A Regression-based Improvement to the Multiple Criteria ABC Inventory Classification Analysis

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ABSTRACT: The aim of this paper is to propose a regression-based approach for obtaining a set of weights for multi-criteria ABC inventory analysis, which differ across classification criteria but are common across inventory items and follow a predetermined descending ordering scheme regarding the relative importance of classification criteria. The proposed alternative is based on the Inequality Constrained Least Squared model and is to be compared with the existing linear and non-linear programming models available in the literature.

KEYWORS: Multi-criteria analysis; ABC inventory; Regression methods

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1. Introduction

The ABC analysis is perhaps the most widely used method in inventory management aiming to classify items into three ordered classes: class A contains a relatively small number (5-10%) of the most important items, class B includes a larger number (20-30%) of items with moderate importance while the remaining (50-70%) items with relatively little importance belong to class C.[1] Initially, the ABC analysis was based on a single classification criterion, namely the annual dollar usage, but soon it was recognized that a number of other criteria, such as inventory cost, lead time, and several others listed in Hu *et al*. (2018) Appendix B, may also be useful for obtaining a satisfactory classification of inventory items. Flores and Whybark (1986, 1987) are the first to consider the ABC analysis as a multi-criteria problem, where a score summarizing achievements across the considered criteria is used first to rank all items and then to classify them into three classes. Since then, several alternative approaches (accompanied with a large number of empirical studies)[2] have been used to deal with the multi-criteria ABC inventory classification problem, which according to Douissa and Jabeur (2020) can be grouped into four categories: (*i*) those using mathematical programming; (*ii*) those relying on artificial intelligence and meta-heuristics; (*iii*) those employing multi-criteria decision making techniques; and (*iv*) those based on hybrid methods.

Our work in this paper is linked with the literature in the first category, where alternative linear programming models are used to estimate items' score by means of a weighted average of the considered criteria measures. This may be accomplished by using either the conventional (Ramanathan, 2006) or the normalized (Ng, 2007) Benefit-of-the-Doubt (BoD) model.[3] In the normalized BoD model, the measures of all criteria (which may not be in a common scale) are normalized in such a way that their values range between zero and one and their aggregation weights are forced to add up to one while neither of these is necessary in the conventional BoD model. In practical terms, the main advantage of the normalized BoD model is that is less computationally demanding as it contains a smaller number of constraints than the conventional BoD model. In addition, if the classification criteria can be ranked in a descending order of importance then as Ng (2007) showed one can obtain the scores of the inventory items (which have values between zero and one) without a linear optimizer but rather based on the maximum partial average. Besides that, we can impute the weights of every classification criteria associated with an item's score. In contrast, Hadi-Vencheh (2010, p. 962) claimed that "the Ng-model leads to a situation where the score of each item is independent of the weights obtained from the model. That is, the weights do not have any role for determining total score of each item."

The first objective of this paper is to examine this issue. For this purpose, in the second section of the paper we show how to derive explicitly the weights assigned to various classification criteria and we verify that they are directly related to an item's score. These weights may be seen as quasi flexible in the sense that, even though they differ across classification criteria and items, there is only a limited number of weights profiles, the maximum number of which is equal to the number of classification criteria considered in the analysis. One such a weights profile, that results in the maximum score of unity for an inventory item performing well in terms of only one criterion, which happens to be the most valuable one, assigns a weight of one to that criterion and zero to all others. More generally, for some items, a number of criteria is making no contribution to the resulting score and this number may range from 1 to $J-1$, where J is the number of the considered classification criteria.

On these grounds, Hadi-Vencheh (2010) refinement of the Ng (2007) model could be proved useful as it overcomes the aforementioned shortcomings. Using non-linear programming, Hadi-Vencheh (2010) proposed a model that delivers a set of weights that differ across classification criteria but are common across inventory items.[4] This common-weights scheme avoids the problem of obtaining a deceiving score of one for an item that performs well in terms of only one criterion. In addition, as a common-weights scheme can, according to Kao and Hung (2005a) and Wang, Luo and Liu (2011), be used both to compare and to rank inventory items as all of them are evaluated on the same basis. Another advantage of the common-weights scheme is that it can be applied to assess performance for inventory items not being in the sample (Kao and Hung, 2007).

