness.[22] The regulator defines optimal output and abatement level solely based on the environmental damage parameter z, ignoring consumer's effort to internalize the externality.

3 Second best

3.0.1 Game Structure and Government Intervention

We represent the market outcome as a perfect information two-stage game between the regulator and the monopolist. At the first stage the regulator chooses a tax base–either net emissions (emissions tax), or total output produced (output tax)–and the corresponding tax level, in order to maximize social welfare. At the second stage, taking the tax type and rate as given the monopolist chooses abatement and output in order to maximize its own profit.[23]

Under the assumption that the production of one unit of output creates one unit of emissions, *for any exogenously given level of abatement* the two types of taxation are equivalent, since the effect of one can be replicated by the other through an appropriate adjustment of the tax-rate. This equivalence may no longer hold when the abatement decision becomes endogenous.[24]

3.0.2 Second Stage equilibrium: the firm's choice of output and abatement.

Assuming a tax rate t on output, the monopolist's objective function is,

$$\pi=\left(p-t\right)q-v^{2}, \tag{11}$$

[22] Had we instead assumed that $W=\widetilde{U}-D-C$, all the expressions would depend on ϕ. Maximized welfare would then be $\widetilde{W}=-\frac{1}{2}q^{2}(2z+2\phi+1)+q(v(2z+\phi)+1)-v^{2}(z+1)$ with $d\widetilde{W}/d\phi=-\left(q-v\right)<0$, implying that increases in environmental sensitivity reduce the maximum welfare that a society can attain! This is due to the fact that computing the value of consumption from (4) implies that increases in environmental consciousness reduce the value of consumption.

A similar pattern holds if instead of *emissions haters* one considers environmentally sensitive consumers as being *abatement lovers,* as in footnote 10. If in the social welfare function u is replaced by $\widehat{U}$, the first-best maximum welfare becomes an increasing function of ϕ. For $\phi>0$ abatement produces a *prozac effect,* since it increases *ceteris paribus* the value of consumption.

[23] Implicit in the game structure is the assumption that the regulator can credibly commit to a single tax rate before the monopolist decides on abatement and output.

[24] See Cremer and Thisse (1999) and Constantatos and Sartzetakis (1999) for the case of output taxation in markets characterized by pure vertical-differentiation.

whereas if the tax is on emissions, the profit function is,

$$\pi = pq - t\,(q - v) - v^2. \tag{12}$$

Maximizing the appropriate profit function yields optimal decisions under each tax regime. Letting the digit "2" at the superscript indicate second-stage equilibrium values, and distinguishing hereafter output-tax from emissions-tax outcomes by the digit "O" or "E", respectively, at the superscript, we have,[25]

$$q^{2O} = \frac{2\,(1-t)}{B},\; v^{2O} = \frac{\phi\,(1-t)}{B}\;, \tag{13}$$

in case of output tax, and

$$q^{2E} = \frac{2 - t\,(2-\phi)}{B},\; v^{2E} = \frac{\phi + t\,(2+\phi)}{B}\;, \tag{14}$$

in case of an emissions tax, where $B \triangleq 4 + (4-\phi)\phi$, with $B > 0$, $\forall \phi < 4$. Taking into account that $t < \alpha = 1$ (otherwise the market closes down), inspection of (13) and (14) reveals that $\phi \leq 2$ is the necessary and sufficient condition for $q^{2O} \geq 0$, $v^{2O} \geq 0$, and $q^{2O} - v^{2O} \geq 0$, and since the emissions tax rate cannot be negative, for $v^{2E} > 0$, and $q^{2E} > 0$. In order to avoid degenerate solutions, hereafter, we maintain $\phi \leq 2$. Nevertheless, this assumption does not guarantee that net emissions are positive in the case of an emissions tax. For $q^{2E} - v^{2E} = \frac{2-\phi-4t}{B} \geq 0$, it must also be that $t < 1/2$, and $\phi \leq 2\,(1-2t)$, *i.e.*,

$$t \leq (2-\phi)\,/4\;. \tag{15}$$

These conditions are checked later on at the optimal value of the emissions tax rate.

4 The Unregulated Market case

In this section we assume that the government does not intervene and we examine the impact of environmental awareness on the free market equilibrium. Besides representing a benchmark for the effects of taxation, this section sheds light on two questions related to the use of long-term increase in environmental awareness as a substitute to tax policy. First, can increases in environmental consciousness yield the first-best outcome in the absence of any other intervention? Second, does an increase in environmental awareness always improve welfare, as has been often implied in the literature?[26] The answer to both questions turns out to be negative.[27]

[25] Second order conditions can be easily shown to hold.

[26] See for example Endres (1997), Petrakis *et al.* (2009)). More recently environmental agencies, among which the European Environmental Agency, have recognized "awareness raising" as a policy instrument (http://www.eea.europa.eu/themes/policy/intro).

[27] Amir et al. (2019) also compares monopolist's choice of price and green quality to the first-best, pointing to the importance of log-supermodularity of demand in determining the outcome.

Setting $t = 0$, in either (13) or (14) obtains:

$$q^F(\phi) = \frac{2}{B},\ v^F(\phi) = \frac{\phi}{B}. \tag{16}$$

where the superscript F denotes the free market equilibrium. Obviously, $\forall\phi \in [0,2]$, $v^F(\phi)$ is increasing concave, $q^F(\phi)$ is decreasing convex, and $q^F(\phi) > v^F(\phi) \geq 0$ (net emissions are non negative). It can be easily shown that the condition $1 - \phi e^F > 0$ is met for all $\phi \in [0,2]$. Substituting v^F, q^F from (16) into the social welfare function (7) we obtain after simplification,

$$W^F(\phi; z) = \frac{2(3-2z) + 4(2+z)\phi - (3+z)\phi^2}{B^2}. \tag{17}$$

Note that for $\phi = 0$, $W^F = (3-2z)/8$ which may be negative if society places significant importance on emissions ($z > 3/2$): in the absence of any intervention this market produces negative net social value, due to the environmental externality.[28] We define $\overline{\phi} = \min\{z, 2\}$, and limit $\phi \in [0, \overline{\phi}]$. The following proposition summarizes the main results concerning social welfare and environmental damage in the case of no intervention.

Proposition 1 *Under our assumptions i) the unregulated welfare does not monotonically increase with increases in environmental consciousness, but instead peeks at some value of ϕ, call it $\phi^F(z) \in [0, \overline{\phi}]$; ii) the maximum welfare level that the unregulated market can reach falls short of the first best welfare level, i.e., $W^{F*} < W^*$, where $W^{F*} = W^F\left(\phi^F(z); z\right)$.*

Proof. See the appendix. ■

The proposition shows that when environmental consciousness is at low levels, increases in ϕ are desirable, but when consciousness is already developed at sufficiently high levels, making consumers even more conscious may have detrimental effects on welfare. By punishing the polluting firm, consumers induce simultaneously an increase in abatement and a reduction in total quantity. The former is obviously welfare enhancing, but lower production has an ambiguous effect on welfare due to cleaner environment but also reduced consumption. At high levels of $\phi > \phi^F$ where the production is already "too low", further increases in consumer awareness are welfare reducing. The second part of the proposition shows that first-best welfare cannot be attained by simply improving consumers' environmental awareness without the imposition of some form of regulation.

Figure 3 is drawn assuming $z = 3/2$, so that $W^F(0) = 0$ and $\overline{\phi} = 3/2$. The W^* (green) line represents first-best welfare, which is independent of ϕ, and the W^F (brown) curve shows welfare when the market is unregulated, as function of ϕ. The maximum value of W^F is attained at $\phi^F \approx 0.917 \leq \overline{\phi} = 3/2$.

Turning to the environmental damage resulting from the production of X, the following Proposition summarizes some key results that help us understand the welfare results presented above.

[28] Nevertheless, private surplus and profits are positive, keeping this market open.

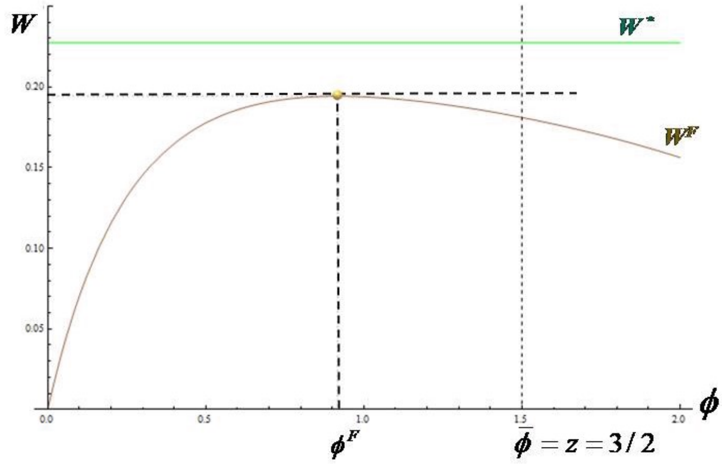


Figure 3: Social welfare without government intervention.

Proposition 2 *Emissions, and therefore environmental damage in the unregulated market are always decreasing in ϕ and for sufficiently high levels of ϕ ($\phi > \widetilde{\phi}^F(z) = \frac{1}{2}\left(3z + 5 - \sqrt{3}\sqrt{3z^2 + 2z + 11}\right) \in \left[0, \overline{\phi}\right]$) the unregulated market may produce less emissions and environmental damage than what is observed at first-best. At the value $\phi = \phi^F$ that allows the highest possible welfare in the unregulated market, emissions are already lower than their first-best level.*

Proof. See the appendix. ■

Proposition 2 shows that in an unregulated market increases in ϕ alone are able to lead to lower emissions than those at first-best. Hence, the reason of why changes in ϕ unassisted by some tax fail to reach first-best is not to be found in the environment, but rather on their effect on quantity, as suggested in the discussion of Proposition 1. Figure 4, drawn for $z = 3/2$, depicts the environmental damage at the first best, D^* (green horizontal line), and at the unregulated market equilibrium D^F (brown downward sloping curve), as functions of ϕ. The vertical dashed line indicates $\overline{\phi}(z = 3/2) = 3/2$.

When $\phi \geq \widetilde{\phi}^F = 0.805$, the unregulated market leads to lower environmental damage than its level at the first-best solution, and note that $\widetilde{\phi}^F < \phi^F$. Increases in environmental consciousness above ϕ^F improve the environment, but only at the expense of lower welfare.

