REVIEW

In memory of Professor Nenad Mladenović (1951–2022)

Angelo Sifaleras[1]

Abstract

This article is dedicated to the memory of Professor Nenad Mladenović. It provides a short biography, a description of his main scientific achievements, and also testimonials from scholars all over the world who were collaborators for several years. This peer recognition highlights not only his exceptional academic career but also the integrity of his character.

Keywords Operations Research · Optimization · Variable Neighborhood Search · Less is More

1 Short Biography

Prof. Nenad Mladenović was born in 1951 in Jagodina, located in central Serbia. He earned his B.Sc. Degree in Mathematics and M.Sc. and Ph.D. Degrees in Operations Research from the University of Belgrade. Prof. Mladenović held the position of Professor of Industrial and Systems Engineering at Khalifa University (KU), Abu Dhabi, United Arab Emirates (UAE). Before joining KU, he taught at several universities in different countries, including France, Canada, Belgium, the UK, and Serbia.

Prof. Nenad Mladenović served as a program committee chair of several international conferences on optimization and operations research, such as the Yugoslav Symposium of Operational Research (SYM-OP-IS) in 2003 and in 2009, the EURO Mini Conference (MEC XXVIII) on Variable Neighborhood Search in 2012, and the XI Balkan Conference on Operational Research in 2013. He had also been frequently invited to give plenary or tutorial talks on various international conferences such as the Discrete Optimization and Operations Research (DOOR'07) in 2007, the XIV International Baikal Conference on Optimization Methods and Applications (Baikal 2008) in 2008, and others.

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Prof. Nenad Mladenović (see Fig. 1) was a member of the Scientific Society of Serbia since 2004. He was also a founding editorial board member of the Yugoslav Journal of Operations Research (YUJOR), and recently in 2022 he was appointed as the sole Editor-in-Chief of YUJOR. Prof. Vera Kovačević-Vujčić, a former co-Editor-in-Chief of YUJOR and a long-time collaborator and friend of Prof. Nenad Mladenović, also prepared an article in his memory [1].

Prof. Nenad Mladenović was also a member of the European Academy of Sciences (Academia Europea) and led or participated in several international research projects in the UK, France, China, Russia, Kazakhstan, and Serbia. In 2020 he was recognized as the #2 top scientist in the UAE according to the Guide2Research 2020 Top Computer Science and Electronics Scientists Ranking and he gave an inspiring interview available online at: https://www.ku.ac.ae/ku-professor-recognized-as-one-of-the-top-scientists-in-the-uae.

Nenad passed away on 7 May 2022 at the Cleveland Clinic in Abu Dhabi and his funeral took place in Belgrade, Serbia, on 17 May 2022.

Fig. 1 Professor Nenad Mladenović during ICVNS 2019 at Rabat, Morocco

2 Research Work

The research interests of the late Prof. N. Mladenović were always around efficient methods for optimization of complex problems. In particular, his focus was on simpler and user-friendly principles for solving mathematically and computationally difficult problems.

Prof. Mladenović was a highly cited researcher and had more than 26,000 citations with an h-index equal to 61, according to the Google Scholar at the time of writing this article. He had proposed several solution approaches and a few general such techniques that have been very well received by scholars. Variable neighborhood search, formulation space search, and the less is more approach were the three most cited approaches that he had developed. It should be noted that these general techniques have also been used successfully by a large number of practitioners and organizations all over the world. Examples include Ultramar Canada, Avis Car Rentals, and Hydro-Quebec Canada. Among other examples of industrial use, his techniques were applied for the berthing allocation in the Hong Kong harbor, the Petro-Brazilian over rig transportation, and also for the human nutrition problem in the Yugoslav army.

His main research contribution is the creation of the well-known Variable Neighborhood Search (VNS) metaheuristic together with Prof. P. Hansen, which gained a lot of popularity due to its simplicity and efficiency. Their seminal paper on VNS [2] was published in the Computers & Operations Research journal, in 1997. Variable Neighborhood Search is a metaheuristic that solves combinatorial and global optimization problems, the main concept of which is to systematically change the neighborhood structures both within a descent phase to find a local optimum and in a perturbation phase to get out of the corresponding valley. Nowadays, several VNS schemes have been proposed, such as Reduced VNS (RVNS), Variable Neighborhood Descent (VND), General VNS (GVNS), Skewed VNS (SVNS), Variable Neighborhood Decomposition Search (VNDS), and others. The VNS methodology was applied for the solution of a large number of optimization problems, such as exact solution of large-scale location problems by primal-dual VNS, generation of solutions to large mixed integer linear programs by hybridization of VNS and local branching, generation of solutions to very large mixed integer programs using VNS decomposition and exact solvers, etc.

His paper on the VNS metaheuristic [3] with P. Hansen was selected among the 20 most influential articles published in the European Journal of Operational Research (EJOR) in the 40-year history of EJOR and ranked eighth most cited in 40 years of EJOR (https://www.journals.elsevier.com/european-journal-of-operational-research/news/celebrate-40-years-european-journal-operational-research.

Prof. Mladenović had multiple academic positions in different countries across the world and collaborated with many research groups. Thus, some of his colleagues and research partners from the research community of optimization who collaborated with Prof. Mladenović, share their testimonials in the following section.