The second objective of this paper is to provide an alternative for obtaining a common-weights scheme, inspired by Kao and Hung (2003) compromise solution and based on a linear regression model. As we explain in the third section of the paper, the proposed alternative proceeds in two steps: in the first step, we use the maximum partial average to obtain the scores of inventory items based on the Ng (2007) model and then in the second stage, we use the inequality constrained least squared (ICLS) model (see Judge and Takayama, 1966; Liew, 1976; Judge *et al*., 1985), to regress the scores of inventory items from the first stage on all partial averages under the restrictions that the estimated parameters are non-negative and sum up to one. These estimated parameters are then used to obtain a new score. As with Hadi-Vencheh (2010) approach, we are able to obtain scores of inventory items based on a set of common weights.

The main advantages of the proposed approach compared to that of Hadi-Vencheh (2010) are: *first*, we can examine the statistical significance of the assumed descending ordering restrictions and infer on whether they are supported by the data at hand. In the case that they are statistically rejected, alternative descending ordering restrictions may be introduced and tested. *Second*, we are able to provide not only point estimates of item's scores but also their lower and upper bounds that can be used to construct confidence intervals. Thus, we can judge in statistical grounds the classification status of inventory items by examining whether the lower and the upper bounds of scores result in the same class categorization. Both statistical significance and confidence intervals are inherent features of econometric but not of linear or non-linear programming models. Their aim in the multiple criteria ABC classification analysis is to provide useful insights, based on statistical grounds, about the assumed descending ordering scheme, the estimated aggregation weights and the classification status of the inventory items.

2. Derivation of the Criteria Weights

Consider the following multiple criteria ABC inventory classification model used by Ng (2007):

$$S^k = \max_{w_j^k} \sum_{j=1}^{J} w_j^k y_j^k$$
$$st \sum_{j=1}^{J} w_j^k = 1 \qquad (1)$$
$$w_j^k \geq 0 \qquad j = 1, \dots, J$$
$$w_j^k - w_{j+1}^k \geq 0 \qquad j = 1, \dots, J-1$$

where *S* refers to the score of the inventory items, *y* to the normalized measure of the *j*th classification criteria defined as $y_j^k = \left(I_j^k - \min_k I_j^k\right)/(\max_k I_j^k - \min_k I_j^k)$, *w* to classification criteria weights, and *k=1,…,K* is used to index inventory items and *j=1,…,J* classification criteria. This is essentially the normalized BoD model augmented with the last inequality constraint that ranks the classification criteria in a descending order of importance. Its solution results in a set of inventory-specific classification criteria weights that lie within the bounds imposed by the three constraints in (1) and maximize the score of the evaluated item.

Ng (2007) showed however that there is no need to solve (1) as the scores of the inventory items can obtained by using the maximum of partial averages, namely $S^k = \max_j \left\{\left(\frac{1}{J}\right) \sum_{j=1}^{J} y_j^k, j = 1, .., J\right\}$. To do this, one has to rewrite (1) as:

$$S^k = \max_{u_j^k} \sum_{j=1}^{J} u_j^k x_j^k$$
$$st \sum_{j=1}^{J} j u_j^k = 1 \qquad (2)$$
$$u_j^k \geq 0 \qquad j = 1, \dots, J$$

when $u_j^k = w_j^k - w_{j+1}^k, u_J^k = w_J^k$ and the x_j^k's are the partial sum of the y_j^k's, namely $x_1^k = y_1^k, x_2^k = \sum_{j=1}^{2} y_j^k$, …, and $x_J^k = \sum_{j=1}^{J} y_j^k$. From (2) one can derive both the score of the inventory item and the weights of the classification criteria associated with this score as follows: if $S^k = \max_j \left\{\left(\frac{1}{J}\right) \sum_{j=1}^{J} y_j^k, j = 1, .., J\right\} = y_1^k$ then $u_1^k = 1$ and $u_{j>1}^k = 0$, which in turn implies that $w_1^k = 1$ and $w_{j>1}^k = 0$ and thus, the resulting weights profile is: $\{1,0, \dots ,0\}$. If $S^k = \max_j \left\{\left(\frac{1}{J}\right) \sum_{j=1}^{J} y_j^k, j = 1, .., J\right\} = \frac{1}{2}(y_1^k + y_2^k)$ then $u_2^k = 1/2$ and $u_1^k = u_{j>2}^k = 0$, which in turn implies that $w_1^k = w_2^k = 1/2$ and