5 Optimal taxation

In this section we derive the optimal tax rate levied on either output, or emissions. The optimal output-tax rate may be negative (subsidy), but we do not

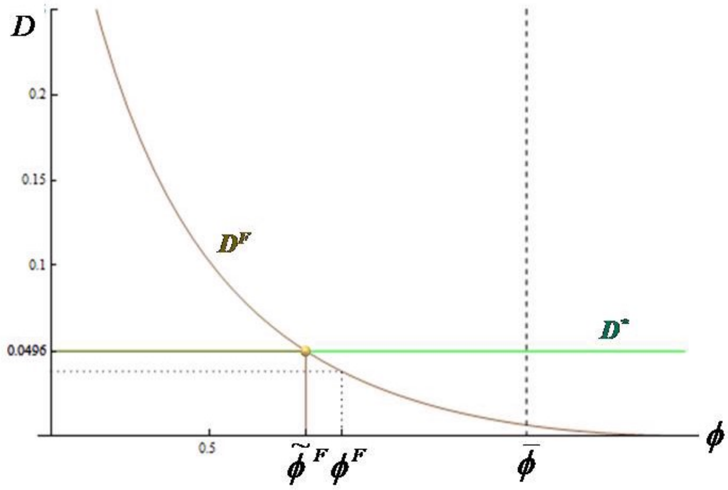


Figure 4: Environmental damage without government intervention.

allow for "emissions subsidies": when the optimal emissions-tax rate is negative we consider it equal to zero (unregulated market).

Substituting optimal abatement and quantity, first from (13), and then from (14) into (7) yields social welfare as function of t, in the case of output and emissions tax respectively. Maximizing welfare in each case with respect to t yields the respective optimal tax rates, defined in the following Lemma (proof in the appendix).

Lemma 1 *i) The optimal output-tax rate is,*

$$t^O = \frac{2(2z-1) - 4(z+1)\phi + (z+2)\phi^2}{2(2z+1) - 4z\phi + (z+1)\phi^2}; \tag{18}$$

ii) the optimal emissions-tax rate is

$$t^E = \frac{4(4z-1) - 2(4z+5)\phi + 4\phi^2 - \phi^3}{4(8z+3) + 4\phi + 3\phi^2}; \tag{19}$$

iii) both derivatives $\partial t^O/\partial\phi$ and $\partial t^E/\partial\phi$, are negative $\forall\phi \in [0, \overline{\phi}]$ and $z \geq 1$.

While tedious, it can be verified that $t^E \leq (2-\phi)/4$, guaranteeing that when applying the optimal emissions-tax, net emissions are nonnegative, according to (15). Part *iii*) of the Lemma proves the intuitively expected result that optimal tax rates are reduced following an increase in environmental consciousness.

Since the optimal output-tax rate can be either positive or negative, the following lemma determines that either both tax-rates will be of the same sign, or the optimal output tax rate may be negative while the optimal emissions tax rate positive (proof in the appendix). Therefore, it is impossible to have a situation where the choice is between an output tax and an emissions subsidy. Setting the optimal tax rates in (18) and (19) equal to 0 and solving for z we obtain, respectively:

$$z_E = \frac{4 + 10\phi - 4\phi^2 + \phi^3}{8(2-\phi)}, \quad z_O = \frac{2\left(1 + 2\phi - \phi^2\right)}{(2-\phi)^2}. \tag{20}$$

Lemma 2 *For all $z \geq 1$ there exist two values of $\phi \in \left[0, \overline{\phi}\right]$, call them $\phi_O \equiv z_O^{-1}(z)$ and $\phi_E \equiv z_E^{-1}(z)$ with $\phi_O < \phi_E$, such that: i) $\forall \phi \leq \phi_O$, both t^O and t^E are non negative, ii) for $\phi \in (\phi_O, \phi_E)$, $t^O < 0$ while $t^E > 0$, and iii) $\forall \phi \geq \phi_E$, $t^O < 0$, but the optimal emissions-tax rate is equal to zero (corner solution since no emissions subsidies are allowed).*

While ϕ_O is simply an auxiliary variable informing whether at a given (z, ϕ) combination the output tax-rate is positive or negative, ϕ_E represents an important critical value of ϕ in order to avoid the absurdity of emission subsidies. With respect to ϕ_O we can easily show that,

$$\phi_O = 1 - \frac{\sqrt{2(4+z)}}{(2+z)}, \tag{21}$$

but the explicit determination of ϕ_E is omitted since it is quite complex, involving the solution of a third degree equation.

6 Optimal values under the two tax regimes and comparisons

In this section we derive the optimal values of abatement, quantity, environmental damage and total welfare under the two tax regimes. We also examine the impact of increases in ϕ in the two benchmark cases (first best and no intervention) and the two tax regimes. Whether the tax is on output or emissions, in all cases we consider that it has been adjusted at its optimal level, given in (18) and (19), respectively.

6.1 Quantity

By introducing the optimal tax rate from (18) and (19) into the first elements of (13) and (14) respectively, and simplifying we obtain optimal quantities under each tax type:

$$\begin{aligned} q^O(\phi, z) &= \frac{2}{2(1+2z) - 4z\phi + (1+z)\phi^2}, \\ q^E(\phi, z) &= \frac{8(1+z) - 2\phi + \phi^2}{4(8z+3) + 4\phi + 3\phi^2}. \end{aligned} \tag{22}$$

The following proposition determines the impact of changes in ϕ on quantity for the case of emissions- and output-tax.

Proposition 3 *When the tax is on output, i) as ϕ increases the per capita consumption initially increases, reaches a peak at $\phi = \phi^* \triangleq 2z/(1+z) \in (0, \overline{\phi})$ and then decreases; ii) at $\phi = \phi^*$ the optimal output tax rate is negative (subsidy). When the tax is on emissions, the per capita consumption is monotonically decreasing in ϕ.*

Proof. See the appendix. ■

It is clear from part *iii)* of Lemma 1 that as ϕ increases, the optimal tax rate, whether on output or emissions, becomes smaller. Proposition 3 shows that when the regulator uses an output tax, unless ϕ is already too high (above ϕ^*), consumption increases as consumers become more conscious and the regulator reduces the tax rate. On the contrary, when the tax is levied on emissions, lower tax rates resulting from increased environmental consciousness, lead to *lower* output. This is a rather surprising result, since one expects that output increases following a reduction of a tax rate related to its production. However, lower emissions tax-rates induce lower abatement (see next proposition), thus leading increasingly conscious consumers to lower their consumption. At $\phi = 0$ the emissions tax leads to less output than the first-best and as consciousness increases the gap between q^E and q^* becomes larger. On the contrary, an output subsidy –part *ii)* of the Proposition shows that $t^O(\phi^*) < 0$– is able to bring consumption at its first-best level.

The monopolist's output as function of ϕ under the two tax-bases is depicted on the following diagram along with the two "benchmark" cases, first-best and unregulated market. Figure 5 is plotted for $z = 3/2$, which yields $\phi^* = 1.20$, at which q^O (blue line) reaches its maximum value, which coincides with the first-best quantity. The q^E (cyan) line, on the other hand is monotonically decreasing, implying that under an emissions tax, further increases in environmental consciousness reduce quantity, even if the tax-rate is optimally adjusted. The values of ϕ_O and ϕ_E, at which t^O and t^E become zero respectively, are also marked on the figure, and since $\phi^* > \phi_O \approx 0.0524$, the maximum quantity is obtained using a subsidy. At ϕ_O the line q^O intersects q^F and $\forall \phi > \phi^O$, the optimal output tax is a subsidy, leading to higher quantity than the unregulated market. At ϕ_E the q^E (cyan) line intersects with q^F but $\forall \phi > \phi_E$, the two lines, q^E and q^F coincide, due to the fact that the optimal emissions tax cannot be negative.

When the market is unregulated, increases in social responsibility obviously reduce equilibrium output, i.e., q^F is continuously decreasing in ϕ. In the presence of an optimally selected tax the negative relation between ϕ and q cannot be guaranteed, for increases in ϕ result in reductions of the optimal tax rate (or increases in the subsidy rate), which, in turn, stimulate output. The following decomposition of the total effect of ϕ on output shows that it depends on a

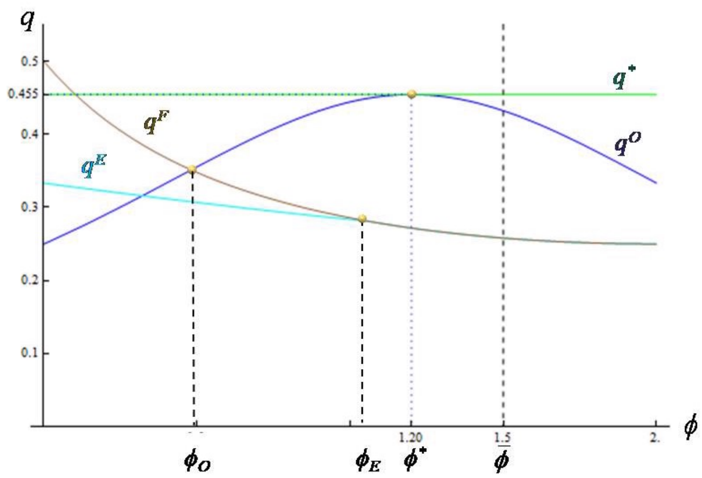


Figure 5: Equilibrium quantities.

direct effect plus an indirect effect through the optimal-tax-rate adjustment:

$$\frac{dq^i(\phi, t(\phi))}{d\phi} = \underset{(-)}{\frac{\partial q^{2i}(\phi, t)}{\partial \phi}}|_{t=t^i(\phi)} + \underset{(-)}{\frac{\partial q^{2i}(\phi, t)}{\partial t}} \underset{(-)}{\frac{\partial t^i(\phi)}{\partial \phi}}, \; i = O, E. \tag{23}$$

On the RHS of the above, the first term shows the direct effect and is the only one present in the unregulated market case; it is shown in the appendix to be negative in all cases. The second term shows the indirect effect to be the product of the impact a change of the tax-rate has on quantity times the impact of ϕ on the optimal tax rate. Both these impacts are negative independently of the tax base, as shown in Lemma 1, therefore in both O, E, cases the indirect effect works towards increasing quantity. Proposition 3 shows that in the emissions-tax case the direct effect always dominates, whereas in the output-tax case the indirect effect dominates, unless ϕ is higher than ϕ^*.