3 Testimonials from Some Colleagues

Personally, I first met Prof. Mladenović during the 10th Balkan Conference on Operational Research, which was held in Thessaloniki, Greece, in 2011. We were honored to have him as an invited speaker and he gave a plenary talk on "Variable neighborhood search for the traveling salesman problem and its variants". This was my first experience with the VNS metaheuristic, and soon after in 2012, I also attended the second VNS meeting "EURO Mini Conference (MEC XXVIII) on Variable Neighborhood Search" [4] at Herceg Novi, Montenegro. From then on, we started our collaboration to organize the "International Conference on Variable Neighborhood Search" (ICVNS) meetings during the following years. Our first joint publication came a few years later with a successful application of VNS to inventory optimization problems [5] with very good results against other state-of-the-art methods. I consider myself very lucky to have met Prof. Mladenović, since not only we had excellent cooperation and friendship, but also because through him I met a large number of other researchers from the VNS community worldwide and also built very good relationships and strong collaborations.

- **Professor Jack Brimberg**, *Royal Military College of Canada.*

  The first time I met Professor Nenad Mladenović was at a mini-conference held by the EURO Working Group on Locational Analysis from 9 to 11 May 1994, in Brussels, Belgium (EWGLA VII). I was relatively new to the academic world, having received my Ph.D. a few years earlier. The number of participants was relatively small, but this allowed an excellent opportunity to mingle in an informal way with colleagues with similar research interests. At that time, Nenad had moved to Montreal with his young family and was working with the renowned Professor Pierre Hansen at the Hautes Études Commerciales (HEC, Université de Montréal). It turned out that we were the only two participants from Canada, which in turn led to a natural inclination to meet. After my presentation, Nenad approached me, introduced himself as a fellow Canadian, and asked me rather bluntly if I had tried using Tabu Search to solve the presented problem. My answer was an emphatic "no", followed by a suggestion that we could work together on this idea when we returned home. We did in fact meet soon after to follow up on this suggestion, which resulted in our first publication together (Studies in Locational Analysis, Vol. 8, 23–32 (1996)).

  This turned out to be the beginning of a long working relationship and a close friendship. For those interested, have a look at the EWGLA web site, where you can find a group photo of the EWGLA VII meeting. Nenad and I are standing next to each other at the far left of this photo. Time really does fly; we were both a lot younger then.

  One year earlier (in June 1993), I had the opportunity to attend the ISOLDE VI conference in Greece. This was my first ISOLDE, and a memorable experience. Nenad was not at this conference, but Professor Hansen was. During a conversation with Professor Hansen, he suggested that I apply for membership to GERAD (Groupe d'études et de recherche en analyse des décisions). I followed

up and eventually became an associate member of this group, which I continue to be to this day. Working at the Royal Military College of Canada (RMC) in Kingston placed me at about a three-hour drive from Montreal, a very reasonable drive by Canadian standards. Now I had a good excuse to visit HEC and collaborate with Professor Hansen. As mentioned above, Nenad was also there, so, when I visited, which was quite often, I would always meet the two of them. Nenad would be my host throughout the visit and I would have his attention exclusively during the day and often into the evening. Professor Hansen, who had many other commitments in parallel, would visit to see how we were doing and to offer advice. Sometimes these visits would last a few hours. Although I thoroughly enjoyed the sessions, the experience could be quite daunting. Professor Hansen is so incredibly knowledgeable that it can be scary. His grasp of the literature and understanding of the theory are so deep that it is difficult not to feel ignorant in front of him. Nenad, on the other hand, seemed quite comfortable. At the end of the day I would feel completely drained, but it was well worth it. What a privilege it was for me to work with these two gentlemen!

Nenad is best known in the Operations Research field as the founder of the Variable Neighborhood Search (VNS) metaheuristic. It was during one of my visits to Montreal that Nenad introduced me to this new methodology. At that time, the idea was not catching on. He had first introduced it at the Optimization Days 1995 conference at HEC, Montreal. The title of his presentation was: Variable Neighborhood Algorithm - A New Meta-Heuristic for Combinatorial Optimization. The abstract of the talk is rather short. Let me quote the key sentence of this abstract: "Escaping from the local optima trap is done by changing the neighborhood (metric) in a systematic way". At this time, the standard practice upon reaching a local optimum was to restart the improvement algorithm (or local search) from a new, feasible and randomly selected point. In fact, random multi-start local searches are still popular to this day and are often used as a benchmark to compare other more sophisticated algorithms. Thus, the underlying concept and principal contribution of VNS is to define a sequence of neighborhood structures to replace the random selection of a new starting solution. A random point(s) is selected in the first (or closest) neighborhood of the current local optimum (which is also the best solution obtained up to this time), and the improvement step begins from there. If a better local solution is obtained, it becomes the new anchor point; otherwise, a random point(s) is selected in the next (further) neighborhood of the current local optimum to conduct the improvement step, and so on. In this manner, some (or most) of the good attributes of the current anchor point are retained instead of starting anew from a completely random point. The fact that the VNS concept was slow to catch on at that time perhaps was due to the simplicity (and elegance) of the concept. And yet, isn't "simple to understand" one of the main attractions of a good heuristic?