$w^k_{j>2} = 0$ and thus, the resulting weights profile is: $\left\{\frac{1}{2}, \frac{1}{2}, 0, \ldots, 0\right\}$. If instead $S^k = \max_j\left\{\left(\frac{1}{J}\right)\sum_{j=1}^{J} y^k_j, j = 1, \ldots, J\right\} = \frac{1}{3}(y^k_1 + y^k_2 + y^k_3)$ then $u^k_3 = 1/3$ and $u^k_1 = u^k_2 = u^k_{j>3} = 0$, which in turn implies that $w^k_1 = w^k_2 = w^k_3 = 1/2$ and $w^k_{j>3} = 0$ and thus, the resulting weights profile is: $\left\{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, \ldots, 0\right\}$, and so on up to the case where $S^k = \max_j\left\{\left(\frac{1}{J}\right)\sum_{j=1}^{J} y^k_j, j = 1, \ldots, J\right\} = \left(\frac{1}{J}\right)\sum_{j=1}^{J} y^k_j$. Then, $u^k_J = 1/J$ and $u^k_{j<J} = 0$, which in turn implies that $w^k_1 = \cdots = w^k_J = 1/J$ and thus, the resulting weight profile corresponds to equal weights, namely $\left\{\frac{1}{J}, \ldots, \frac{1}{J}\right\}$.

From the above we can derive a number of useful practical rules: *first*, the number of possible weights profiles the Ng (2007) model is equal to the number of classification criteria. *Second*, if all classification criteria contribute to the score of an inventory item then we end up with an equal-weights scheme with the weights being equal to *1/J*. *Third*, if only some of the classification criteria contribute to the score of an item, all classification criteria contributing to the score of an item receive the same weight and the remaining receive no weight. The magnitude of the weights depends on which partial average defines the maximum. *Fourth*, if the normalized measure of the most valuable criteria (i.e., the one that is ranked higher than the others) is larger than those of the other criteria, then the score of this inventory item is equal to the normalized measure of the most valuable criteria, which receives a weight that is equal to one and consequently, no weight is assigned to the remaining criteria. This practical rule does not hold however for the lower ranked criteria. *Fifth*, an inventory item reaches the maximum score of unity only in the case that has the maximum possible normalized value for the most valuable criteria.

Based on the above, one can derive the weights profile associated with items' scores using the partial averages reported by Ng (2007, p. 351) in his Table 2. There are eight inventory items, namely #1, #3, #4, #5, #6, #7, #8 and #11, with a weights profile of (1, 0, 0), five inventory items, namely #2, #10, #25, #27, and #30, with a weights profile of (1/2, 1/2, 0), and the remaining 34 items have a weights profile of (1/3, 1/3, 1/3). Thus, the classification of more than two thirds (72%) of the considered inventory items is based on an equal-weights scheme. A closer look of Ng (2007) results, in his Tables 2 and 3, reveals that half of the inventory items in class A, namely #1, #3, #4, #5, and #6, base their scores in only one well-performing criterion, which is also considered to the be the most valuable. In contrast, the vast majority of items in classes B and C base their scores in the equal-weights scheme. These findings cast some doubts about the classification ranking of items obtained by using the quasi-variable weights scheme of (2). They also point towards the need for a common-weights scheme that resolves these shortcomings.

3. A Regression-based common weights model

In an attempt to this direction we propose a regression-based common-weights model as an alternative to the non-linear programming model of Hadi-Vencheh (2010). The proposed model proceeds in two steps: in the first, we estimate the inventory items scores using Ng (2007) procedure of partial averages and then in the second step, we use a regression model of the form

$$S^k = \sum_{j=1}^{J} u_j x_j^k + \varepsilon^k \tag{3}$$

to obtain a set of common weights under the restrictions that $\sum_{j=1}^{J} j u_j = 1$ and $u_j \geq 0$ by assuming that $\varepsilon^k \sim N(0, \sigma^2)$.[5] Note that (3) is a linear regression model without an intercept term, where the classification criteria weight differences, i.e., the u_j's, are common across items parameters to be estimated. As a result, (3) may be seen as the least squared difference (also referred to as dissatisfaction by Kao and Hung (2005b)) between the variable- and the common-weights score of an inventory item under the assumed descending ordering scheme.