6.2 Abatement

Inserting the optimal tax rate from (18) and (19) into the second elements of (13) and (14) respectively, and simplifying we obtain the optimal abatement under each tax type:

$$\begin{aligned} v^O(\phi, z) &= \frac{\phi}{2(1+2z) - 4z\phi + (1+z)\phi^2} > 0, \\ v^E(\phi, z) &= \frac{-2(1-4z) - \phi + \phi^2}{4(3+8z) + 4\phi + 3\phi^2} > 0. \end{aligned} \tag{24}$$

The following proposition determines the impact of changes in ϕ on abatement for the case of emissions- and output-tax.

Proposition 4 *i) Under an optimally selected output tax, equilibrium abatement is initially increasing in ϕ, peaking at some value of $\phi = \phi^{vO} \triangleq \sqrt{2}\sqrt{\frac{2z+1}{z+1}}$; at $\phi = \phi^*$, along with quantity, abatement reaches also its first-best level, while for $\phi \in \left(\phi^*, \min\left\{\phi^{vO}, \overline{\phi}\right\}\right)$ abatement is higher than its first-best level. ii) Under an optimally selected emissions tax, abatement is monotonically decreasing in ϕ until $\phi = \phi_E$, where the optimal emissions-tax rate becomes zero. iii) While for low values of ϕ abatement is higher under an emissions tax, at the neighborhood of ϕ_E the optimally selected output subsidy produces higher abatement.*

Proof. See the appendix. ■

Proposition 4 shows a striking difference in the producer's response to higher environmental consciousness and the resulting lower tax rates. When the tax is on output, lower tax rates stimulate abatement indirectly, through increases in output, which in turn induces the producer to further increase its product's attractiveness (quality) by investing in abatement. On the contrary, lowering the emissions tax rate reduces quantity, thus making such investment less attractive. As consumers' appreciation of the product is lowered, a further reduction in output is observed; as ϕ increases, both output and abatement converge towards their free market levels.

Proposition 4 is illustrated on Figure 6 where the monopolist's abatement is depicted as function of ϕ for both kinds of the tax-base, as well for the two benchmark cases, first-best and unregulated market. The figure is drawn for $z = 3/2$. If the market is unregulated, abatement increases with ϕ but never reaches its first-best level. For low values of ϕ —up to ϕ_O, where $t^O = 0$— the abatement under an output tax (v^O) is below its free-market level. For $\phi > \phi_O$ — t^O becomes now a subsidy— v^O keeps increasing, becoming equal to the first best level ($v^O = v^*$) at ϕ^*, and still increasing for $\phi > \phi^*$ at the neighborhood of ϕ^*. Abatement under an emissions tax, denoted by v^E, is continuously decreasing, reaching its unregulated-market level at ϕ_E, where $t^E = 0$. For $\phi > \phi_E$, we set $t^E = 0$, to avoid emission subsidies and thus $v^E = v^F$.

Recalling that as we move along the ϕ-axis the output tax is reduced, one concludes that abatement rises with tax-reductions! This seeming paradox is due to the fact that, instead of direct causality we have here an indirect relation where both variables–abatement and optimal tax–are simultaneously determined by ϕ: as ϕ departs from zero the presence of more aware consumers induces the firm to increase its abatement and at the same time the regulator to reduce the tax. In the same vein, an output tax at a positive rate induces less abatement than the unregulated market, since by reducing quantity, it also reduces the firm's incentive to invest in a fixed cost.

What is really surprising though is that under an optimally selected emissions tax, *as ϕ increases abatement is reduced!* This counter-intuitive result is

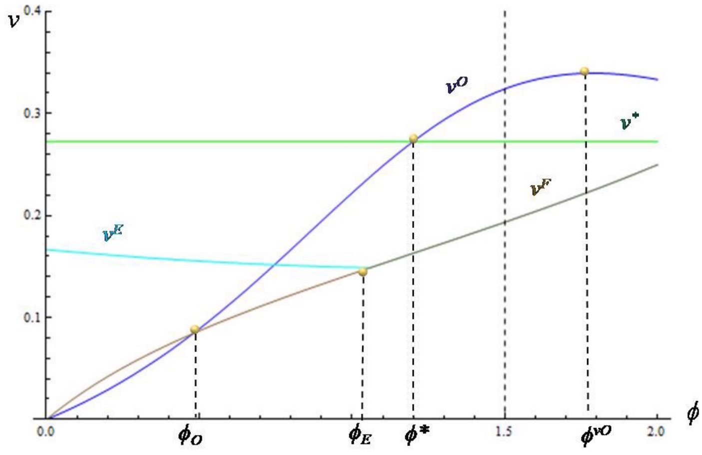


Figure 6: Equilibrium abatement.

obviously related to the fact that increases in ϕ cause the optimal emissions-tax rate t^E to decrease. In order to better understand why rises in environmental consciousness *undermine* the workings of an emissions tax, while they enhance the efficiency of an output-based tax/subsidy intervention as instrument for environmental protection, we perform a decomposition of the total impact of ϕ on abatement in two parts, a direct and an indirect effect:

$$\frac{dv^i(\phi, t(\phi))}{d\phi} = \underset{(+)}{\frac{\partial v^{2i}(\phi, t)}{\partial \phi}}|_{t=t^i(\phi)} + \underset{(+/-)}{\frac{\partial v^{2i}(\phi, t)}{\partial t}} \underset{(-)}{\frac{\partial t^i(\phi)}{\partial \phi}}, \; i = O, E. \tag{25}$$

The first term represents the *direct effect* of ϕ on the firm's abatement decision, evaluated at the optimal tax-rate. This term, evaluated at $\phi = 0$ is equal to $v^F(0)$. The direct effect is positive, since a rise in consumers' consciousness induces the firm to do more abatement. The second term in (25) is the *indirect effect* of ϕ on abatement, through the induced change in the optimal tax-rate. The indirect effect is in turn decomposed in two parts, the second being the same as the corresponding part in (23), and negative, from lemma 1. For the first term of the indirect effect we use the second elements of (13) and (14) respectively, to get,

$$\frac{\partial v^{2E}(\phi, t)}{\partial t} = \frac{\phi + 2}{4 + (4 - \phi)\phi} > 0,$$

while,

$$\frac{\partial v^{2O}(\phi, t)}{\partial t} = \frac{-\phi}{4 + (4 - \phi)\phi} < 0.$$

The indirect effect of a change of the emissions-tax rate on abatement is negative ($\frac{\partial v^{2E}}{\partial t} > 0$ and $\frac{\partial t^i}{\partial \phi} < 0$) since an increase in the tax rate reduces quantity, thus making the fixed investment in abatement less interesting. The indirect effect dominates the direct positive effect, as proved in part *iii*) of Proposition 4, thus making the response of firms to the optimal emissions tax decreasing in ϕ. When the tax is on output, the indirect effect (which is positive since $\frac{\partial v^{2O}}{\partial t} < 0$ and $\frac{\partial t^i}{\partial \phi} < 0$) reinforces the direct effect, thus making the abatement function increasing, and at a higher rate compared to the unregulated market case.

As a conclusion, Proposition 4 shows that *increases in consumers' environmental awareness are able to reach the first-best level of abatement only when coupled with an output-subsidy, whereas they are welfare reducing under an emissions-tax regime.*

6.3 Environmental damage

Recall that $D^i = z \cdot \left(e^i\right)^2$, where $e^i = q^i - v^i$, $i = *, F, O, E$, for the cases of first-best, unregulated market, output tax, and emissions tax, respectively. Substituting q^i, v^i, from (22) and (24) we obtain the second-best equilibrium net emissions:

$$e^O = \frac{2-\phi}{\phi^2(z+1) - 4\phi z + 4z + 2},\ e^E = \frac{10-\phi}{3\phi^2 + 4\phi + 32z + 12}, \tag{26}$$

and total environmental damage in each case, D^O and D^E, is determined accordingly. It is clear from the above two expressions that the effect of an increase in ϕ on emissions is unambiguously negative under an emission tax, $\frac{de^E}{d\phi} < 0$, while it could be either negative or positive under an output tax. We can decompose the total effect of ϕ on emissions, taking into account that emissions depend both on output and abatement and following the same decomposition analysis we used for output and abatement. For $i = O, E$, we can write,

$$\frac{de^i(\phi, t(\phi))}{d\phi} = \underset{(-)}{\frac{\partial e^{2i}(\phi,t)}{\partial \phi}}|_{t=t^i(\phi)} + \underset{(+/-)}{\frac{\partial e^{2i}(\phi,t)}{\partial t}} \underset{(-)}{\frac{\partial t^i(\phi)}{\partial \phi}}$$

$$= \left(\underset{(-)}{\frac{\partial q^{2i}(\phi,t)}{\partial \phi}} - \underset{(+)}{\frac{\partial v^{2i}(\phi,t)}{\partial \phi}} \right)_{|t=t^i(\phi)} + \left(\underset{(-)}{\frac{\partial q^{2i}(\phi,t)}{\partial \phi}} - \underset{(+/-)}{\frac{\partial v^{2i}(\phi,t)}{\partial \phi}} \right) \underset{(-)}{\frac{\partial t^i(\phi)}{\partial \phi}}.$$

We have already shown that the direct effect of ϕ on output is negative and on abatement positive, therefore the direct effect of ϕ on emissions is negative, $\frac{\partial e^{2i}}{\partial \phi} < 0$,[29] which is consistent with our findings about environmental damage in the unregulated market, D^F.[30] In the case of an output tax, the sign of the indirect effect is ambiguous, depending on whether the output or the abatement effect dominates, leaving the overall effect to be determined. Similarly, the sign of the overall effect under an emissions tax cannot be determined from the above,

[29] See (23) and (25), respectively.

[30] See proposition 2.