After my initiation to VNS, I suggested to Nenad that we try to apply it to the multi-source Weber problem, also known as the continuous location-allocation problem. This problem is a continuous version of the classical p-median problem on a graph. The problem may be simplified by noting that the fixed points (or customers) are assigned to their closest facilities in any optimal solution. Since there

is a finite (but very large!) number of possible partitions of the customer set, the problem falls within the realm of combinatorial optimization. My suggestion to Nenad led to the first publication of a paper dealing with VNS, entitled "A variable neighborhood algorithm for solving the continuous location-allocation problem", which appeared in Studies in Locational Analysis, vol. 10, 1–12 (1996). A later paper, "Improvements and comparison of heuristics for solving the uncapacitated multisource Weber problem", which included coauthors Professor Hansen and Eric Taillard, applied several new variants of VNS to this same problem; it appeared in Operations Research, vol. 48, 444–469 (2000). The computational results in this paper were outstanding! Several new best-known solutions were found on benchmark instances, and the improvements were quite significant. A new shaking step was used that proved to be quite powerful. Instead of perturbing the current solution by changing some customer assignments from one facility to another, some facilities are randomly selected, the number depending on the specific neighborhood in use, and these selected facilities are then moved to random locations. A nice feature of VNS is that its components (improvement step, neighborhood change, and shaking) provide a great deal of flexibility in building a suitable algorithm for a given type of problem.

Nenad remained in Montreal for a few years. During this period, I made regular trips to HEC to visit him. I also met his wife Branka and their children, Marko and Jelena. He had moved to Canada not only for professional reasons, but also because he wanted his family to be safe. Recall that at this time there was a lot of unrest in and around Serbia. When Nenad moved his family back to Belgrade, my regular meetings with him unfortunately came to an abrupt stop. Nenad's popularity in the Operations Research community had been on the rise, and he accepted a faculty position at Brunel University London. I recall that he told me that he would take an airplane to Belgrade on most weekends to be with his family as much as possible. The income he was earning in England also helped, as salaries in Serbia were very low in comparison. This must have been a very stressful time for him: teaching, supervising graduate students, research, and then frequent flights to Belgrade and back. Fortunately for us, Nenad and Branka would often return to Canada for a short visit during summer breaks. Nenad would spend a week at HEC in Montreal as a research guest of Professor Hansen, and the second week at RMC in Kingston as my guest. Branka became a good friend of my wife, Sharon.

Nenad visited me a few times when I was at the University of Prince Edward Island. On one trip, while waiting to pick him up at the small airport near Charlottetown, he called to let me know that he had missed his connecting flight to Halifax. Apparently, he was hungry and had decided to walk out to grab a sandwich. He did not make it back in time to catch his flight. Meanwhile, he wanted to know if I could pick him up at Halifax airport, where he was now stranded. I explained that this would require a four- or five-hour one-way road trip in the middle of the night. So, we decided that it was best for him to get some sleep in a nearby hotel and catch an early flight to Charlottetown the next morning. This turned out to be quite inconvenient, but all is well and ends well. He made it to Charlottetown the next morning, and from what I gathered, that infamous sand-

wich was quite delicious! On another visit from Nenad, I was able to arrange for a mutual colleague and friend, Professor Said Salhi, to drop in at the same time. It is impossible to be serious with this fellow. We had many laughs! Over the years, while enjoying good comradery, the three of us have managed to work together on some interesting projects.

   Nenad and I co-authored a good number of papers, just the two of us, or with other colleagues joining, that covered a wide range of topics. I was fortunate as well to meet and work with some of his colleagues, as he was with mine. This was a great experience for me. A sample of papers is given below to illustrate the different topics we worked on together:

– *Attraction probabilities in variable neighborhood search*, J. Brimberg, P. Hansen, and N. Mladenović, 4OR, vol. 8, 181–194 (2010). This paper tries to explain why VNS consistently outperforms random multi-start algorithms using the same local search. The argument centers on the distribution of the attraction basins around a current solution as a function of the distance from the solution. Empirical evidence is given showing that attraction probabilities for improved solutions are orders of magnitude larger in neighborhoods that are close to the current solution.
– *Degeneracy in the multi-source Weber problem*, J. Brimberg and N. Mladenović, Mathematical Programming vol. 85 (1), (1999). This paper shows that a significant portion of solutions obtained by local search can be degenerate (i.e., some facilities are out of use; i.e., they are not serving any customers) when large numbers of new facilities are being considered. A simple modification of the local search results in significant improvements in solution quality.
– *Primal-Dual Variable Neighborhood Search for the Simple Plant-Location Problem*, P. Hansen, J. Brimberg, D. Urošević, and N. Mladenović, INFORMS Journal on Computing, vol. 19, 552–564 (2007). *Solving large p-median clustering problems by primal-dual variable neighborhood search*, P. Hansen, J. Brimberg, D. Urošević, and N. Mladenović, Data Mining and Knowledge Discovery, vol. 19, 351–375 (2009). In each of these two papers, a VNS-based heuristic is applied to the primal problem to find high-quality solutions. VNS is also applied to a dual relaxation, which allows us to obtain additional useful information in the form of guaranteed bounds.
– *The multi-source Weber problem with constant opening cost*, J. Brimberg, S. Salhi, and N. Mladenović, Journal of the Operational Research Society, vol. 55, 640–646 (2004). This paper introduces a constant fixed cost of establishing a facility, resulting in a continuous analog of the simple plant location problem (SPLP). The solution method developed uses a multi-phase heuristic that first solves a discrete SPLP by existing methods to obtain an estimate of the optimal number of facilities.
– *Decomposition strategies for large-scale continuous location-allocation problems*, J. Brimberg, P. Hansen, and N. Mladenović, IMA Journal of Management Mathematics, vol. 17, 307–316 (2006). This paper examines new ways to decompose the location-allocation problem. These strategies are then used in a

variable neighborhood decomposition search (VNDS) heuristic and tested on large instances of this problem.