The above regression model with one equality and three inequality constraints in our case can be estimated by ICLS (Liew, 1976). However, the estimation burden may be reduced to a great extent by using the equality constraint to eliminate one of the u_j's and then estimating the resulting model with ICLS.[6] Solving $\sum_{j=1}^{J} j u_j = 1$ for let say $u_1 = 1 - \sum_{j=2}^{J} j u_j$ and by substituting it into (3) results in:

$$\underline{S}^k = \sum_{j=2}^{J} u_j \underline{x}_j^k + \varepsilon^k \tag{4}$$

where $\underline{S}^k = S^k - x_1^k$ and $\underline{x}_j^k = x_j^k - j x_1^k$ for $j = 2, 3$. Then, (4) is estimated by ICLS under the two inequality constraints $u_j \geq 0$ for $j = 2, 3$.

The ICLS estimator is obtained as follows (see Judge *et al*. 1985, pp.62-64): if all the inequality constraints are redundant then the ICLS estimator coincides with the ordinary least squared (OLS) estimator. On the other hand, if all of the inequality constraints are binding then the ICLS estimator coincides with the restricted OLS estimator. If however only some of the inequality constraints are binding then the ICLS estimator is given by the restricted OLS estimator imposing those constraints that are binding. This means that, for J independent constraints, the ICLS estimator is determined by a choice rule that selects among (at most) the 2^J different restricted and unrestricted estimators. In (4) we have $J=2$ and thus there are at most four possible solutions, of which only three are feasible: (a) neither inequality constraint is binding and thus, we use the unrestricted OLS estimator; (b) only the first inequality constraint is binding and thus, we use the restricted OLS estimator with the first constraint imposed; and (c) only the second inequality constraint is binding and thus, we use the restricted OLS estimator with the second constraint imposed. The fourth possible solution, in which both inequality constraints are binding, is not feasible because, in the absence of an intercept term, no independent variable (i.e., regressor) is left in (4). If the estimated parameters of more than one of the above regressions satisfy the inequality constraints then we select the one yielding the lowest weighted sum of squared residuals (Wolak, 1989).

Based on the statistical significance of the estimated parameters in (3) we can test whether the importance of the corresponding classification criteria is different than zero and thus justify in statistical grounds their inclusion in the analysis. In addition, using the standard deviation of the estimated parameters in (3) we can construct confidence intervals for the common-weights scores of the inventory items $\tilde{S}^k = \sum_{j=1}^{J} w_j y_j^k$ that provide useful information about the robustness of the resulting ABC classification. On the other hand, we can also test whether the data at hand support the parametric restrictions, i.e., $\sum_{j=1}^{J} w_j = 1$ and $w_j \geq w_{j+1}$ for $j=1,..,J-1$, imposed in (2) and are thus consistent with the proposed model specification.

4. Empirical Results

To illustrate our approach we are considering the same multi-criteria inventory classification problem as Ng (2007) and Hadi-Vencheh (2010), based on data given by Flores, Olson and Doral (1992). The data refers to an inventory with 47 items and following Ng (2007) three classification criteria, namely annual dollar usage, average unit cost, and lead time, are used, all of which are positively related to the score of the inventory items. Their values are presented in the first four columns of Table 1. In addition, for comparison purposes, we maintain the same distribution of class A, B and C items as in Ng (2007) and Hadi-Vencheh (2010), namely 10 items in class A, 14 in class B and 23 in class C.

The estimated parameters required to compute the values of item scores based on our regression-based common weights model are reported in Table 2, where in its first two columns we present the unrestricted OLS estimates of (3), in the next two the unrestricted OLS estimates of (4), and in the last two the ICLS estimates of (4). The unrestricted OLS estimation of (3) results in a negative but statistically insignificant estimated value of u_2 that violates one of the inequality restrictions and in a value of $\sum_{j=1}^{J} ju_j$ that is greater than one. Consequently, the resulting criteria weights, reported at the bottom of Table 2, violate the restriction that $w_2 \geq w_3$ and also they do not sum up to one. On the other hand, the unrestricted OLS estimation of (4) results in a negative and statistically significant estimated value of u_2 that violates one of the inequality restrictions and results into a negative value for w_2. Nevertheless, the estimated value of u_3 is positive and thus, it is in accordance with the second inequality constraint.

