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Spatio-temporal efficiency measurement under undesirable outputs using multi-objective programming: a GAMS representation

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Abstract
Time series data in DEA often represent successive versions of the same unit (DMU). In order to assess efficiency of each DMU, DEA techniques have been employed. One of the problems that conventional DEA models face is that the reference set, when dealing with time series data, is not constructed correctly. This is attributed to the fact that conventional DEA models examine the DMUs and extract their efficiency scores based only the spatial dimension. However, when dealing with time series data for DMUs in the DEA context, the temporal dimension should be also taken into account. This paper is based on Spatio-Temporal DEA (ST-DEA) model (Petridis et al. in Ann Oper Res 238(1–2):475–496, 2016) and presents a GAMS representation of the model for the solution and explanation of ST-DEA model through an illustrative example. The scope of the paper is to analyze the concept of ST-DEA model and demonstrate its applicability via an application explained in GAMS optimization software.

Keywords Data envelopment analysis · Computational mathematics · MOP · Spatio-temporal efficiency · GAMS

1 Introduction

Each entity (a hospital, a school, an industry, a business etc) consumes inputs (raw material, labor etc) to produce outputs (products, services, etc). In economic terms, to measure the efficiency of these units is given by the following formula Efficiency $= \frac{Outputs}{Inputs}$ (Charnes et al. 1978).

In the presence of multiple inputs and outputs, the efficiency is calculated with Data Envelopment Analysis (DEA) which is a non parametric technique that uses Linear Programming. The first DEA models have been introduced by Charnes et al. (1978) with Constant Returns to Scale (CRS) formulation. The original DEA–CRS formulation is given below:

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$$\begin{aligned}
& max\ h_0 = \frac{\sum_{r=1}^{s} u_r \cdot y_{r,0}}{\sum_{i=1}^{m} v_r \cdot x_{i,0}} \\
& s.t. \\
& \frac{\sum_{r=1}^{s} u_r \cdot y_{r,0}}{\sum_{i=1}^{m} v_r \cdot x_{i,0}} \leq 1 \\
& u_r, v_i \geq 0, i = 1, .., m, r = 1, .., s
\end{aligned} \tag{1}$$

In the LP formulation, u_r and v_i are multipliers that are associated with the outputs and inputs respectively and are provided by solving DEA model for each Decision Making Unit (DMU). The DEA model initially as described in the previous LP model is called CCR model.

The CRS model has been extended by Banker et al. (1984) to variable returns to scale (VRS). The corresponding DEA model is the following:

$$\begin{aligned}
& max\ h_0 = \frac{\sum_{r=1}^{s} u_r \cdot y_{r,0}}{\sum_{i=1}^{m} v_r \cdot x_{i,0}} \\
& s.t. \\
& \frac{\sum_{r=1}^{s} u_r \cdot y_{r,0}}{\sum_{i=1}^{m} v_r \cdot x_{i,0}} \leq 1 \\
& \sum_{i=1}^{m} v_r \cdot x_{i,0} = 1 \\
& u_r, v_i \geq 0, i = 1, .., m, r = 1, .., s
\end{aligned} \tag{2}$$

Since the introduction of the CCR and BCC models, there have been proposed all possible applications and extensions of DEA models to almost all scientific areas. In the construction/manufacturing area during the production process, except for desirable outputs (for example energy), undesirable outputs are produced as well (GHG emissions, waste etc). Initially, Range Adjusted Measure (RAM) have been proposed to approach the phenomenon of undesirable outputs (Cooper et al. 2001). The RAM models have been extended to measure the efficiency of DMUs in the presence of undesirable outputs (Sueyoshi and Sekitani 2007; Sueyoshi and Goto 2011).

One of the major deficiencies of conventional DEA is the ability to construct the reference set of DMUs if each DMU is temporally allocated. An index that measures the level of change in inputs and outputs over a finite time horizon is Malmquist Index defined as follows (Caves et al. 1982):

$$M_t^{t+1} = \sqrt{\frac{D_0^t(x^{t+1}, y^{t+1}) \cdot D_0^{t+1}(x^{t+1}, y^{t+1})}{D_0^t(x^t, y^t) \cdot D_0^{t+1}(x^t, y^t)}}$$

Applications of Malmquist Index are presented in vehicle inspection services (Odeck 2000), in efficiency measurement of electricity distribution utilities (Førsund and Kittelsen 1998) and on a wide variety of scientific areas and disciplines.