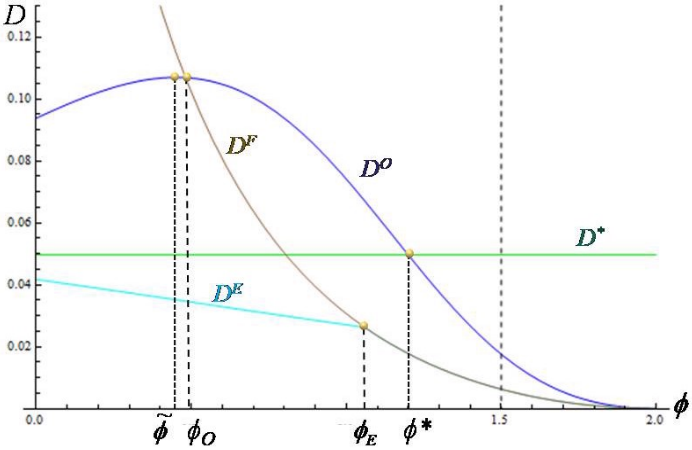


Figure 7: Equilibrium damage.

since the indirect effect is positive (both parts of indirect effect are negative) while the direct effect is positive. The following proposition determines the impact of changes in ϕ on total environmental damage for the case of emissions- and output-tax.

Proposition 5 *Under an optimally selected output tax, as ϕ rises environmental damage peaks at some $\phi = \widetilde{\phi} < \phi^O$ (the tax rate is still positive) and is decreasing thereafter; at $\phi = \phi^*$, damage is at its first-best level while for $\phi < (>)\, \phi^*$ environmental damage is higher (lower) than its first-best level. Under an emissions tax, environmental damage is decreasing in ϕ as long as the corresponding optimal tax-rate is nonnegative.*

Proof. See the appendix. ■

The content of Proposition 5 is illustrated on Figure 7 where the environmental damage is represented as function of ϕ, for $z = 3/2$. The D^*, D^F lines show environmental damage in the benchmark cases of first-best and unregulated market, respectively, while the lines D^O and D^E, (blue and cyan lines, respectively) represent damage under the two types of tax.

An emissions tax at an optimally selected rate is the most efficient instrument in terms of environmental protection since it reduces damage to levels below the first-best one even when consumers do not care at all about the externality they create ($\phi = 0$). Moreover, as ϕ increases, the impact of the quantity reduction described in Proposition 3 more than outweights the impact of the reductions in abatement described in Proposition 4, making D^E a monotonically decreasing function of ϕ. Somewhat more complex is the nature of the $D^O(\phi)$ function: at low initial levels of ϕ further increases in consumers' environmental consciousness end-up increasing equilibrium damage! This should not come as a surprise after Propositions 4 and 3: more aware consumers induce higher abatement which leads the planner to lower the output-tax rate and induce in turn higher output. For low levels of ϕ the output effect dominates, but as consumers' consciousness increases D^O starts decreasing, eventually passing at levels below D^*. That $D^O(\phi^*) = D^*$ is also an immediate consequence of the fact $q^O(\phi^*) = q^*$ and $v^O(\phi^*) = v^*$, as shown in the respective propositions. Note finally that for $\phi > \phi_O$, $D^O > D^F$, which means that when optimal, output subsidization reduces environmental quality compared to the unregulated-market case.

6.4 Social welfare

In this Section we compare social welfare under different regimes. In the unregulated market case social welfare is expressed in (17). Substituting the optimal tax rates from (18) or (19) into the social welfare function (29) and performing simplifications (see the appendix for details) yields the maximized welfare corresponding to the use of an output- or emissions tax.

Lemma 3 $\forall z \geq 1$, $\phi \in \left[0, \overline{\phi}\right)$ *, social welfare under an optimally selected output - and emissions-tax rate is,*

$$W^O(\phi, z) = \left[2(1+2z) - 4z\phi + (1+z)\phi^2\right]^{-1}, \tag{27}$$

$$W^E(\phi, z) = \frac{1}{2}\frac{10(1+z)+\phi^2}{4(3+8z)+4\phi+3\phi^2},$$

respectively.

We now turn our attention to the effect of ϕ on social welfare and the comparison of social welfare under the two tax regimes. Starting with the case of an output tax, the following Proposition summarizes the effect of ϕ on social welfare under both types of tax.

Proposition 6 *i) For any level of social consciousness the market produces higher welfare if social consciousness is paired with a tax, whether the latter is on output or emissions. ii) With an optimally selected output tax rate welfare peaks at $\phi = \phi^*$ where it reaches its first-best level; when $\phi \leq (\geq)\, \phi^*$ increases in social consciousness improve (deteriorate) welfare. iii) With an optimally selected (positive) emissions tax rate, welfare is constantly declining with increases in social consciousness, never reaching the first-best level. iv) There is a consciousness level $\mathring{\phi}$ such that, when $\phi \leq (\geq)\, \mathring{\phi}$, the emissions (output) tax produces higher welfare.*

Proof. See the appendix. ■

While the detailed proof of the proposition is relegated to the appendix, the following figure illustrates Proposition 6. Figure 8 illustrates welfare as function of ϕ for the cases of output tax (blue line W^O), emissions tax (cyan line W^E), unregulated market (brown line W^F), and first-best (green line W^*)

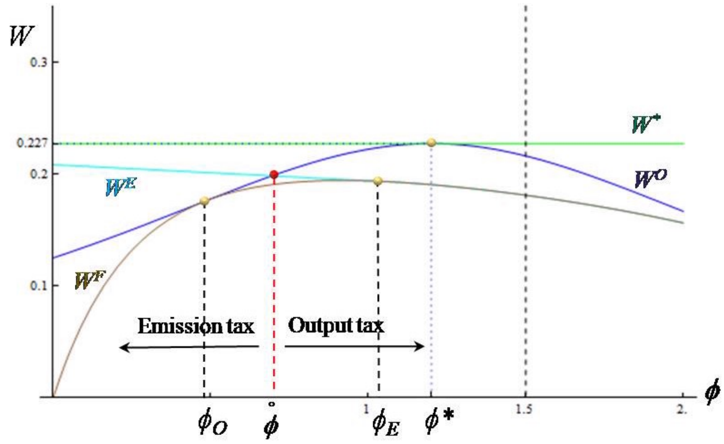


Figure 8: Equilibrium welfare.

for $z = 3/2$, which represents also the highest admissible value of ϕ, marked by a dashed vertical line.

At ϕ_O and ϕ_E, $t^O = 0$ and $t^E = 0$, hence $\forall \phi > \phi^O$, the optimal output tax is a subsidy, while $\forall \phi > \phi^E$, $W^E = W^F$. The W^O line reaches its maximum at $\phi = \phi^*$, where $W^O = W^*$. For $0 \leq \phi < \mathring{\phi}$, an optimally selected emissions tax is the best policy but it is unable to reach first-best welfare. For $\mathring{\phi} < \phi < \phi_E$ the best policy is an output subsidy. For $\phi \geq \phi_E$ the comparison no-longer makes sense, but an output subsidy is the right policy even for $\phi > \phi^*$ since it yields higher welfare than the unregulated market.

Increases in social consciousness may reduce optimal tax rates but do not eliminate the need for intervention: no matter the level of social consciousness, when combined with some form of intervention it may produce a higher level of welfare relative to the unregulated market with the same level of ϕ (parts *i*), *ii*) and *iii*) of Proposition 6). Part *iv*) of the proposition shows that for low levels of social consciousness, an emissions tax generates higher welfare, but for high values of ϕ ($\phi > \mathring{\phi}$) an output subsidy is to be preferred.

When the tax is on output, as ϕ increases social consciousness takes in charge the environmental problem leaving the tax/subsidy to deal with the monopoly induced quantity distortion. Thus, the resulting reduction of the output-tax rate corresponds to a smooth switch of the regulator's target, from focusing primarily on environmental protection –when ϕ is very low and the tax rate positive– towards generating higher levels of quantity. The higher levels of abatement resulting from higher quantity also help to reach both targets and attain first-best welfare at some finite and admissible value of ϕ ($\phi = \phi^* < \overline{\phi}$).

The picture is completely different with an emissions tax. Since this type of tax targets emissions directly, the reductions in $t^E(\phi)$ following an increase in consumers' consciousness are unable to stimulate quantity, and since they also discourage abatement (see Proposition 4), they end-up reducing welfare! Intuitively, the emissions tax targets exactly the same thing as social consciousness and increases in the latter make the workings of the tax problematic. This is not to say though that the emissions tax is inefficient. For low levels of ϕ the emissions tax provides both, a cleaner environment and superior welfare relative to the output tax (part *iv*) of Proposition 6). It is only at higher levels of ϕ, where the attention of public policy must turn towards stimulating quantity, that the emissions tax loses its efficiency in terms of welfare. Since $\mathring{\phi} < \phi_E$, it is important to note that $t^E > 0$ in the interval $\left(\mathring{\phi}, \phi_E\right]$, while an output subsidy produces higher welfare. This implies that if social consciousness rises steadily, the policy change from emissions tax to output subsidy must occur abruptly at some critical value of ϕ, rather than waiting for the optimal emissions-tax rate to become zero. It also illustrates the fact that the superiority of the output subsidy is not due to the fact that t^E eventually becomes equal to 0 and is not allowed to reach negative grounds, but rather on its adverse impact on quantity and abatement.[31]

7 Robustness

In this section we examine the robustness of our main results, focusing on how they are affected when: *i)* waiving the simplifying assumptions on parameter values and *ii)* modifying the way we treat social consciousness. Starting from the first, recall that we have assumed $\alpha = n = \delta = k = 1$, otherwise some of the proofs would have been, at best, very cumbersome–due to many different cases to be considered–and at worst only numerical. Since α affects only the vertical intercept of the demand function, setting $\alpha = 1$ is a mere normalization, without which quantities and abatement levels are multiplied by α, while social welfare and environmental damage levels, by α^2.