Based on these, we impose the restriction that $u_2 = 0$ and we re-estimate (4) to obtain the ICLS estimates. In this case, the estimated value of u_3 is positive and statistically significant, the resulting value of u_1 is also positive, and $\sum_{j=1}^{J} ju_j = 1$ (see Table 2).[7] To derive the corresponding criteria weights and their statistical significance notice, on the one hand, that $u_j^k = w_j^k - w_{j+1}^k$ and $u_J^k = w_J^k$ imply in our case that $w_3 = u_3$, $w_2 = u_3$ and $w_1 = u_1 + u_3$ since $u_2 = 0$. On the other hand, $\sum_{j=1}^{J} ju_j = 1$ which implies that $u_1 = 1 - 3u_3$ and thus, $w_1 = 1 - 2u_3$. Then, the resulting common-weights profile is give as (0.529, 0.239, 0.239) and it is consistent with both the assumed descending ordering scheme of $w_1 \geq w_2 \geq w_3$ and the adding-up restriction $\sum w_j = 1$. Moreover, from the above it is clear that the statistical significance of the resulting criteria weights hints on the statistical significance of u_3, which as we can see from Table 2 it is statistically significant. In addition, following Kode and Palm (1986), we test statistically the hypotheses that $u_3 \geq 0$ and that

$u_2 = 0$ and $u_3 \geq 0$.[8] At the 5% level of significance, we cannot reject the first of these hypotheses but we reject the second one. This means that the descending ordering scheme of $w_1 \geq w_2 \geq w_3$ used in previous as well as the present studies is not actually supported by Flores, Olson and Doral (1992) data.[9]

The resulting common-weights profile (0.529, 0.239, 0.239) is then used to compute the values of the item's scores and their classification status, reported in the last two columns of Table 1. In Table 3 we provide confidence intervals of these scores using the lower and the upper bounds of the aforementioned classification weights, which are obtained by adding and subtracting the standard deviation of u_3 to its ICLS-based estimated value and by using the relations $w_1 = 1 - 2u_3$ and $w_2 = w_3 = u_3$. The resulting bounds are [0.478, 0.566] for w_1 and [0.217, 0.261] for w_2 and w_3. From the results reported in Table 3 and portrayed in Figure 1 we can see that there is only little variation around the estimated items' scores and more importantly, there are no changes in the classification status of items, except for items #22 and #34. Consequently, our classification results can be considered as being rather robust.

Compared to previous studies, we see that despite the substantial differences in item scores there are only few differences in the classification status of items. In particular, the differences between Ng (2007) and our results concern the classification of items #6 and #8 in class A or in class B and the classification of items #15 and #33 in class B or in class C while the differences between Hadi-Vecheh (2010) and our results concern the classification of items #8 and #14 in class A or in class B and the classification of items #15 and #33 in class B or in class C.[10] All in all, the differences are related to five items; namely items #6, #8, #14, #15 and #33. Item #6 appears as an A-class item according to Ng (2007) but as a B-class item according to the other two studies; item #8 appears as an A-class item according to our results but as a B-class item according to the other two studies; item # 14 appears as an A-class item according to Hadi-Vecheh (2010) but as a B-class item according to the other two studies; item #15 appears as a B-class item according to our results but as a C-class item according to the other two studies; and item #33 appears as a C-class item according to our results but as a B-class item according to the other two studies.

We may also examine how the classification status of these five items changes according to the assigned weight criteria. Notice that in Ng (2007) the weight profile assigned to items #6 and #8 is (1, 0, 0) while to items #14, #15 and #33 is (1/3, 1/3, 1/3). Thus, with the common-weights profile, item #6 turns into class B while by weighing only the most valuable criterion it may turn into class A. On the other hand, with the common-weights profile, item #8 may turn into either class A or class B (depending on whether its score is obtained respectively by regression analysis or quadratic programming) while it turns into class B by weighting only the most valuable criterion. Similarly, with the equal-weights profile, item #15 turns into class C while with the common-weights profile it may turn into either class B or class C (depending on whether its score is obtained respectively by regression analysis or quadratic programming). Lastly, with the equal-weights profile, item #33 turns into class B while with the common-weights profile it may turn into either class B or class C (depending on whether its score is obtained respectively by quadratic programming or regression analysis).