– *Less is more: Solving the Max-Mean diversity problem with variable neighborhood search*, J. Brimberg, N. Mladenović, R. Todosijević, and D. Urošević, Information Sciences 382, 179–200 (2017). This paper presents a new application of the general variable neighborhood search in order to solve the given problem. Despite the simplicity of the method, significantly better results are obtained compared to a more complex heuristic that represents the state-of-the-art.

Two authors listed above, Raca Todosijević and Dragan Urošević, are former PhD students of Nenad. The four of us have been working together on various hub location problems over the past few years. We were also working on a book on VNS applications in hub location when Nenad passed away.

Nenad moved from Brunel after about seven years to take a position as a senior researcher in Computer Science at the Université Polytechnique Hauts-de-France in Valenciennes. After some four years there, he accepted a position as Professor of Industrial and Systems Engineering at Khalifa University in Abu Dhabi, United Arab Emirates. He was a man of the world with colleagues and friends in every corner. He was always on the go and juggling numerous projects and different teams all at the same time. He served as Editor-in-Chief of the Yugoslav Journal of Operations Research for many years, and was on the editorial board and/or guest editor of several other journals including, for example, the European Journal of Operational Research, Computer Science and Operations Research, Optimization Letters, and the International Journal of Metaheuristics. In addition to all this, Nenad would not shy away from administrative responsibilities, including program committee work for many, many conferences. Highlights include the Yugoslav Symposium of Operations Research (SYMOPIS), where he served as Program Chair twice, and the International Conference on Variable Neighborhood Search, which takes place annually, where Nenad has served as a founder and conference chair. Nenad invited me to be a plenary speaker at a SYMOPIS conference, which took place in Montenegro. Sharon and I had a wonderful time there. The nature was very beautiful and the people were very friendly. I had one little mishap though upon arrival at the hotel. It was late afternoon and the sea looked so tempting that I had to go swimming. Well, I forgot to take my glasses off before diving into the water (a clear aqua blue sea) from a small pier near the hotel. My glasses were never found. As expected, I had to deal with much teasing from Nenad and his friends! By the way, I did not have a spare set with me. That night Nenad's friends managed to find an optometrist whose store was still open, and one, possibly two days later, I could see clearly again!

The times I shared with Nenad were not all work. We had many meals together and many glasses of beer. We would talk about many subjects: family life, travels, politics, sports, etc. He was very keen about tennis, and would be quite vocal when the Serbian champion, Novak Djokovic, made bad plays. Over the years, Nenad and I became very good friends. However, during the last few years, we started to lose touch. Perhaps this was due to Nenad's incredibly busy schedule. (I often wondered how he managed.) The two-year

period of COVID, when travel was difficult or impossible, did not help either. We did continue to communicate by email, though, and to continue working on some projects, but this also slowed down. The last work that we completed together (and with Professor Zvi Drezner) dealt with two book chapters in the recently published Palgrave Handbook of Operations Research (editors, Said Salhi and John Boylan), dealing with Formulation Space Search, and the Less-is-More principle in heuristic design. These are both relatively new areas of study that can be added to Nenad's long list of achievements.

Like many others, I was shocked to learn about Nenad's passing away. It was very sudden and unexpected. Although he is gone, his memory will live forever through his work, his family, his students, colleagues, and friends.

Rest in peace, my friend. We miss you.

- **Professor Abraham Duarte** and **Professor Eduardo García Pardo**, *Universidad Rey Juan Carlos, Spain.*

  Back in May 2011, Prof. Mladenović visited the Universidad Rey Juan Carlos (Madrid, Spain). It was the first time we met Prof. Mladenović. At that time, Prof. Pardo was just a Ph.D. student, co-advised by Prof. Duarte and Prof. Pantrigo, and it was only one week before Prof. Pardo defended his Ph.D. Thesis. A small dark room in the basement of a building at Universidad Rey Juan Carlos was holding a seminar on VNS, given by Prof. Mladenović. He was invited by some colleagues from the Department of Statistics and Operations Research and he was presenting the foundations of Variable Neighborhood Search and several applications of the methodology. Both of us attended the seminar with great interest. He was scarce in words but bright in ideas. At the end of the seminar, we had the opportunity to meet him.

  Less than a year later, Prof. Pardo visited Prof. Mladenović at Brunel University (London, UK) for a few months. During his research stay, Prof. Duarte visited them together with Prof. Pantrigo. After the time they spent together, they were no longer Profs. Mladenović, Pardo, Duarte, and Pantrigo, but Nenad, Eduardo, Abraham, and Juanjo. Nenad used to work in his office until late at night and Eduardo used to visit him downstairs regularly before going home, to discuss some ideas or share new results. That was just the beginning; Nenad and the members of the GRAFO Research team collaborated actively for more than a decade. Among the collaborations we have made together, we are particularly proud of three methodological contributions to the literature: the first was the Variable Formulation Search (VFS) [6] to deal with problems that present a flat landscape, such as min/max or max/min problems. The second was the Multi-objective Variable Neighborhood Search [7], a methodology based on VNS to deal with multi-objective optimization problems. And the third one was the Parallel Variable Neighborhood Search [8], a new paradigm to parallelize VNS in order to increase the exploration speed of the method.