The role of the other variables is more complex, since–roughly speaking–they affect the relative weights of consumption benefits, environmental damages, and abatement costs. While abatement and individual consumption are affected, it can be shown that changing any of these parameters has no effect upon the essence of any of the propositions, provided of course that the second order conditions are respected: instead of simply requiring that $\phi \leq 2$, we need now that $\phi \leq 2k\delta$. In what follows, we present the general expressions for *per capita* consumption and total abatement in the first best,

$$q^* = \frac{k+z}{k+2\delta^2 knz+z},$$
$$v^* = \frac{\delta nz}{k+2\delta^2 knz+z},$$

which they collapse to the expressions in (8), assuming $n = \delta = k = 1$. The

[31] As a matter of fact, allowing for negative values of t^E (emissions subsidy) would reduce welfare to levels even below that of the unregulated market.

general expressions for *per capita* consumption and total abatement at the equilibria with optimally selected output or emissions tax rates are,[32]

$$q^O = \frac{2k^2}{k^2\left(4\delta^2 nz+2\right)-4\delta knz\phi+n(k+z)\phi^2},$$

$$q^E = \frac{4(k+z)\left(\delta^2 kn+1\right)-2\delta kn\phi+n\phi^2}{\left[4k\left(\delta^2 kn+2\right)\left(2\delta^2 nz+1\right)+8z\right]+4\delta kn\phi+n\left(2\delta^2 kn+1\right)\phi^2},$$

$$v^O = \frac{kn\phi}{k^2\left(4\delta^2 nz+2\right)-4\delta knz\phi+n(k+z)\phi^2},$$

$$v^E = \frac{2\delta n\left(2\delta^2 knz+2z-k\right)+n\left(1-2\delta^2 kn\right)\phi+\delta n^2\phi^2}{\left[4k\left(\delta^2 kn+2\right)\left(2\delta^2 nz+1\right)+8z\right]+4\delta kn\phi+n\left(2\delta^2 kn+1\right)\phi^2}.$$

Assuming $n=\delta=k=1$, the above expressions collapse to the ones presented in (22) and (24).[33]

Each of the above expressions is a division of polynomes in ϕ, none of them exceeding the 2nd degree. While the algebraic structure of these equilibrium values is simple in principle, extracting results is extremely tedious, especially once the above are inserted into the social welfare function in order to determine the behavior of total welfare. However, using the "Reduce" function of the *Mathematica* software we are able to verify all the results stated in the paper in this more general context. In what follows we show that the main result of our paper holds. Taking the derivative of the optimized social welfare function with respect to ϕ in both cases, of emission and output tax, yields,

$$\frac{\partial W^O}{\partial \phi} = \frac{2k^2n^2\left[2\delta kz-\phi(k+z)\right]}{\Delta_O^2},$$

$$\frac{\partial W^E}{\partial \phi} = -\frac{2n^2\left(4\delta^3k^2n+6\delta k-\phi\right)\left(\delta kn\phi+k+z\right)}{\Delta_E^2},$$

where the denominators are positive real numbers.[34] Considering the restriction

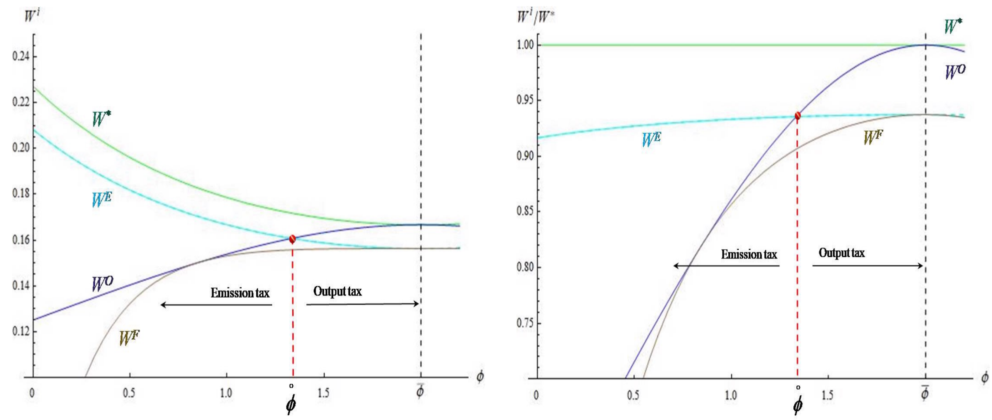
[32] The comparative statics with respect to $\{n, \delta, k \text{ and } z\}$ are left out since they present no particular interest.

[33] While positivity of quantities and abatement is guaranteed for any set of parameters, we must also have $e = \delta nq - v \geq 0$, in all cases, otherwise there is a corner solution with $v = \delta nq$. In the simplest case of first best, nonnegativity of e is guaranteed if $k > k^F = (n-1)z$; this condition is fulfilled $\forall k$ when $n = 1$. Nonnegativity of emissions imposes lower bound to k also under output or emissions tax. However in these cases the constraint can also be expressed in terms of ϕ, since in the output-tax case, $k^O = \frac{n\phi}{2\delta}$, while with tax on emissions the lower bound of k, $k^E(\phi, \cdot)$, is the larger root of a 2^{nd} degree equation in k involving all the remaining parameters in a very complex way. If we admit very low values of k (yet higher than k^F) while placing the restriction on ϕ it may be possible that some of ϕ's critical values (like for instance, $\phi^*(k,\cdot)$) lie outside the range of admissible values, without nevertheless affecting the shape of the figures. By setting $k \geq \max\{k^F, k^O, k^E\}$, we avoid having to discuss special cases of corner solutions due to the restricted range of ϕ.

[34] To be precise, $\Delta_O = \left(k^2\left(4\delta^2nz+2\right)+kn\phi(\phi-4\delta z)+nz\phi^2\right)$ and $\Delta_E = \left(n\phi^2\left(2\delta^2kn+1\right)+4\delta kn\phi+4k\left(\delta^2kn+2\right)\left(2\delta^2nz+1\right)+8z\right)$

$\phi \leq 2k\delta$, it is obvious that $\partial W^O/\partial\phi$ is positive for small values of ϕ up to $\phi = 2\delta kz/(k+z)$ and negative thereafter, while $\partial W^E/\partial\phi < 0$ for all admissible values of ϕ. Hence, Proposition 6 (a core result of the paper) holds for any admissible parameters constellation and Figure 8 keeps its "main shape": W^O is bell curved reaching the first-best welfare, while W^E is monotonically decreasing in ϕ.

The second robustness check concerns the situation in which individuals are directly harmed from pollution. To illustrate the situation, assume that instead of emissions we deal with the use of fertilizers that harm simultaneously both, the environment and consumer's health. Instead of measuring social consciousness ϕ now measures the intensity of an additional *objective* damage which must make direct part of the regulators objective function. Instead of using U we must now insert $\widetilde{U}$ from (4) into the social welfare function, exactly as in the hedonic approach. Figure 9 shows the results, assuming that $z = 3/2$.



Actual values of W^i, $i = O, E, F$ Values of W^i/W^*, $i = O, E, F$

Figure 9: Equilibrium welfare using $\widetilde{U}$ for calculating W

The LH panel illustrates W^*, W^F, W^O and W^E, with the usual colors (green, brown, blue, cyan, respectively). In the cases of unregulated market and output tax, welfare increases with ϕ up to a point, whereas W^E is always decreasing, exactly like in our previous analysis. What changes is that first-best welfare W^* becomes a decreasing function of ϕ since the latter measures the intensity of a real damage, exactly like z does.[35] The RH panel of Figure 9 shows $w^i \triangleq W^i/W^*$, $i = O, E, F$, thus measuring relative performance in terms of first-best welfare. Note now that the relative-welfare performance of an emissions tax ameliorates with ϕ, because the emissions tax targets directly the new damage

[35] The social welfare function becomes like that of the hedonic approach which is also decreasing in ϕ, as described in footnote 17 in section 2.3. Contrary to the latter, ϕ now measures the intensity of an *objective* damage inflicted upon the individual, rather than the intensity of an individual's *sensitivity* to a collectively suffered damage. For this reason, the negative relation between ϕ and social welfare in now justified.

related to ϕ. At least up to a point, relative welfare increases in all cases since the private damage leads to partial internalization of the externality.

8 Conclusions

The present paper examines the effect that the presence of environmentally aware consumers has on the choice between output and emission tax. We first find that in the absence of policy intervention, reaching high levels of consumers' environmental awareness could reduce welfare when market imperfections are quite prominent. We also find that while an emissions tax increases welfare relative to the free-market case, the overall welfare performance of an optimally chosen such tax is monotonically decreasing in consumers' environmental sensitivity. Furthermore, an output tax is also welfare improving relative to free market, but when it is imposed in markets where consumers' environmental awareness is not developed, its result is welfare inferior to that of an optimally chosen emissions tax. However, social welfare under an optimally chosen output tax is monotonically increasing in the level of environmental awareness up to a certain level. Eventually, in markets where consumers' environmental awareness has reached high levels, an output tax/subsidy welfare dominates an emission tax (both selected optimally).

While the assumptions discussed in the previous section are mostly simplifying ones, the scope of our results is limited to the case where a) consumers care about total emissions, and b) abatement effort eliminates an amount of emissions. Alternatively, consumers could care about *emissions per unit of output* and/or technology could be such that any amount spent on abatement reduces a *percentage* of emissions (or the *emissions per unit of output*). While all four possible combinations represent interesting cases each reflecting particular situations, some preliminary efforts have shown that the three combinations left beyond the scope of the paper present serious tractability issues. Finally, the combination of output or emissions taxes with other instruments in situations where consumers show environmental responsibility must receive more attention in the future.

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10 Appendix

Proposition 1 *i)* $\forall z \geq 1$, $\exists \phi^F(z) \in (0, \overline{\phi})$, *such that* $W^{F*} \triangleq W^F\left(\phi^F\right)$ *represents the unique maximum of* W^F *in the admissible interval of* ϕ; *ii)* $W^{F*} < W^*$.