5. Concluding Remarks

In this paper, we propose a regression-based approach for obtaining a set of weights for multi-criteria ABC inventory analysis, which differ across classification criteria but are common across inventory items and follow a predetermined descending ordering scheme regarding the relative importance of classification criteria. The proposed alternative is based on the ICLS model where the scores obtained by means of the Ng (2007) model (i.e., based on the maximum partial average of criteria values) are regressed on all partial averages of criteria measures under the restrictions that the estimated parameters are non-negative and sum up to one. These estimated parameters are then used to obtain a new score, which is essentially a common-weights metric allowing for complete comparison and ranking. The main advantage of the proposed approach compared to Hadi-Vencheh (2010) non-linear programming one is that we can provide statistical inference with respect to both the significance of each classification criteria and the estimated items' scores. Nevertheless, the accuracy of the estimated aggregation weights depends on, as it is the case in all econometric models, the underlying assumptions, namely that of uncorrelated and orthogonal to the error term regressors, and the degrees of freedom, i.e., the difference between the number of observations and the number of explanatory variables. In our case, the accuracy of the estimated aggregation weights may be questioned when the difference between the number of evaluated inventory items and the number of the considered criteria is less than twenty; i.e., the degrees of freedom are less than twenty--a well-known econometric criterion.

Our empirical results confirm the importance of the three classification criteria considered under the assumed descending ordering scheme, namely that annual dollar usage is at least as important as average unit cost, which is turn is at least as important as lead time. In addition, the confidence interval for the estimated scores is quite tided providing evidence about the robustness of the implied classification status of the inventory items. On the other hand, despite the substantial differences in items scores compared to previous studies, we find few differences in the classification status of items, which are related to only five out of the forty-seven evaluated items.

The applicability of the proposed regression-based approach is not limited to the ABC inventory classification analysis but it can be extended into a number of other multiple criteria problems such as supplier selection examined for example by Ng (2008), assessment of faculty members' research productivity considered for example in Karagiannis and Paschalidou (2017), and countries' overall ranking in Olympic games as long as a descending order of importance can be assigned to the evaluation criteria. For example, journal publication are usually considered more important than chapters in books, which in turn are considered more important than conference proceedings and in a similar fashion, gold metals are more prestige than silver metals which in turn are more prestige than brose metals.

Table 1: Data and Empirical Results

Item	Annual dollar usage	Average unit cost	Lead time	Ng (2007) results		Hadi-Vencheh (2010) results		Proposed model results	
1	5840.64	49.92	2	1.00	A	1.0379	A	0.6141	A
2	5670	210	5	0.99	A	1.5457	A	0.9050	A
3	5037.12	23.76	4	0.86	A	0.9558	A	0.5911	A
4	4769.56	27.73	1	0.82	A	0.8273	A	0.4522	A
5	3478.8	57.98	3	0.59	A	0.7225	A	0.4513	A
6	2936.67	31.24	3	0.50	A	0.5964	B	0.3715	B
7	2820	28.2	3	0.48	B	0.5720	B	0.3574	B
8	2640	55	4	0.45	B	0.6901	B	0.4124	A
9	2423.52	73.44	6	0.53	A	0.9064	A	0.4941	A
10	2407.5	160.5	4	0.58	A	0.9666	A	0.5146	A
11	1075.2	5.12	2	0.18	C	0.2164	C	0.1341	C
12	1043.5	20.87	5	0.31	B	0.5369	B	0.2691	B
13	1038	86.5	7	0.52	A	0.9064	A	0.4248	A
14	883.2	110.4	5	0.44	B	0.7678	A	0.3591	B
15	854.4	71.2	3	0.27	C	0.4561	C	0.2312	B
16	810	45	3	0.22	C	0.3752	C	0.1966	C
17	703.68	14.66	4	0.22	C	0.3868	C	0.1915	C
18	594	49.5	6	0.38	B	0.6639	B	0.3020	B
19	570	47.5	5	0.32	B	0.5600	B	0.2577	B
20	467.6	58.45	4	0.28	C	0.4849	C	0.2214	C
21	463.6	24.4	4	0.22	C	0.3868	C	0.1813	C
22	455	65	4	0.29	C	0.4965	C	0.2279	C
23	432.5	86.5	4	0.32	B	0.5600	B	0.2510	B
24	398.4	33.2	3	0.18	C	0.3059	C	0.1459	C
25	370.5	37.05	1	0.11	C	0.1555	C	0.0682	C
26	338.4	33.84	3	0.18	C	0.3002	C	0.1413	C
27	336.12	84.03	1	0.22	C	0.3111	C	0.1199	C
28	313.6	78.4	6	0.41	B	0.7159	B	0.3105	B
29	268.68	134.34	7	0.56	A	0.9641	A	0.4116	A
30	224	56	1	0.14	C	0.1979	C	0.0772	C
31	216	72	5	0.34	B	0.5946	B	0.2545	B
32	212.08	53.02	2	0.14	C	0.2482	C	0.1125	C
33	197.92	49.48	5	0.3	B	0.5311	B	0.2266	C
34	190.89	7.07	7	0.35	B	0.6004	B	0.2561	B
35	181.8	60.6	3	0.21	C	0.3637	C	0.1584	C
36	163.28	40.82	3	0.18	C	0.3002	C	0.1337	C
37	150	30	5	0.27	C	0.4716	C	0.1995	C
38	134.8	67.4	3	0.22	C	0.3752	C	0.1621	C
39	119.2	59.6	5	0.32	B	0.5542	B	0.2313	B
40	103.36	51.68	6	0.36	B	0.6177	B	0.2605	B
41	79.2	19.8	2	0.08	C	0.1443	C	0.0618	C
42	75.4	37.7	2	0.11	C	0.1962	C	0.0823	C
43	59.78	29.89	5	0.26	C	0.4618	C	0.1913	C
44	48.3	48.3	3	0.18	C	0.3117	C	0.1321	C
45	34.4	34.4	7	0.38	B	0.6581	B	0.2740	B
46	28.8	28.8	3	0.15	C	0.2598	C	0.1076	C
47	25.38	8.46	5	0.23	C	0.3983	C	0.1632	C