  In addition to the research we have done together, we actively collaborated with Nenad in the development of the International Conferences on Variable Neighborhood Search (ICVNS) over many years, with special attention to the one celebrated in Malaga (Spain) in 2016. A place by the sea to celebrate the conference was always a must for him. Also, we co-edited some book chapters

and special issues together. During all these years, Nenad visited us in Madrid in many occasions for different events, so we had the opportunity to talk with him at great length about science, politics, and life, among others.

In summary, Nenad was more than just a colleague to us; he was also a mentor who helped us deepen our understanding of our field and, even more, a friend to be remembered.

- **Professor Saïd Hanafi**, *Université Polytechnique Hauts de France, France.*

    I have known Nenad for more than 30 years. Since 2007, Nenad has made several stays as a visiting professor at the University of Valenciennes (now called the Université Polytechnique Hauts-de-France (UPHF)). We collaborated to obtain the first international chair of the UPHF co-funded by the Hauts-de-France region and the Institute of Technological Research RAILENIUM on rail transport from 2013 to 2016. During this period in Valenciennes (France), with Nenad, we collaborated on VNS with more than 25 researchers, the majority of whom are young people, some of whom have become confirmed researchers.

    Nenad is a lively and pleasant person to be with, he is well appreciated for his open mind to communicate and collaborate easily with people from different cultures. Nenad applied optimization and his VNS metaheuristics in real life. He often surprised me with his innovative choice alternatives to solve some practical problems concerning him, especially to minimize the cost, with subtle relaxations and unlikely combinations.

    The VNS metaheuristic is also strongly intertwined in Nenad's scientific and practical life, and it is not easy to make him deviate from this subject, which can also be called the "Variable Nenad Search". At the scientific level, his papers prove quite well the diversity and variants and hybridizations of VNS. At the level of daily life, Nenad often changed geographical and social locations, be it for work or for a stay.

    We have lost a close friend and a great scientist for the scientific community and especially for Serbia.

- **Professor Bassem Jarboui**, *Higher Colleges of Technology, Abu Dhabi, United Arab Emirates*

    I have had the honor of knowing Nenad Mladenović since 2009, when we were both invited as visiting professors at the University of Valenciennes by professor Said Hanafi. Before then, I knew him as the founder of the Variable Neighborhood Search Algorithm and one of the best researchers in the field of Metaheuristics. After that, we shared many research interests and published many papers. Over time, our collaboration grew further and we became very close since 2018 when he came to the United Arab Emirates, and we worked together at the Emirates College of Technology, one year before he moved to Khalifa University. Nenad was not only a colleague, but also a good friend. He was not only a great scientist, but also a great honest and modest man. His death was a great loss for the scientific community and for me. He will remain in our memory and in our work as a source of inspiration.

- **Professor Pierre Hansen**, *HEC Montréal, Canada.*

    I first met Nenad at the home of our colleague Jean-Pierre Brans in the Belgian countryside in the 1970s. Jean-Pierre and his wife Claire were kindly hosting a gathering of about twenty young researchers in Operations Research who

represented around fifteen European countries. Nenad was one of those young researchers in attendance and played very energetic football in the garden. That day, we spoke at length about mathematical programming and I found him bright, engaged, and friendly. We stayed in touch and met at conferences over the years until he joined me at GERAD in Montreal for his sabbatical year in the early 1990s. Nenad remained at GERAD for several years and we began our productive lifelong collaboration. We wrote several articles together until, in 1997, we published our most important paper on Variable Neighborhood Search. The wide practical applicability of VNS opened up many research opportunities and led to many more articles by Nenad and me up to 2019. Our foundational VNS paper has been cited close to 5,000 times and led to the formation of an independent field of research that continues to host a yearly international conference to this day. At any of those VNS conferences around the world, one could always enjoy Nenad's boundless energy, enthusiasm, and insights that pushed forward the field of VNS all these years and led us to work with many scientific communities from Brazil to Greece to Hong Kong. Nenad was an esteemed colleague but also a dear friend and will be greatly missed. His prolific publications span across many fields, from chemistry to location theory to cluster analysis. His achievements continue to live on.

- **Professor Vera Kovačević-Vujčić**, *University of Belgrade, Serbia.*

Serbian OR community is still in disbelief that a dear friend and respected colleague Nenad is gone forever. Because he often had longer or shorter stays abroad, but he always returned, full of energy, enthusiasm, and new ideas. Only three months ago, we met and discussed a new organization of Yugoslav Journal of Operations Research. He agreed to be the only editor-in-chief and has already received the consent of several distinguished colleagues from the country and abroad to be area editors. Soon after that we heard the terrible news that he was in the hospital and that they were fighting for his life, and not long after that he passed away. The Mathematical Institute of Serbian Academy of Sciences and Arts commissioned me to compile Nenad's biography and bibliography, and I found a kind of consolation in collecting data on Nenad's life and work. The details can be found at the address: http://www.mi.sanu.ac.rs/novi_sajt/members/in_memoriam.php. Let me share here some personal memories.

I have known Nenad ever since he joined the Faculty of Organizational Sciences of Belgrade University in 1985 as a dynamic hard-working teaching assistant. In 1988, I had the honor to be a member of the committee for the evaluation of his doctoral dissertation "New nonlinear programming methods with application in location, allocation, and transportation problems" and already then, I knew that he would leave a trace in the field of operations research.