The DEA models that have been proposed in the relevant literature approach the measurement of efficiency by only one dimension at a time. The conventional DEA models which are time invariant assume that DMUs represent homogeneous units on the same time horizon whereas if the temporal dimension is introduced, then Malmquist Index is used which measures the rate of change of inputs to outputs over two consecutive time periods.

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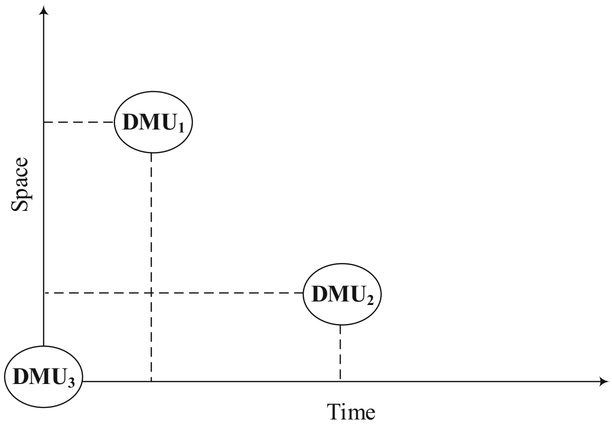

Fig. 1 Reference set of $DMU_{\tau=3}$ as per spatial and temporal dimension

Aim of the proposed paper is to provide a model where both temporal and spatial dimension are taken into account for the construction of the reference set (Petridis et al. 2016). Assuming that there are three DMUs ($DMU_{\tau=1,2,3}$) which are temporally allocated with $DMU_{\tau=3}$ to be closer to present date and $DMU_{\tau=1}$ to be the furthest from the present date. If $DMU_{\tau=2}$ is the fully efficient then the reference set will be constructed as per $DMU_{\tau=1}$ and $DMU_{\tau=3}$, therefore. If $DMU_{\tau=2}$ represents a hospital, a school or an economy, then the interpretation of the reference set would lead to the comparison of this entity at time $\tau = 2$ with the same entity as measured in the previous year ($\tau = 1$) and the same entity in the next year ($\tau = 3$). To ensure that the reference set will be constructed based on the temporal sequence of DMUs, then DEA model should be solved for each time point by adding DMUs that preceded the DMU under investigation. The latter is expressed in terms of the VRS constraint, as follows for $DMU_{\tau=3}$ and $DMU_{\tau=2}$.

for $\tau = 2, \ \sum_{j=1}^{\tau=2} \lambda_j = 1 \rightarrow \lambda_1 + \lambda_2 = 1$

for $\tau = 3, \ \sum_{j=1}^{\tau=3} \lambda_j = 1 \rightarrow \lambda_1 + \lambda_2 + \lambda_3 = 1$

The temporal construction of the reference set, solves partly the problem of $DMU_{\tau=3}$, however, except for the temporal dimension, the spatial dimension should also be considered. In the example of reference set of $DMU_{\tau} = 3$, it can be seen that $DMU_{\tau=1}$ is spatially closer to $DMU_{\tau=3}$ and $DMU_{\tau=2}$ is temporally closer to $DMU_{\tau=3}$. Since DEA models handle only one of the two dimensions for the construction of reference set and calculation of efficiency measures, then a new mathematical formulation is needed to provide a unique peer selection in terms of both the spatial and temporal dimensions (Fig. 1).

The proposed model provides a solution to the aforementioned problem of reference set construction. Such model has not yet been proposed in the relevant literature. The rest of the paper is structured as follows; in Sect. 2 the literature review summarizes all the models proposed in the relevant DEA literature. The model formulation and corresponding GAMS code are presented in Sect. 3, and results are presented in Sect. 4. The paper concludes in conclusions (Sect. 5).