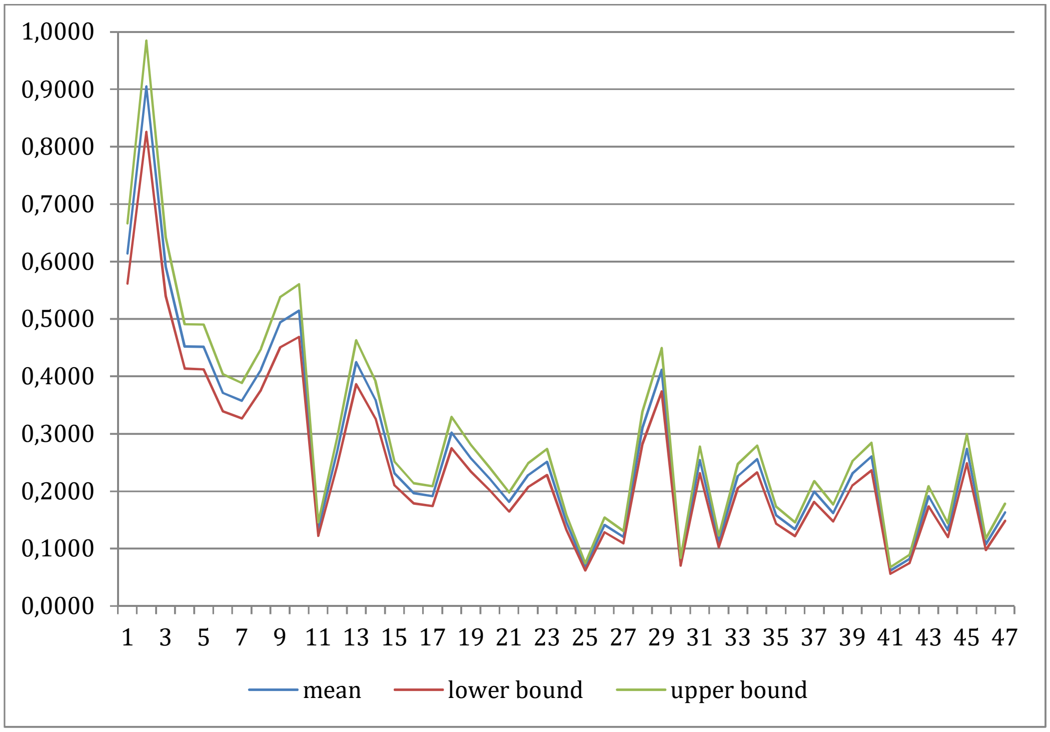
Table 2: Regression Results

	Value	t-statistics	value	t-statistic	value	t-statistic
u_1	0.576	5.83	0.696		0.282	
u_2	-0.131	-1.47	-0.417	-4.48	0	
u_3	0.323	12.87	0.379	10.38	0.239	10.62
adj. R^2	0.981		0.791		0.710	
RSS	0.146		0.412		0.596	
w_1	0.768		0.658		0.522	
w_2	0.192		-0.037		0.239	
w_3	0.323		0.379		0.239	
$\sum w_j$	1.283		1.000		1.000	