In 1991, at the invitation of Professor Pierre Hansen, Nenad went to the University of Montreal for scientific training, but instead of the planned year, he stayed there until 1997. In Montreal, he worked as a researcher at one of the world's largest centers for operational research — the GERAD Institute. Working on the practical problem of designing an oil pipeline in South Gabon, Nenad came up with the idea to use the fact that the local minimum with respect to one neighborhood structure (induced from a certain metric) is not necessarily the local minimum

with respect to another type of neighborhood. In addition, the global minimum, which is almost impossible to find in real time, is local with respect to all types of neighborhoods. Following these simple facts, Nenad first independently, and then together with Professor Hansen, developed the Variable Neighborhood Search, an original approximate methodology for solving combinatorial and global optimization problems. Nenad proposed the VNS in 1995, and since then it has gained the status of one of the leading general heuristic methods. We heard about the VNS before it became generally accepted because Nenad, during one stay in Serbia, gave us a lecture at the Seminar for Applied Mathematics of the Mathematical Institute. He spoke with great enthusiasm, and we saw that the result was extraordinary. At the time, the most popular method for finding a local minimum of a differentiable function was the variable metric method, and I remember that Nenad told us — I am proposing the variable neighborhood search method that is applicable in both discrete and continuous optimization.

Nenad came back to Belgrade in 2000, where he got a job at the Mathematical Institute as the leader of several scientific projects related to mathematical models and methods of optimization. Due to Nenad's intensive scientific activity, Belgrade became one of the centers for the development of the VNS. I and several other colleagues worked with Nenad on the application of the VNS to global optimization of a continuous function, both unconstrained and constrained. It was one of many successful applications of the VNS.

Since the Mathematical Institute allows its associates longer stays abroad, in the period 2006–2018 Nenad was a visiting professor at the University of Birmingham, Brunel University, and University of Valenciens. At the same time, Nenad participated in numerous other scientific research projects and gave lectures in Canada, Spain, France, Russia, Germany, Kazakhstan, Brazil, and China. Such intensive scientific activity has significantly contributed to the dissemination of VNS and its numerous applications. Nenad remained at the Mathematical Institute until his retirement in 2018. From 2018 to 2022, he was hired at Khalifa University of Science and Technology in Abu Dhabi.

From Nenad's bibliography, it can be seen that in less than 6 years, from 2017 to 2022, he has published 60 articles in scientific journals, which is almost a third of all articles he has published. This is the best illustration of the fact that death interrupted him in full creative momentum. But death cannot and will not erase the Variable Neighborhood Search and its numerous applications.

- **Professor José A. Moreno-Pérez**, *Universidad de La Laguna, Spain*.

I met Nenad at the First EURO Summer Institute in July 1984 in Brussels, where we also met Pierre Hansen. There started a great friendship and fruitful collaboration with Nenad. During this event Nenad introduced me to who was later my deceased wife Marina. He found her lost on the VUB campus during the night, and he tried to help her find her room. He couldn't and had to look for me to help him, and finally we found it. We got married two years later and spent part of our honeymoon journey with Nenad and his wife Branka in Herceg-Novi (for the II Meeting of the EWG on Locational Analysis organized by Nenad) and in Belgrade in their small flat. We met again at several meetings around the world. I visited him at GERAD in Montreal and at Brunel University in London,

and he visited me at the Universidad de La Laguna in Tenerife several times. He also came to Tenerife with his whole family, and we all enjoyed several tourist trips on the island; they had two children of a similar age as our two children. He told me some years later that when we went to the Pico del Ingles, the highest viewpoint in the Anaga mountains, at the north-east of the island, where one can see a set of lower peaks around it, he was already thinking on VNS. Few years ago I tried to get a stay for several months for him in the University of La Laguna but I did not get it. He also told me that he had been considering the idea of retiring on the southern beaches of Tenerife, but it will not be possible.

Nenad and I started to collaborate in the field of Location Analysis. Our most relevant contribution was the joint paper that we published, with Pierre Hansen and Jack Brimberg, with the title "The p-median problem: A survey of metaheuristic approaches" in 2007 in European Journal of Operational Research that has been cited 283 times (Scopus). He also gave me the opportunity to cooperate with him on VNS and we published, also with Pierre Hansen, the paper "Variable neighbourhood search: Methods and applications" first in 4OR in 2008 and an updated version in Annals of Operations Research in 2010. These two articles have been cited 220 and 598 times, respectively. We also edited a special issue in the European Journal of Operations Research in 2008 devoted to VNS and, also with Pierre. Pierre Hansen and Nenad made the chapter on VNS in the first Handbook of Metaheuristics edited in 2003, and Nenad, Pierre, Jack, and I made the updated VNS chapter of the 2010 and 2019 editions of the Handbook. This chapter has a number of citations similar (in Google Scholar) to the previous paper in Annals of Operations Research. I am also very happy to have written with him and Pierre Hansen the chapter on parallel VNS in the book of "Parallel Metaheuristics: A New Class of Algorithms" in 2005 and the first paper on VNS in Spanish in the journal "Revista Iberoamericana de Inteligencia Artificial" in 2003.

We also cooperate in the organization of meetings, first in the field of Location and later on VNS. Nenad and I belong to the EWG on Locational Analysis since it was created after the first ESI in 1984 where we participated. Nenad organized the second meeting of the EURO Working Group in Herceg-Novi in 1986 and I organized the 12th EURO Summer Institute also on Location Analysis in Tenerife in 1995. The 18th Mini EURO Conference on VNS that we organized in Tenerife in 2005 can be considered the first edition of the series of International Conferences on VNS. However, it was from the 28th Mini EURO Conference on VNS that we also organized in 2012 in Herceg-Novi that was the second ICVNS when the series started to be organized with some regularity.