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2 Literature review

2.1 A general overview

In real life cases, industries produce, except for desirable outputs, undesirable outputs as well, since industries consume raw material to produce final goods. Initially, DEA models considered only desirable outputs to measure the efficiency of DMUs. New extensions to assess the efficiency of undesirable outputs among with the desirable ones, over time, have been proposed in the relevant literature (Chung et al. 1997). In recent DEA review papers, new trends and extensions have been proposed throughout the years. Recent studies suggest that the number of publications utilizing DEA technique has grown "exponentially" from less than 100 in 1978 where the first DEA models have been introduced to approximately 1100 publications in 2016. Cumulatively, the number of publications in DEA from 1978–2016 is approximately 10,300 (Emrouznejad and Yang 2018). Due its simplicity of use, Data Envelopment Analysis technique has been applied in a wide selection of scientific areas, from supply chain design integrated with Mixed Integer Linear Programming models (Petridis et al. 2016; Grigoroudis et al. 2014), to the study of complex Energy & Environmental (E& E) issues (Giannakis et al. 2005; Petridis 2019; Abbott 2005). Especially, recent literature reviews (Sueyoshi et al. 2017) in the area of energy and environment indicate that the papers dealing with undesirable outputs have risen over the years.
The literature review section is divided into three parts; the papers dealing with DEA method for measuring efficiency considering undesirable outputs over a specific time point and the papers dealing with DEA model measuring the evolution of efficiency over time.

2.2 Efficiency measurement with undesirable outputs

Models measuring efficiency of units which consume inputs to produce desirable and undesirable outputs have been proposed in the literature. The introduction of 'bad' or undesirable outputs in the production process, has been proposed by Färe et al. (1989) (Färe 1993). In their work, a non-linear model has been proposed maximizing desirable and minimizing undesirable outputs. Several formulations have been proposed in order to handle undesirable outputs. One of them is to set undesirable outputs as inputs in the production process (Koopmans 1951; Berg et al. 1992). Except for the additive inverse ($-y^{und}$), the multiplicative inverse ($1/y^{und}$)has been also applied to deal with undesirable outputs (Golany and Roll 1989; Lovell et al. 1995; Athanassopoulos and Thanassoulis 1995). Another option regarding the undesirable outputs is the inclusion of a sufficient large number M added to the undesirable output ($M - y^{und}$) (Seiford and Zhu 2002).
Generally, DEA models with undesirable outputs have been used for efficiency measurement in energy production considering environmental consequences regarding harmful emissions during the production process (e.g. CO_2 emissions). 'Bad' or undesirable outputs are commonly used in coal-fired power plants (Yang and Pollitt 2009; Liu 2015; Song et al. 2014; Jie 2017) and in energy production where undesirable outputs can be energy loses, system failures etc (Petridis 2019). One of the main characteristics of undesirable outputs is the measurement of the efficiency in the case of services. Airport services have been examined with data regarding cargo movements, aircraft movements, and undesirable outputs regarding flight delays (Lozano et al. 2013). Advanced in DEA models handling undesirable outputs extend the ini-

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tial mathematical formulations (Liu 2010; An 2015). Network DEA formulations have been proposed in the literature simulating the multiple stages of a production process.

2.3 Measuring evolution of efficiency

In this section, the papers which deal with dynamic DEA formulations are presented. When time dimension is introduced into efficiency measurement dynamic DEA formulation (Emrouznejad and Thanassoulis 2005) since conventional DEA models fail to incorporate temporal dimension. Generally there are multiple DEA formulations when handling DMUs in a specific time horizon horizon. These formulations correspond to productions processes which can vary if there is a single period, multi-period without inter-temporal input–output dependence and multi-period with inter-temporal input–output dependence (Kao 2013). Dynamic models have been applied in all areas and disciplines to measure efficiency. The initial formulation introducing the dynamic aspect of DEA was proposed by Fare and Grosskopf (Färe and Grosskopf 1997). Since then dynamic DEA models have evolved incorporating uncertainty of input prices (Sengupta 1994, 1999). The temporal dimension of units is generally utilized in the banking sector (Avkiran 2015; Yu et al. 2019; Kweh 2018). Except for dynamic DEA models, efficiency in the presence of temporal data is calculated using Malmquist index (Caves et al. 1982). Since the index utilizes the evolution of the inputs and outputs of each DMU, several applications are proposed in finance (Tohidi et al. 2012, 2014), in Energy & Environmental studies (Sueyoshi and Goto 2013; Zhou et al. 2010; Pozo 2019). The essence of evolving units taking into consideration the temporal dimension find numerous applications ranging from the first flights of aircrafts and jets to wireless technologies (Durmuşoğlu and Dereli 2011; Inman et al. 2005). This formulation has a lot of advantages, nevertheless is applied only to technological forecasting assuming the superiority of a technology over other similar technologies in order to measure the efficiency of all units over time. Also, this technique does not take into account the spatial dimension in comparison to the proposed ST-DEA.