Proof

Taking the derivative of (17) we obtain:

$$\begin{aligned} W^{F'} &= \frac{\partial W^F}{\partial \phi} \\ &= \frac{-2(z+3)\phi^3 + 12(z+2)\phi^2 - 8(5z+4)\phi + (3z-1)\,16}{[4+(4-\phi)\phi]^3} \end{aligned} \tag{28}$$

Since $\phi < 2$, the denominator of the above is positive. Its numerator, call it $N_{W^{F'}}(\phi, z)$, is a third degree polynome in ϕ, with $N'_{W^{F'}} = -8(5z+4) + 24(2+z)\phi - 6(3+z)\phi^2$. The latter is a trionyme with discriminant $\Delta_{N'} = -192z(7+2z) < 0$, it is therefore everywhere negative, hence $N_{W^{F'}}$ is monotonically decreasing, with $N_{W^{F'}}(z)\,|_{\phi=0} = (3z-1)\,16$, which is positive $\forall z > 1/3$. Also, recall that if $z \geq 2$, ϕ is bounded at $\phi = 2$, and $N_{W^{F'}}(\phi = 0) = -32 < 0$, whereas, if $z < 2, \phi$ is bounded by z, and

$$\begin{aligned} N_{W^{F'}}(\phi = z) &= -2\left(z^4 - 3z^3 + 8z^2 - 8z + 8\right) \\ &\propto -\left(z^2 + 3z - 8\right) z^2 - 8(z-1) \ . \end{aligned}$$

Since $z \geq 1$, for the above to be negative it suffices that $\left(z^2 + 3z - 8\right) > 0$, which holds $\forall z$, since the discriminant of the trionyme is equal to $-23 < 0$. Since the derivative $\partial W^F / \partial \phi$ is continuous, monotonically decreasing and positive at $\phi = 0$ while negative at $\phi = \min\{z, 2\}$, it follows that it becomes zero at some unique value of $\phi \equiv \phi^F(z) < \min\{z, 2\}$. The exact value of $\phi^F(z)$ is found by setting $N_{W^{F'}}(\phi) \equiv 0$, but it is difficult to obtain analytically, due to the cubic power of ϕ. The proof of the second part of the proposition is deferred to the proof of Proposition 6 where it is shown that $\forall \phi$, $W^F < W^O \leq W^*$.

Proposition 2 *i)* $\forall z \geq 1$, $\exists \widetilde{\phi}^F(z) = \frac{1}{2}\left(3z + 5 - \sqrt{3}\sqrt{3z^2 + 2z + 11}\right) \in (0, \overline{\phi})$, *such that* $\forall \phi \lesseqgtr \widetilde{\phi}^F(z)$, $D^F\left(\widetilde{\phi}^F\right) \gtreqless D^*\left(\widetilde{\phi}^F\right)$, *where* D^* *is the environmental damage at the first best given in (9); ii)* $\forall z > 1$, $\widetilde{\phi}^F \leq \phi^F$.

Proof

Substituting q, v, from (16) into the definition of social damage yields:

$$D^F(\phi) = z\left[\frac{2-\phi}{4(\phi+1)-\phi^2}\right]^2 .$$

It can be shown straightforwardly that $D^F(\phi)/\partial\phi < 0,$ as expected. Using (9) we can write:

$$\frac{D^F}{D^*} = \left[\frac{(3z+1)(2-\phi)}{4(\phi+1)-\phi^2}\right]^2.$$

Ignoring the square of the bracketed term, $D^F \geq D^* \iff (3z+1)(2-\phi) - \left[4(\phi+1)-\phi^2\right] = \phi^2 - (3z+5)\phi + 6z - 2 \geq 0$. The discriminant of the trionyme is $33 + 3z(2+3z) > 0$, and its two roots are $\phi = \frac{1}{2}\left(3z + 5 \mp \sqrt{3}\sqrt{3z^2+2z+11}\right)$. The large root is clearly greater than one, therefore rejected; the smaller root, call it $\widetilde{\phi}^F$, can be shown to be positive and smaller than $\min\{z, 2\}$, since a) for $z = 1$, $\widetilde{\phi}^F(1) = 2\left(2-\sqrt{3}\right) \approx .536$, b) $\widetilde{\phi}^F(z)$ is monotonically increasing since:

$$\begin{aligned}\frac{\partial\widetilde{\phi}^F}{\partial z} &= \frac{1}{2}\left[3 - \frac{(6z+2)\sqrt{3}}{2\sqrt{11+2z+3z^2}}\right] = \frac{\sqrt{3}\left(\sqrt{9z^2+6z+33} - 3z - 1\right)}{2\sqrt{3z^2+2z+11}} \\ &\propto \sqrt{9z^2+6z+33} - 3z - 1,\end{aligned}$$

which is positive if $\sqrt{9z^2+6z+33} > 3z+1 \iff 18z^2+12z+34 > 0$, which holds obviously, and c) $\lim_{z\to\infty} \widetilde{\phi}^F(z) = 2$.

For part *ii)*, after replacing ϕ by $\widetilde{\phi}^F$ into $\partial W^F/\partial\phi$ in (28) and performing some extremely tedious calculations, we obtain:

$$W^{F'}|_{\phi=\widetilde{\phi}} = \frac{9z^3+15z^2-9z+177+\left(3z^2+4z-31\right)\sqrt{9z^2+6z+33}}{32(3z+1)^3}.$$

Note first that, for all $z \geq 1$, $15z^2 - 9z > 9z^2 - 9z \geq 0$. The term $\left(3z^2+4z-31\right)$ can be easily shown to be positive $\forall z > \frac{1}{3}\left(\sqrt{97}-2\right) \approx 2.6163$; in this case, since also the root is positive and $9z^3 > 0$, we have that $W^{F'}\left(\phi = \widetilde{\phi}\right) > 0$. Turning to the case where $z \in \left[1, \frac{1}{3}\left(\sqrt{97}-2\right)\right]$, the positivity of $W^{F'}\left(\phi = \widetilde{\phi}\right)$ is equivalent to $9z^3 + \left(3z^2+4z-31\right)\sqrt{9z^2+6z+33} \triangleq \psi(z) > 0$, which holds $\forall z \in \left[1, \frac{1}{3}\left(\sqrt{97}-2\right)\right]$, but this can only be shown numerically.

Proof of Lemma 1

Substitution of the optimal abatement and quantity under each type of tax into (7) allows to write the social welfare function as,

$$W^{2i} = \frac{y_0^i + y_1^i t + y_2^i t^2}{\left[4(\phi+1)-\phi^2\right]^2},\ i = O, E, \tag{29}$$

where all the y-coefficients are polynomes, linear in z and quadratic in ϕ, defined below. Since the denominator of the above is strictly positive and contains no t, maximizing W^{2O} and W^{2E}, as expressed in (29) obtains $\partial W^{2i}/\partial t = y_1^i + 2y_2^i t =$

$0 \Leftrightarrow t^{MAi} = -\frac{y_1^i}{2y_2^i}$, $i = O, E$, which after replacement of the y-coefficients yields, almost immediately, the expressions in (18) and (19). Second order conditions hold whether the tax is on output or emissions, since $\partial^2 W^{2i}/\partial t^2 = 2y_2^i < 0$, due to the fact that both components of y_2^i are negative whether the tax is on output or emissions (see their definitions below).

The coefficients in (29) are as follows: $y_0^O = 6 + (8-3\phi)\phi - (2-\phi)^2 z$, $y_0^E = 2y_0^O$, and $y_j^i = \left(y_{j0}^i + y_{j1}^i z\right)$, $j = 1, 2$, $i = O, E$, with:

$y_{10}^O = -4\left(1 + 2\phi - \phi^2\right)$ $\quad y_{11}^O = 2(2-\phi)^2$

$y_{20}^O = -\left(2 + \phi^2\right)$ $\quad y_{21}^O = -(2-\phi)^2$

$y_{10}^E = -2\left[4 + \phi\left(10 - 4\phi + \phi^2\right)\right]$ $\quad y_{11}^E = 16(2-\phi)$

$y_{20}^E = -\left(12 + 4\phi + 3\phi^2\right)$ $\quad y_{21}^E = -32$

For the second part of the lemma we must examine the sign of the optimal-tax derivative with respect to ϕ. Since $t^{MAi} = -\frac{y_1^i}{2y_2^i}$,

$$\frac{\partial t^i(\phi, z)}{\partial \phi} \propto -y_1^{i\prime} y_2^i + y_2^{i\prime} y_1^i = H^i \ .$$

The derivatives $y_1^{i\prime}$ and $y_2^{i\prime}$ are also linear functions of z, of the form $y_j^{i\prime} = w_{j0}^i + w_{j1}^i z$, $i = O, E$, $j = 1, 2$. The following table shows all the coefficients of these functions:

$w_{10}^O = -8(1-\phi)$ $\quad w_{11}^O = -4(2-\phi)$

$w_{20}^O = -2\phi$ $\quad w_{21}^O = 2(2-\phi)$

$w_{10}^E = -20 + 2(8-3\phi)\phi$ $\quad w_{11}^E = -16$

$w_{20}^E = -2(2+3\phi)$ $\quad w_{21}^E = 0$

Thus, H^i can be written as,

$$H^i = -\left(w_{10}^i + w_{11}^i z\right)\left(y_{20}^i + y_{21}^i z\right) + \left(w_{20}^i + w_{21}^i z\right)\left(y_{10}^i + y_{11}^i z\right), \tag{30}$$

which in case of output tax the above translates to,

$$\begin{aligned} H^O &= -\left[-8(1-\phi) - 4(2-\phi)z\right]\left[-\left(2+\phi^2\right) - (2-\phi)^2 z\right] \\ &\quad + \left[-2\phi + 2(2-\phi)z\right]\left[-4\left(1+2\phi-\phi^2\right) + 2(2-\phi)^2 z\right] \\ &= 4z^2(\phi-2)^3 + 12z\left[(\phi-2)\phi + 2\right](\phi-2) + 8\left[\phi^3 - \phi^2 + 2(\phi-1)\right] \\ &\quad -4z^2(\phi-2)^3 - 4z\left[3(\phi-2)\phi - 2\right](\phi-2) - 8\left[\phi^3 - 2\phi^2 - \phi\right]) \\ &= -32(2-\phi)z + 8\left[\phi^2 + 3\phi - 2\right] = 8\phi^2 + (32z+24)\phi - 8(8z-1) \ . \end{aligned}$$

The last expression above is a trionyme that is negative between its roots, which are $\frac{1}{2}\left[-(3+4z) \mp \sqrt{16z^2 + 56z + 17}\right]$. The smaller of these roots is always negative but the larger, call it ϕ^{tO}, can be shown to always be less than 2. It is a bit tedious but straightforward to show that when $1 \leq z \leq \frac{1}{10}\left(5 + \sqrt{65}\right) \approx 1.306$, $\phi^{tO} \geq z$, and the trionyme is always negative since its higher root lies outside the admissible interval of ϕ values. When $z > \frac{1}{10}\left(5 + \sqrt{65}\right)$, $\partial t^O(\phi, z)/\partial\phi < (\geq)\, 0$ iff $\phi < (\geq)\, \phi^{tO}$.