From my personal point of view the most interesting idea of Nenad is the "less is more" approach. This idea was in our conversations from the beginning of our relationship. He always liked simple things rather than complicated and sophisticated ones. He liked the p-median problem for its simplicity and the possibility of testing different optimization strategies. He always defended the simplicity and naturality of VNS as one of the main advantages, without lack of efficiency. He usually did not like it when I proposed any extension or new ingredients for the VNS, or some hybrid. I have been following his research in the last few years and have seen several recent papers on this approach. I deeply regret not being

able to collaborate with him on this line. I truly believe that the less is more approach shows his idea of life.

- **Distinguished Emeritus Professor Panos Pardalos**, *University of Florida, USA.*

    I first met Nenad in the 1980s, and since then we have been scientific collaborators and friends. Nenad enjoyed traveling and exploring new places and cultures. It was very sad news for me when I heard about his unexpected death. We had planned to gather this fall for the VNS conference in the United Arab Emirates, where he had been working the last few years. Nenad was full of life and energy and enjoyed every moment of every day. Although most people know Nenad for the VNS heuristic, he had extensive knowledge of combinatorial optimization and modeling of industrial applications. His work in recent years has focused on the "Less is More" approach, and one of our last articles was [9]. In [9], the dominance relation is defined between two algorithms that includes the simplicity as well, and the general LIMA algorithm is proposed for the first time, where automatic ways are discussed to include common ingredients of all search algorithms. This work was finished in 2019, and online published in 2021 in Optimization Letters. Some years ago, while visiting Jun Pei in China, we started working on applying VNS to solving general nonlinear systems of equations. This resulted in an influential paper [10].

- **Emeritus Professor Said Salhi**, *University of Kent, UK.*

    This is with great sadness that I am trying to put pen to paper as I gather myself to write about the Great Man.

    I first met Nenad at the EWGLA 8 in Kaiserslautern (Germany) in 1995 where we went together for a drink (long one, I mean) and had a wonderful time. It was so good that the next morning he was supposed to present his work on VNS for a p-center or p-median problem, which I don't remember. He was a bit late but happy go lucky as usual. He and I would always talk about that funny morning and we always have a giggle as if it were yesterday!

    We then continued meeting each other, including initially at the INFORMS/CORS in Montreal in 1998. At this time, he was working at the University of Montreal with Professor Pierre Hansen.

    In 2004, I was head of the Group of Management Mathematics at the University of Birmingham, and our group was in the process of recruiting two additional people at the rank of associate professors. I was extremely pleased that Nenad applied for the job, and with his already strong CV, it was easy to draw a line connecting him and a job offer.

    After only a year there for Nenad, after 15 years for me in Birmingham before leaving to Kent, an opportunity arose for Nenad at the University of Brunel (Uxbridge London), where Professor John Beasley was currently located, having moved there from Imperial College. As expected, Nenad proved to be a very strong researcher and was also very popular with Ph.D. students. Professor of Applied Mathematics, Julius Kaplunov, Head of School at that time, was a great supporter of his work. Things did not go as planned after Julius left the Headship. Nenad was on the move again in 2013 but this time to Valenciennes in France, while also staying as a research fellow in Brunel for one extra year. By that time, Nenad had been working very closely with my also dear friend, Pro-

fessor Said Hanafi, for the latter few years, and even supervised a PhD student with him. Nenad was then offered a 3 years senior research post, the University International Chair, working on the freight transportation. After a break in Belgrade, in 2018, he joined Professor Bassem Jarbaoui in Abu Dhabi in the UAE at the Technology College. But after a year, in September 2019, Nenad obtained a full professor position at the renowned research university of the UAE, namely Khalifa University, where he enjoyed his life to the full until his death. While there, he remained a very close friend to Bassem. Thank you Bassem for being there for him and keeping us informed of his final sad days. Your thoughtfulness is greatly appreciated.

Nenad was kind enough to invite me to give talks at BALCOR and VNS conferences several times, including the latest face-to-face 7th ICVNS, in October 2019 in Rabat. I remember well spending a good time with Nenad at the delicious conference dinner on a boat. After that, Nenad, Professor Martine Labbé who also knows him for over 30 years, and I decided that it was not time to head back to the hotel. Instead, we visited a pub where we had a few drinks and pleasant chats, and above all, a lot of laughter. The next day I had my talk and enjoyed lunch with the big man, not knowing that it would be the last time we would have a proper laugh together, although we stayed in contact virtually all the time until a few weeks before his death.

Nenad was a great friend and a great mentor and teacher to many. He was an innovative and prolific researcher whose work will provide inspiration for years to come.

Rest in peace, my friend. You will always be remembered in my heart and in my soul. God be with you Boss, as we have always called each other!

- **Professor Marc Sevaux, EURO President**, *Université de Bretagne-Sud - Lorient, France.*

  Our late colleague Nenad Mladenović is known to be the father of the Variable Neighborhood Search metaheuristic created jointly with Pierre Hansen [2]. This new method was published at the end of the 1990s, when metaheuristics were shown to be efficient and competitive methods for solving hard combinatorial optimization problems. According to the history of metaheuristics, it falls into Period 3: *The framework-centric* period [11].