Table 3: Lower and Upper Bounds of Items' Scores

Item	Lower bound		Upper bound	
1	0.5616	A	0.6666	A
2	0.8256	A	0.9844	A
3	0.5402	A	0.6420	A
4	0.4139	A	0.4906	A
5	0.4122	A	0.4905	A
6	0.3393	B	0.4036	B
7	0.3265	B	0.3884	B
8	0.3762	A	0.4485	A
9	0.4503	A	0.5379	A
10	0.4689	A	0.5603	A
11	0.1225	C	0.1457	C
12	0.2450	B	0.2932	B
13	0.3864	A	0.4632	A
14	0.3267	B	0.3916	B
15	0.2105	B	0.2519	B
16	0.1791	C	0.2142	C
17	0.1744	C	0.2087	C
18	0.2746	B	0.3294	B
19	0.2343	B	0.2810	B
20	0.2013	C	0.2415	C
21	0.1649	C	0.1977	C
22	0.2072	C	0.2486	B
23	0.2282	B	0.2738	B
24	0.1327	C	0.1591	C
25	0.0622	C	0.0743	C
26	0.1285	C	0.1541	C
27	0.1091	C	0.1308	C
28	0.2821	B	0.3389	B
29	0.3739	A	0.4493	A
30	0.0702	C	0.0841	C
31	0.2312	B	0.2778	B
32	0.1022	C	0.1227	C
33	0.2058	C	0.2473	C
34	0.2327	B	0.2796	C
35	0.1440	C	0.1729	C
36	0.1215	C	0.1459	C
37	0.1813	C	0.2178	C
38	0.1473	C	0.1770	C
39	0.2101	B	0.2525	B
40	0.2366	B	0.2844	B
41	0.0561	C	0.0674	C
42	0.0748	C	0.0899	C
43	0.1737	C	0.2089	C
44	0.1200	C	0.1442	C
45	0.2488	B	0.2992	B
46	0.0977	C	0.1175	C
47	0.1482	C	0.1783	C

Figure 1: Mean and Bounds of the Estimated Items' Scores



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Footnotes

[1] Multi-criteria classification is part of spare parts management along with demand forecasting, inventory optimization and supply chain system simulation (see Hu *et al*., 2018).

[2] Douissa and Jabeur (2020) in the most recent review of the subject surveyed 83 studies.

[3] Cherchye *et al*. (2007) provided an introduction to the BoD model. See Kao *et al*. (2008), Karagiannis and Paschalidou (2017) and Karagiannis (2020) for the relation of the conventional with the normalized BoD model.

[4] There is an extensive discussion in the literature about the different problems of the estimated weights in the BoD model that is summarized in Greco *et al*. (2019). Therein, the interest reader can also find the alternative approaches that have been used to derive common weights (p. 74), which are directly applicable to Ramanathan (2006) but not to Ng (2007) model. The relevant literature for the normalized BoD model used by Ng (2007) is, to the best of our knowledge, limited to Hadi-Vencheh (2010) and the present study.

[5] Due to the equivalence of (1) and (2), the regression model could alternatively be stated in terms of classification weights, i.e., w_j's, rather than the u_j's. However, as one can verify, the computation burden would be much greater in this case as the inequality restrictions $w_j \geq w_{j+1}$ for *j=1,…,J-1* are more complicated to deal with econometrically than the inequality restrictions $u_j \geq 0$.

[6] As it will be evident below, we have at most sixteen regressions to run with (3) but only four with (4).

[7] Notice that the estimated value of u_1 in both the OLS and the ICLS estimation of (4) is obtained by using the equality restriction $\sum_{j=1}^{J} j u_j = 1$.

[8] See also Vancey, Judge and Bock (1981) and Wolak (1989) for testing inequality and equality and inequality restrictions.

[9] In principle, all other possible descending ordering schemes, namely $w_1 \geq w_3 \geq w_2$, $w_2 \geq w_1 \geq w_3$, $w_2 \geq w_3 \geq w_1$, $w_3 \geq w_1 \geq w_2$ and $w_3 \geq w_2 \geq w_1$, can be tested in a similar fashion. However, as this is not our primarily task in this paper, we leave it for future work. Here we focus only on comparing the regression-based results with those of previous studies for a given descending ordering scheme, namely that $w_1 \geq w_2 \geq w_3$.

[10] Consequently, the differences between Ng (2007) and Hadi-Vecheh (2010) only concern the classification of items #8 and #14 in class A or in class B.