It can be seen from the literature review that the papers published propose methods which consider spatial or temporal dimension. A single formulation which will measure the efficiency of each DMU and construct its corresponding reference set taking into account both dimensions has not yet been proposed.

3 Methodology

3.1 Model formulation

In this section the model is formulated to introduce spatial and temporal dimensions in efficiency measurement. The model that is extended to calculate spatio-temporal efficiency is based on radial measurement of efficiency under desirable and undesirable outputs (Seiford and Zhu 2002; Sueyoshi and Goto 2014). The basic formulation is presented in the next LP formulation:

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Table 1 Notation of the variables, sets and parameters of the model

Index	**Set**
$j = 1, \ldots n$	DMUs
$\tau = \mu, \ldots, \text{n} \mid \mu = \max\{\text{n} \cdot \text{m}, 3 \cdot (\text{n} + \text{m})\}$	Subset of DMUs
$i = 1, \ldots, m$	Inputs
$r_1 = 1, \ldots s_1$	Desirable outputs
$r_2 = 1, \ldots s_2$	Undesirable outputs
$l = 1, \ldots, n$	Reference Set
$s = 1, \ldots, SC$	Iterations
Parameters	
w_{sp}^s	Weight of spatial criterion at iteration s
w_t^s	Weight of temporal criterion at iteration s
$y_{r_1,j}$	Desirable output r_1 of DMU j
λ_j^*	Optimal solutions of lambdas for DMU j
λ_j^*	Optimal solutions of lambdas for DMU j
ORD($\star$)	Function that attributes the order of set $\star$
$\boldsymbol{A}$	Spatial dimension matrix
$\boldsymbol{\Delta}$	Temporal dimension matrix
λ_τ^{max}	Maximum lambda value
δ_τ^{min}	Minimum temporal distance
Continuous Variables	
λ_j	Peer of each DMU
β	Inefficiency measure
$\hat{\beta}$	Spatio-Temporal Inefficiency
Binary Variables	
ζ_l	1 if lambda l is selected, 0 otherwise

$$
\begin{aligned}
& max\ \beta \\
& \quad s.t. \\
& \quad \sum_{j=1}^{n} \lambda_j \cdot x_{i,j} \leq x_{i,0}, i = 1, \ldots, m \\
& \quad \sum_{j=1}^{n} \lambda_j \cdot y_{r_1,j} \leq (1 + \beta) \cdot y_{r_1,0}, r_1 = 1, \ldots, s_1 \\
& \quad \sum_{j=1}^{n} \lambda_j \cdot y_{r_2,j} = (1 - \beta) \cdot y_{r_2,0}, r_2 = 1, \ldots, s_2 \\
& \quad \sum_{j=1}^{n} \lambda_j = 1 \\
& \quad \lambda_j \geq 0,\ j = 1, \ldots, n \\
& \quad \beta free
\end{aligned}
\tag{3}
$$

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In formulation 3, aim of the model is to maximize inefficiency represented by variable β. A full list of the parameters and variables of the proposed model is shown in Table 1. Nevertheless, to adjust the proposed model to the Spatio–Temporal framework for the reference set construction, then the model should be solved for a subset of the total of the DMUs. The parameters of the LP model are the inputs i for each DMU j $(x_{i,j})$, the desirable outputs r_1 for each DMU j $(y_{r_1,j})$ and the undesirable outputs r_2 for each DMU j $(y_{r_2,j})$. Assuming there are $DMUs$ which consume 1 input to produce 2 desirable outputs and 1 undesirable output, then $j = 1, .., 10$, $r_1 = 1, 2$ and $r_2 = 1$.