When the tax is on emissions, (30) becomes:

$$
\begin{aligned}
H^E &= -\left[-20+2\left(8-3\phi\right)\phi-16z\right]\left[-\left(12+4\phi+3\phi^2\right)-32z\right] \\
&\quad +\left[-2\left(2+3\phi\right)\right]\left[-2\left[4+\phi\left(10-4\phi+\phi^2\right)\right]+16(2-\phi)z\right] \\
&= z^2\left(-80\phi^2-240\phi-180\right)+z\left(-8\phi^4-48\phi^3+14\phi^2-72\phi-136\right) \\
&\quad -2\left(9\phi^4-12\phi^3+34\phi^2-56\phi+120\right)+32(\phi-2)(3\phi+2)z \\
&\quad +4\left(3\phi^4-10\phi^3+22\phi^2+32\phi+8\right) \\
&= -512z^2-16\left[60-\phi(20-9\phi)\right]z-2\left[3\phi^4+8\phi^3-10\phi^2-120\phi+104\right] \\
&= -16(4z+1)(8z+13)+80(4z+3)\phi+(20-144z)\phi^2-6\phi^4-16\phi^3 \\
&< -16(4z+1)(8z+13)+80(4z+3)\phi+(20-144z)\phi^2 \ .
\end{aligned}
$$

Again we deal with a trionyme that is negative between its roots, which are,

$$
\frac{2\left[5\left(4z+3\right)\mp\sqrt{2}\sqrt{576z^3+800z^2-216z-145}\right]}{36z-5} \ .
$$

The smaller root can be shown to be smaller than 0, whereas the larger root can easily be shown to be > 2.

Proof of Lemma 2

Evaluating $\partial W^{2i}/\partial t$ at $t=0$ yields $\partial W^{2i}/\partial t|_{t=0} \propto y_1^i$, hence $y_1^i \geq (\leq)0$ implies that the optimal tax rate is positive (negative) which holds iff $z \geq (\leq) -\frac{y_{10}^i}{y_{11}^i}$. In case of output tax, $-\frac{y_{10}^O}{y_{11}^O} = \frac{4\left(1+2\phi-\phi^2\right)}{2(2-\phi)^2} = z_O$, while in case of emissions tax $-\frac{y_{10}^E}{y_{11}^E} = \frac{4+\phi\left(10-4\phi+\phi^2\right)}{16(2-\phi)} = z_E$.

For $\phi \in \left[0, \overline{\phi}\right]$, both z_E and z_O can be shown to be continuous and monotonically increasing in ϕ, with $z_E(0) = 1/4 < z_O(0) = 1/2 < 1$, and $\lim_{\phi\to 2} z_E = \lim_{\phi\to 2} z_O = \infty$. Thus, $\forall z \geq 1$ it is possible to define the inverse functions $\phi_O \equiv z_O^{-1}(z)$, and $\phi_E \equiv z_E^{-1}(z)$. In order to have $\phi_O < \phi_E$ it must be that $z_O > z_E$, which holds iff:

$$
\begin{aligned}
\frac{-y_{10}^O}{y_{11}^O}\frac{y_{11}^E}{-y_{10}^E} &= \frac{4\left(1+2\phi-\phi^2\right)16(2-\phi)}{2(2-\phi)^2\left[4+\phi\left(10-4\phi+\phi^2\right)\right]} > 1 \Leftrightarrow \\
h^E &\triangleq \phi^4-6\phi^3+2\phi^2+16\phi+8>0;
\end{aligned}
$$

h^E is a polynomial with four roots: $2\left(1-\sqrt{2}\right) \approx -.828$, $1-\sqrt{3} \approx -.732$, $1+\sqrt{3} \approx 2.73$, and $2\left(1+\sqrt{2}\right) \approx 4.828$. The range of admissible values of ϕ lies entirely between the 2nd and the 3rd root, therefore the polynomial has everywhere the same sign, which is the sign it of $h^E(0) = 8 > 0$, which show that $\forall\phi \in \left[0,\overline{\phi}\right]$, $z_O > z_E$, and therefore $\forall z \geq 1$, $\phi_O(z) < \phi_E(z)$.

Proposition 3 *i) In the output-tax case,* $\exists \phi^* = 2z/(1+z) \in \left(0, \overline{\phi}\right)$ *such that,* $q^O(\phi^*) = q^*$, *and* $\forall \phi \leq (\geq) \phi^*$, $\partial q^O / \partial \phi \geq (\leq) 0$; *ii)* $t^O(\phi^*) < 0$; *iii) in the emissions-tax case, the equilibrium quantity is monotonically decreasing in* ϕ, *and* $q^E(0) < q^*$.

Proof

For part i), from the first element of (22) we have,

$$\frac{\partial q^O}{\partial \phi} = \frac{8z - 4(z+1)\phi}{\left(z(\phi-2)^2 + \phi^2 + 2\right)^2} ,$$

which has the sign of its numerator, which is positive (negative) according to whether $\phi \geq (\leq) 2z/(1+z) = \phi^*$. It is easy to see that for all $z \geq 1$, $\phi^* \leq z$, and that $\lim_{z \to \infty} \phi^* = 2$.

For part ii) note that

$$\phi^* - \phi_O = \frac{z^2 + \sqrt{2}\sqrt{z+4}z + z + \sqrt{2}\sqrt{z+4} - 2}{(z+1)(z+2)};$$

the sign of the above expression depends on the sign of its numerator, which is monotonically increasing in z and equal to $2\left(\sqrt{2}-1\right) > 0$ when $z = 0$, hence it is always positive.

For part iii), the derivative of the second element in (22) is,

$$\frac{\partial q^E}{\partial \phi} = \frac{-82(12z+7) + 8(2z-3)\phi + 10\phi^2}{\left[12 + 32z + \phi(3\phi+4)\right]^2}.$$

The numerator of the above is a trionyme with discriminant $256\left(11 + 12z + z^2\right) > 0$, and roots $\frac{2}{5}\left(3 - 2z \mp 2\sqrt{z^2 + 12z + 11}\right)$. The first root is obviously negative, while the second is $\frac{2}{5}\left(3 - 2z + 2\sqrt{z^2 + 6z + 9}\right) = \frac{2}{5}\left[3 - 2z + 3(3+z)\right] > 2$, hence the trionyme is negative $\forall \phi \in \left[0, \overline{\phi}\right)$, and the quantity at the second-best equilibrium with the optimal emissions tax is decreasing in ϕ.

The sign of the terms in (23)

For the *direct effect* in the two tax-cases note from (13) that,

$$\frac{\partial q^{2O}}{\partial \phi} = -\frac{2(t-1)(2\phi-4)}{\left[4(\phi+1) - \phi^2\right]^2} < 0.$$

Similarly, from (14) we have that:

$$\frac{\partial q^{2E}}{\partial \phi} = \frac{t\left[12 - (4-\phi)\phi\right] - 4(2-\phi)}{\left[4(\phi+1) - \phi^2\right]^2} ,$$

which can be positive or negative, depending on the value of t. Evaluating the above at $t = t^E$, and performing simplifications yields

$$\frac{\partial q^{2E}}{\partial \phi}|_{t=t^E} = -\frac{4(4z+9) - 2(4z+7)\phi + 4\phi^2 - \phi^3}{\left[4(\phi+1) - \phi^2\right]\left[3\phi^2 + 4\phi + 4(8z+3)\right]} < 0.$$

The denominator of the above is clearly positive. For the numerator note first that $\forall \phi < 2$, $2\phi^2 - \phi^3 > 0$. The remaining term $4(4z+9) - 2(4z+7)\phi + 2\phi^2$ is positive for values of ϕ outside the roots which are $\frac{1}{2}\left(7 + 4z \mp \sqrt{16z^2 + 24z - 23}\right)$. The larger root is clearly > 2, and the same holds for the small one, since $\frac{1}{2}\left(7 + 4z - \sqrt{16z^2 + 24z - 23}\right) < 2$ implies $(3+4z)^2 < 16z^2 + 24z - 23$, which is impossible for $z \geq 1$. Hence, the numerator is also positive and the entire expression is negative.

The second term of the *indirect effect* has been shown to be negative for both the O and E cases in Lemma 1. Concerning its first term, from (13) and (14) it is easy to see that $\partial q^{2O}/\partial t = -2/B < 0$, and $\partial q^{2E}/\partial t = -(2-\phi)B < 0$, hence the sign of the indirect effect is positive in both cases.

Proposition 4: $\forall z \geq 1$, *i) The function* $v^O(\phi)$ is initially increasing and peaks at some value of $\phi = \sqrt{2}\sqrt{\frac{2z+1}{z+1}} \triangleq \phi^{vO}$; *at* $\phi = \phi^*$, $v^O(\phi^*) = v^*$ *and* $\left(\partial v^O/\partial \phi\right)|_{\phi^*} > 0$. *ii) With an emissions tax,* $v^E(0) < v^*$, *and* $\forall \phi \leq \phi_E$, $\partial v^E/\partial \phi < 0$.

Proof Part i) is obtained by simple replacement of ϕ^* from Proposition 3 into the first expressions in . From (24) we have that $\left(\partial v^O/\partial \phi\right) = \left[2(1+2z) - (1+z)\phi^2\right] / \left[z(\phi-2)^2 + \phi^2 + 2\right]^2$, the denominator of which is positive while the sign of its numerator is positive (negative) when

$$\phi < (>) \sqrt{2}\sqrt{\frac{2z+1}{z+1}} \triangleq \phi^{vO}$$

After substituting ϕ^* for ϕ and in the numerator of $\partial v^O/\partial \phi$ and simplifying, the latter becomes $2(1+2z) - (1+z)\phi^{*2} = \frac{2(3z+1)}{z+1} > 0$.