  At that time, classical metaheuristics like Genetic Algorithms, Tabu Search, and Simulated Annealing, well established, needed to be completed by other frameworks. The idea of using more than one neighborhood was both simple and innovative. As demonstrated in many papers, the reason for being stuck in a local optima is mainly due to the neighborhood. Hence, changing it allows us to escape from the previous local optimum. Together with this simple idea, the authors have proposed a very general framework known as VNS that uses a shaking phase (to favor exploration) and an improvement phase (to favor exploitation). The neighborhood-change phase could be considered as a basis change and opens a new perspective for the search. Also, the simple description of the framework allowed other researchers to refine the method by using other metaheuristics (e.g., Tabu Search) as the improvement phase.

  Google search on *Variable Neighborhood Search* returns more than 32 million hits and still 2 million when restricted to scholar articles. What followed

has shown that this new framework became unavoidable and has been applied in many (if not all) fields of research and applications ranging from graph problem, location, transportation, to many real-life applications.

For those of you who are new to VNS (if one exists), I suggest reading the paper [3] as an introduction to VNS, variants, and applications. The conclusion of this paper cites the main principles and characteristics of good metaheuristics. And you will see that VNS ticks all these boxes.

During all these years, what had struck me was that Nenad Mladenović never stopped applying VNS himself to various problems and had hundreds of collaborations around the world. He was present in many conferences, gave plenary and invited talks, always defending the benefit of VNS. He also persevered in developing variants of VNS, namely VND, RVNS, Skewed VNS, VNDS, Primal-Dual VNS, VNB, or Variable Neighborhood Formulation Space Search. There are very few researchers in our community who have marked it so much.

On a more personal basis, Nenad was a very happy colleague and friend, always smiling and supportive of all initiatives on metaheuristics. We all miss you, Nenad.

- **Dr. Dragan Urošević**, *Mathematical Institute of Serbian Academy of Science and Arts, Serbia.*

  Dragan Urošević started collaboration with Professor Nenad Mladenović in June 1999, a few months before Nenad's employment at the Mathematical Institute of the Serbian Academy of Sciences and Arts. At that time, Dragan Urošević was employed as a researcher assistant at the Mathematical Institute of the Serbian Academy of Sciences and Arts. Previously, he was a Ph.D. student at the Faculty of Mathematics, University of Belgrade. Nenad Mladenović soon became a co-advisor in the preparation of Dragan's Ph.D. dissertation. After successfully defending the dissertation, they continued their cooperation in solving various other combinatorial optimization problems. During this cooperation, several very important methods (results) were obtained and published as follows:

  – The variable branching method [12] that can be applied in solving mixed-integer programming problems by using CPLEX or some other MIP solver to solve subproblems of the basic problem obtained by limiting that the Hamming distance between the currently best known solution and the local optimum is not greater than k (where is a parameter k that changes the value in a defined way).
  – A method known as Formulation Space Search was first applied to the problem of packing circles of equal radius into a unit circle [13]. Professor Frank Plastria also participated in the development of the method. The method was later applied to a number of other combinatorial optimization problems.
  – Nested variable neighborhood descent and mixed variable neighborhood descent proposed as new variants of local search. Mixed variable neighborhood descent was firstly successfully applied for solving the uncapacitated single allocation problem p -Hub median problem (USApHMP for short) [14]. Later, mixed variable neighborhood descent was successfully applied within general variable

    neighborhood search for Pick–up–and–Delivery Traveling salesman problem [15], Traveling deliveryman problem [16], etc.

- One of the most recently proposed methods is the Less is More method, which has been successfully applied for the first time to solve the minimum differential dispersion problem [17]. The method was developed in response to the explosion of various hybrid methods for combinatorial optimization problems that are very often complicated to implement. The main goal of the less is more approach is to develop methods that are as simple as possible (they have very few steps), but can find optimal or almost optimal solutions.

Nenad Mladenović and Dragan Urošević also collaborated in popularizing operations research, especially in Serbia. The Mathematical Institute of the Serbian Academy of Sciences and Arts successfully organized the Yugoslav Symposium on Operational Research (SYM-OP-IS) in 2003, 2009, 2015, and Nenad Mladenović was chairman of the Program Committee. In addition, the Mathematical Institute of the Serbian Academy of Sciences and Arts organized the EURO Mini Conference XXVIII on Variable Neighborhood Search in Herceg Novi (October 04, 2012–October 07, 2012).

4 Concluding Remarks

I was deeply honored by the kind invitation of the Editors-in-Chief of the Operations Research Forum, Professors Marco Lübbecke and Panos M. Pardalos, for proposing this review to honor an internationally recognized scientist and also a very good friend.

To end with, it is my firm belief that all researchers working on VNS topics should carry on the legacy of Professor Nenad Mladenović and continue, and further improve, the established series of the "International Conference on Variable Neighborhood Search" (ICVNS) meetings.

Declarations

Ethics Approval This study does not involve human participants or animals.

Consent to Participate Each colleague mentioned in Sect. 3 was invited by the author and kindly gave his/her consent to share his/her testimonial for Professor Nenad Mladenović.

Consent for Publication Each colleague mentioned in Sect. 3 was informed by the author that all testimonials will be included in a forthcoming article dedicated to the memory of Professor Nenad Mladenović.

Conflict of Interest The author declares no competing interests.

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