Therefore, LP 3 is reformulated as follows:

$$
\begin{aligned}
& for\ t = \mu, ..., n \\
& \quad max\ \beta \\
& \qquad s.t. \\
& \qquad \sum_{j=1}^{t} \lambda_j \cdot x_{i,j} \leq x_{i,0}, i = 1, ..., m \\
& \qquad \sum_{j=1}^{t} \lambda_j \cdot y_{r_1,j} \leq (1+\beta) \cdot y_{r_1,0}, r_1 = 1, ..., s_1 \\
& \qquad \sum_{j=1}^{t} \lambda_j \cdot y_{r_2,j} = (1-\beta) \cdot y_{r_2,0}, r_2 = 1, ..., s_2 \\
& \qquad \sum_{j=1}^{t} \lambda_j = 1 \\
& \qquad \lambda_j \geq 0, j = 1, ..., n \\
& \qquad \beta free \\
& end\ for
\end{aligned}
\tag{4}
$$

In LP formulation 4, the summation index n is replaced by t since LP model is solved sequentially for each DMU rather than for all possible DMUs. For example, for $DMU_{\tau=3}$ the analytical LP model 4 is solved for DMUs 1, 2 and 3 and not for all possible DMUs.

$$
\begin{aligned}
& max\ \beta \\
& \quad s.t. \\
& \quad \lambda_1 \cdot x_{i,3} + \lambda_2 \cdot x_{i,2} + \lambda_3 \cdot x_{i,3} \leq x_{i,3}, i = 1, ..., m \\
& \quad \lambda_1 \cdot y_{r_1,1} + \lambda_2 \cdot y_{r_1,2} + \lambda_3 \cdot y_{r_1,3} \leq (1+\beta) \cdot y_{r_1,3}, r_1 = 1, ..., s_1 \\
& \quad \lambda_1 \cdot y_{r_2,1} + \lambda_2 \cdot y_{r_2,2} + \lambda_3 \cdot y_{r_2,3} = (1-\beta) \cdot y_{r_2,3}, r_2 = 1, ..., s_2 \\
& \quad \lambda_1 + \lambda_2 + \lambda_3 = 1 \\
& \quad \lambda_j \geq 0, j = 1, 2, 3 \\
& \quad \beta free
\end{aligned}
\tag{5}
$$

Once the LP formulation is solved for t = & μ,...,n , then the solutions of 4 are only based on spatial dimension. To construct the Spatio-Temporal reference set, then two matrices are introduced namely $\boldsymbol{A}$ and $\boldsymbol{\Delta}$ representing the spatial and temporal dimension respectively. To provide a better understanding of the construction of table $\boldsymbol{A}$, then assume that in the reference set of $DMU_{\tau=3}$ (Fig. 1), $\lambda_2 = 0.2$ and $\lambda_1 = 0.8$ due to VRS constraint ($\sum_j^t \lambda = 1$). These

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Table 2 Spatial values of table $\boldsymbol{A}$ for example shown in Fig. 1

	DMU_1	DMU_2	...	DMU_n
DMU_3	0.2	0.8	...	0
DMU_4				
$\vdots$				
DMU_n				

Table 3 Temporal values of table $\boldsymbol{\Delta}$ for example shown in Fig. 1

	DMU_1	DMU_2	...	DMU_n
DMU_3	2	1	...	M
DMU_4				
$\vdots$				
DMU_n				

values represent the vertical dashed lines to x-axis (Times). Therefore, for $DMU_{\tau=3}$, $\boldsymbol{A}$ will be the following (Table 2).

The temporal dimension of each DMU is measured as the distance between the time point of the DMU under investigation and the points of the DMUs of its reference set. Assuming that the DMU under investigation is $DMU_{\tau=3}$, therefore the base time point is $\tau = 3$; the time points of its reference set, as described in Fig. 1 are $\tau = 2$ and $\tau = 1$ respectively. The temporal distance are represented as the points at vertical dashed lines to the x-axis (Time) and the axis start. The $\boldsymbol{\Delta}$ matrix is constructed as follows (Table 3).