For part ii) we set first $\phi = 0$ in the expression for v^E in (24) to obtain $v^E(0) = (4z-1)/[2(8z+3)] < z/(1+3z) = v^*$. Concerning the derivative of v^E, we have:

$$\frac{\partial v^E}{\partial \phi} = \frac{-(64z+4) + 4(4z+9)\phi + 7\phi^2}{(32z + \phi(3\phi+4) + 12)^2}, \tag{31}$$

the denominator of which is positive, while the numerator is a trionyme negative within its roots, $\frac{2}{7}\left(-(4z+9) \mp 2\sqrt{2}\sqrt{2z^2 + 23z + 11}\right)$. The small root is obviously negative whereas the large root, call it ϕ_h, is increasing in z and equals 2 for $z = 3$. Hence, for values of $z \geq 3$, the equilibrium abatement is monotonically decreasing in ϕ everywhere, while for $1 \leq z \leq 3$ it is also monotonically decreasing but only for $\phi \leq \phi_h$. What we show next is that ϕ_h is above ϕ_E, the highest value of ϕ that a positive emissions tax exist, and therefore even for low values of z, $\partial v^E/\partial \phi$ cannot be positive.

Note first that,

$$t^E(\phi^*) = -\frac{z^2+4z+1}{8z^4+32z^3+40z^2+19z+3} < 0,$$

which combined with the fact that $\partial t^E/\partial\phi < 0$ (see Lemma 1) shows that $\phi^* > \phi_E$ (at ϕ^* the emissions tax is already zero). What is left to show is that $\phi_h > \phi^*$. Replacing the part of ϕ_h that is under square-root by R yields:

$$\begin{aligned}
\phi_h - \phi^* &= \frac{4(R-8)}{7} - \frac{8z}{7} + \frac{2}{z+1} > 0 \Longleftrightarrow \\
R &> \frac{4z^2+20z+9}{2(z+1)} \Longleftrightarrow \\
0 &< R^2 - \left[\frac{4z^2+20z+9}{2(z+1)}\right]^2 \Longleftrightarrow \\
0 &< \frac{7\left(8z^3+1\right)}{4(z+1)^2},
\end{aligned}$$

which holds $\forall z > 0$. Hence, $\phi_h > \phi^* > \phi_E$, and the negativity of $\partial v^E/\partial\phi$ in (31) is guaranteed even for $z \in [1,3]$.

Proposition 5 *i)* $\exists\phi = \widetilde{\phi}$ *such that* $\forall\phi \leq (\geq)\,\widetilde{\phi}$, $\partial D^O(\phi)/\partial\phi \geq (\leq) 0$; *ii)* $\forall\phi > \phi_0$, $D^O > D^F$ *while at* $\phi = \phi^*$, $D^O(\phi^*) = D^*$; *iii) under an optimal emissions tax,* $\forall\phi \in [0, \phi_E]$, $D^E(\phi) < D^*$, *and* $\partial D^E(\phi)/\partial\phi < 0$.

Proof

Since in all cases $D = z \cdot e^2$, we prove the proposition though e.

For the part of the proposition referring to the damage under an output tax, note first that from (26): $e^O|_{\phi=0} = (2z+1)^{-1} > e^O|_{\phi=\phi^*} = (3z+1)^{-1} = e^*$, hence at ϕ^*, environmental damage is equal to that of first-best.[36] From (26) we get,

$$\frac{\partial e^O}{\partial\phi} = \frac{(z+1)\phi^2 - 4(z+1)\phi + 4z - 2}{\left[z(\phi-2)^2 + \phi^2 + 2\right]^2},$$

which has the sign of its numerator. The latter has discriminant equal to $24(z+1) > 0$ and is positive outside its roots which are $\phi = 2 \mp \sqrt{6(z+1)^{-1}}$. The larger root is clearly greater than 2, while the smaller root–name it $\widetilde{\phi}$– can easily be shown to be positive and smaller than $\min\{2, z\}$, for all $z > 1$.[37] This implies that $\partial e^O/\partial\phi$ changes sign, being positive (negative) $\forall\phi < (>)\,\widetilde{\phi}$. Note that $\widetilde{\phi} < \phi^* \iff \left[2(1+z)^{-1}\right]^2 - 6(1+z) = -2(1+3z)(1+z)^{-2} < 0$.

[36] It could not have been otherwise since $q^O(\phi^*) = q^*$ and $v^O(\phi^*) = v^*$.

[37] As a matter of fact, the smaller root belongs to $[0, 1)$ for all $z \geq 1/2$.

Thus, environmental damage increases with ϕ in the interval $\left[0, \widetilde{\phi}\right)$, reaches a maximum at $\widetilde{\phi} < \phi^*$ and becomes decreasing after that value.

For the part of the proposition referring to the damage under an emissions tax recall from (26) that,

$$e^E = \frac{10 - \phi}{3\phi^2 + 4\phi + 32z + 12}.$$

Evaluating emissions at $\phi = 0$ we can easily verify that $e^E|_{\phi=0} = 10 \cdot [4(8z+3)]^{-1} < (3z+1)^{-1} = e^*$, which represents emissions at first best. Also, while the proof is very tedious, it can be shown that $\forall \phi \in [0, \phi_E]$,

$$\frac{\partial e^E}{\partial \phi} = \frac{-32z + 3(\phi - 20)\phi - 52}{(32z + \phi(3\phi + 4) + 12)^2} < 0.$$

and since e^E is decreasing in ϕ while e^* is independent of ϕ , we have that $\forall \phi$, $e^E < e^*$ and consequently $D^E < D^*$.

Proposition 6 *A) With an optimally selected output tax: i)* $\forall \phi$, $W^O(\phi) \geq W^F(\phi)$*; ii)* $\forall \phi \leq (\geq) \phi^*$, $\partial W^O / \partial \phi \geq (\leq) 0$, *and* $W^O(\phi^*) = W^*$*. B) With an optimally selected emissions tax: i)* $\forall \phi \in [0, \phi_E]$, $W^* > W^E > W^F$*; ii)* $\partial W^E \left(t^E(\phi), \phi\right) / \partial \phi < 0$*. C)* $\forall z \geq 1, \exists \phi = \mathring{\phi}(z) \in [0, \phi_E)$ *such that* $\forall \phi \leq (\geq) \mathring{\phi}$, $W^E(\phi) \geq (\leq) W^O(\phi)$*.*

Proof

Part A).

In order to prove part i) we simply manipulate the difference of the corresponding expressions (27) and (17) to obtain,

$$W^O - W^F = \frac{\left\{z(2-\phi)^2 - 2\left[(2-\phi)\phi + 1\right]\right\}^2}{\left[4 - (4-\phi)\phi\right]^2 \left[\phi^2(z+1) - 4\phi z + 4z + 2\right]}.$$

The sign of the denominator of the above expression depends on the sign of the second expression in square brackets, which is a trionyme in ϕ with discriminant equal to $-8(1+3z) < 0$, therefore positive everywhere. The numerator can never be negative but can become zero when ϕ takes the value $\phi_O = \left[2(z+1) + \sqrt{2(4+z)}\right] / (2+z)$, hence, $\forall \phi \neq \phi_O$, $W^O > W^F$, while for $\phi = 0$ $(t^O = 0)$, $W^O = W^F$.

In order to prove ii) we manipulate the derivative of (27) to obtain:

$$\frac{\partial W^O}{\partial \phi} = -\frac{2(z\phi - 2z + \phi)}{\left(z\phi^2 - 4z\phi + 4z + \phi^2 + 2\right)^2};$$

it can be shown that the sign of the numerator of the above is positive (negative) when $\phi \leq (>) 2z/(1+z) = \phi^*$. Substituting ϕ^* into W^O in (27) and simplifying yields that $W^O(\phi^*) = \frac{1+z}{2(13z)} = W^*$.

Part B).

We can write (27) as,

$$W^E(\phi, z) = \frac{4(1+2z) + 4(1+2z)\phi + \phi^2}{5 + 18z + 2(5+12z)\phi + (8z+10)\phi^2}.$$

For the first part, note that $W^E(0) = [5(1+z)] / [4(3+8z)] < (1+z) / [2(3+z)] = W^*$. Also,

$$W^E(\phi; z) - W^F(\phi) = \frac{\{8z(\phi-2) + \phi[(\phi-4)\phi+10] + 4\}^2}{2[4-(4-\phi)\phi]^2 \left(3\phi^2 + 4\phi + 32z + 12\right)} > 0$$

since the numerator and the first part of the denominator are squares and the second part of the denominator (the one in parenthesis) is a trionym with discriminant equal to $-128(1+3z)$, therefore positive. Turning to the derivative, we obtain:

$$\frac{\partial W^E}{\partial \phi} = -\frac{2(10-\phi)(z+\phi+1)}{(32z + \phi(3\phi+4) + 12)^2} < 0;$$

since $W^E(0) < W^*$, the above implies also that $W^E < W^*$ everywhere.

Part C).

Note that $W^E(0) = [5(1+z)] / [2(1+3z)] > [2(1+2z)]^{-1} = W^O(0)$. As ϕ moves away from 0, W^E decreases while W^O increases eventually reaching W^* at $\phi = \phi^*$. Since $W^* > W^E(0)$, $W^O(\phi^*) > W^E(0)$ therefore the curves $W^O(\phi)$ and $W^E(\phi)$ must cross at least once. It remains to show that this crossing happens while the emissions-tax rate is positive. For this, we replace z from (20) in both W^O and W^E, to obtain after simplifications:

$$W^O(\phi, z_E(\phi)) = \frac{8}{[4-(4-\phi)\phi][(\phi-2)\phi+6]}, \quad W^E(\phi, z_E(\phi)) = \frac{(2-\phi)\phi+10}{8[(4-\phi)\phi+4]}.$$

Thus:

$$W^O(\phi, z_E(\phi)) - W^E(\phi, z_E(\phi)) = \frac{[(2-\phi)\phi+2]^2}{8\left[\phi^2 - 4\phi + 4\right]\left[\phi^2 - 2\phi + 6\right]};$$

the numerator of the above is clearly positive, while the denominator is proportional to the product of the two terms in square brackets, both trionyms in ϕ. The discriminant of the second trionym is equal to -20, so it can be immediately seen that the trionym is everywhere positive. The discriminant of the first trionym is equal to 0, making obvious that the trionyme is positive everywhere except for its root, which is equal to 2, hence $\forall \phi \leq 2$, the above difference is positive. Combining the latter with the fact that both $\partial W^O/\partial\phi > 0$ and $\partial W^E/\partial\phi < 0$, proves that the intersection of the W^O and W^E curves occurs at some $\phi = \mathring{\phi} < \phi_E$.