Since aim of the model is to select the DMU in the reference set which is spatially and temporally closer to the DMU under consideration then the formulation should take into account the maximum λ value or the DMU with the minimum time distance from the one under investigation. To exclude selection of a DMU when constructing the temporal reference set, a very big number denoted as M is introduced.

In the case of the spatial dimension, the maximum λ value is selected and stored in vector λ_τ^{max} which is defined as:

$$\lambda_\tau^{max} = \max_l \lambda_{l,\tau}$$

In the case of the temporal dimension, the minimum temporal distance (defined as δ_τ) value is selected and stored in vector calculated as:

$$\delta_\tau^{min} = \max_j \delta_{l,\tau}$$

Once the $\boldsymbol{A}$ and $\boldsymbol{\Delta}$ matrices are populated with the solutions of λ values from LP model 4 and corresponding spatial λ_τ^{max} and temporal δ_τ^{min} vectors are calculated, then the nature of the problem becomes multi-objective since the decision of the DMU to be selected is based on two dimensions, either on spatial or temporal. Therefore, in order to construct the Spatio-Temporal reference set, then weights on each dimension should be introduced. To readjust the formulation, w_{sp}^s is the weight assigned to the spatial dimension and w_t^s is the weight assigned to the temporal dimension and $w_{sp}^s + w_t^s = 1$. Finally, the Spatio-Temporal reference set is selected based on the DMUs derived from LP formulation 4 based on temporal (higher weight on the temporal dimension and less on the spatial dimension $w_{sp}^s < w_t^s$) or on the spatial

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dimension and less on the spatial dimension $w_{sp}^{s} > s_t$), then binary variable ζ_l is introduced. Since the model is solved for different weight representations s for each dimension and due to the existence of binary variables, the resulting formulation is a Weighted Sum Model–Mixed Integer Linear Programming (WSM-MILP) model.

$$
\begin{aligned}
& for\ t = \mu, ..., n \\
& \quad for\ s = 1, ..., SC \\
& \qquad max\ w_{sp}^{s} \cdot \sum_{l}^{n} \frac{\lambda_{l,\tau}}{\lambda_{\tau}^{max}} \cdot \zeta_l - w_t^s \cdot \sum_{l}^{n} \frac{\delta_{l,\tau}}{\delta_{\tau}^{min}} \cdot \zeta_l \\
& \qquad s.t. \\
& \qquad \sum_{l=1}^{\tau} \zeta_l \cdot x_{l,i} \leq x_{i,0}, i = 1, ..., m \\
& \qquad \sum_{l=1}^{\tau} \zeta_l \cdot y_{l,r_1} \leq (1 + \beta) \cdot y_{r_1,0}, r_1 = 1, ..., s_1 \\
& \qquad \sum_{l=1}^{\tau} \zeta_l \cdot y_{l,r_2} = (1 - \beta) \cdot y_{r_2,0}, r_2 = 1, ..., s_2 \\
& \qquad 1 - \hat{\beta} \geq 0 \\
& \qquad \sum_{l=1}^{\tau} \zeta_l = 1 \\
& \qquad \zeta_l \in \{0, 1\} \\
& \qquad \beta\, free \\
& \quad end\ for \\
& end\ for
\end{aligned}
\tag{6}
$$

In WSM - MILP model 6, the objective function maximizes the outputs as per spatial $w_{sp}^{s} \cdot \sum_{l}^{n} \frac{\lambda_{l,\tau}}{\lambda_{\tau}^{max}} \cdot \zeta_l$ and temporal dimension $-w_t^s \cdot \sum_{l}^{n} \frac{\delta_{l,\tau}}{\delta_{\tau}^{min}} \cdot \zeta_l$. Each factor is divided by the corresponding maximum or minimum vector so that $\frac{\lambda_{l,\tau}}{\lambda_{\tau}^{max}} \cdot \zeta_l, \frac{\delta_{l,\tau}}{\delta_{\tau}^{min}} \cdot \zeta_l \in [0, 1]$. Since the direction of the objective function is maximization, term $-w_t^s \cdot \sum_{l}^{n} \frac{\delta_{l,\tau}}{\delta_{\tau}^{min}} \cdot \zeta_l$ represents the minimum distance from the DMU under investigation and the temporally closer DMU of its reference set. In constraints, λ value is replaced by binary variable ζ since the model selects a DMU from the reference set of the DMU under investigation. Constraint $\sum_{j=1}^{\tau} \zeta_l = 1$, ensures that a single DMU will be selected as per each dimension (spatial or temporal). Due to the latter constraint, a single DMU is selected, therefore, Spatio-Temporal efficiency will receive values greater than or equal to 1. To reject any solutions of the WSM-MILP model 4

3.2 Illustrative example with GAMS code

In this section the application of the Spatio-Temporal DEA model for measuring Spatio-Temporal efficiency and construction of reference set is demonstrated through an example and application to GAMS software analyzing the code.

The declaration of sets in GAMS is the following:

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Table 4 Data for illustrative example

	Input	Output1	Output2	Und Output
DMU1	5	6	14	12
DMU2	6	1	7	2
DMU3	7	3	9	3
DMU4	4	4	11	4
DMU5	5	12	6	11
DMU6	9	15	4	10
DMU7	7	6	12	11
DMU8	4	16	9	5
DMU9	5	10	8	6
DMU10	10	20	3	2

```
SETS t DMUs /DMU1*DMU10/
     kk(t) /DMU4*DMU10/
     j Inputs and Outputs /Dummy, Output1, Output2, UndOutput/
     ji(j) Inputs /Dummy/
     ds(j) Outputs /Output1, Output2/
     und(j) Undesirable output /UndOutput/
     headers /DMU, modelstat, solvestat, objval,temporal,data/;
```

For sake of simplicity, an example considering an input, two desirable outputs and one undesirable output is used. The data for the example are shown in Table 4.

Table 4 which have all the data regarding inputs, and desirable and undesirable outputs are shown in stated in GAMS with the code below. The data include all type of parameters needed for the model $[x_{j,i}, y_{j,r_1}, y_{j,r_2}]$.

```
      TABLE DATA(t,j) inputs and outputs of each DMU
      Dummy  Output1   Output2  UndOutput
DMU1    5      6          14       12
DMU2    6      1           7        2
DMU3    7      3           9        3
DMU4    4      4          11        4
DMU5    7      12          6       11
DMU6    9      15          4       10
DMU7    7       6         12       11
DMU8    4      16          9        5
DMU9    5      10          8        6
DMU10   10     20          3        2;
```

Next step is to declare the variables of the model. Firstly, model 4 is solved for $DMU_{\tau=\mu,\ldots n}$. The corresponding code is shown in GAMS code as follows:

```
VARIABLES
EFFICIENCY
BETA;
POSITIVE VARIABLES
LL(t);
```


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In correspondence with LP model 4, then `BETA` refers to β variable measuring inefficiency of each DMU and `LL(t)` corresponds to λ_j variable.

The parameters of LP model 4 are introduced below; `Y(j)` corresponds to the data of the DMU under investigation namely $x_{i,0}$ for inputs, $y_{r_1,0}$ for desirable outputs and $y_{r_2,0}$ for undesirable outputs. Parameter `eff(t)`, stores the efficiency (or inefficiency) scores to a vector for each DMU. The `Counter` parameter will be used for solving the model sequentially for each DMU as described in LP model 4.

```
Parameters Y(j) slice of data
           eff(t) efficiency report
           Counter;
Counter=4;
```

After the declaration of parameters, then the equations (generally the objective function and constraints of the model) are introduced. `CON1` corresponds to constraint $\sum_{j=1}^{t} \lambda_j \cdot x_{i,j} \leq x_{i,0}$. In the formulation, there is a conditional statement to bound the upper summation for considering only DMUs less than the order of the counter. Similarly, `CON2` corresponds to constraint $\sum_{j=1}^{t} \lambda_j \cdot y_{j,r_1} - (1+\beta) \cdot y_{r_1,0} \leq 0$ regarding desirable outputs and `CON3` corresponds to constraint $\sum_{j=1}^{t} \lambda_j \cdot y_{j,r_1} + (\beta+1) \cdot y_{r_1,0} = 0$ regarding undesirable output. Finally, `CON4` represents the VRS constraint $\sum_{j=1}^{t} \lambda_j = 1$.

```

EQUATIONS CON1(ji)
          CON2(ds)
          CON3(und)
          CON4;

CON1(ji)..  SUM(t$(ORD(t) LE Counter),LL(t)*DATA(t,ji))=L=Y
(ji);
CON2(ds)..  SUM(t$(ORD(t) LE Counter),LL(t)*DATA(t,ds))-Y(ds)
*(1+BETA)=G=0;
CON3(und).. SUM(t$(ORD(t) LE Counter),LL(t)*DATA(t,und))+Y
(und)*(BETA-1)=E=0;
CON4..      SUM(t$(ORD(t) LE Counter),LL(t))=E=1;

PARAMETER REP(kk,headers) solution report summary;
          Alpha(kk,t) Alpha table;

MODEL DEA1/OBJ,CON1,CON2, CON3,CON4/;

alias(kk,kkk);
alias(t,kkk1);

loop(kkk$(ORD(kkk) LE Counter),
   Y(j) = DATA(kkk,j);
   Counter=Counter+1;
   SOLVE DEA1 MAX  USING LP;
   REP(kkk,'DMU') = Counter;
   REP(kkk,'objval') = 1-BETA.l;
   REP(kkk,'solvestat') = DEA1.solvestat;
```


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```
   REP(kkk,'modelstat') = DEA1.modelstat;
   loop(kkk1$(ORD(kkk) LE Counter),
      Alpha(kkk,kkk1)=LL.l(kkk1);
   );
);
```

The Alias command duplicates the set, where in this case is subset kk(t). Model is solved for $DMU_{\tau=4,...10}$ and REP(kkk,'solvestat') store the Solver termination condition and model solution status respectively. The loop is used to solve Model 4 for $DMU_{\tau=4,...10}$. Once the solutions are obtained for each LP solved $\boldsymbol{A}$ table is constructed with the values of the optimal $\lambda_j^{\star}$ for $DMU_{\tau=4,...10}$.

Once the DEA model 4 is solved for each $DMU_{\tau=4,...10}$, the temporal distance is calculated. To find the order value of each DMU of the reference set constructed earlier, `Var(kk)` parameter is calculated. For the calculation, two GAMS functions are used: `CARD() ORD()`. The first function returns the cardinal value of a set.

```
Parameter Var(kk) Positions of DMUs for Delta matrix
Loop(kk,
    Var(kk)=CARD(t)-CARD(kk)+ORD(kk);
);
```

In this instance, since the set is set t is used, and set t includes $DMU_{\tau=1,...10}$, then `CARD(t)` will return 10. Function `ORD()` returns the order of an element of a set.

Below the calculation of $\boldsymbol{\Delta}$ matrix is shown. The $\boldsymbol{\Delta}$ can be only calculated only if the $\boldsymbol{A}$ is calculated.

```
Parameter Delta(kk,t);

set DD(kk,t), DD1(kk,t);

DD(kk,t)$(Alpha(kk,t) NE 0)=YES;
DD1(kk,t)$(Alpha(kk,t) EQ 0)=YES;

Delta(kk,t)$DD(kk,t)=var(kk)$DD(kk,t)-ORD(t)$DD(kk,t);
Delta(kk,t)$DD1(kk,t)= 1e7;
```

Also, two dynamic sets are constructed namely:

- `DD(kk,t)`
- `DD1(kk,t)`

Set `DD(kk,t)` is constructed upon the values of $\boldsymbol{A}$. Assuming that the elements of table $\boldsymbol{A}$ are denoted with $\lambda_{l,\tau}$, then the aforementioned set include the elements of table $\boldsymbol{A} : \lambda_{l,\tau} \neq 0$. Similarly, dynamic set `DD(kk,t)`, include the elements of table $\boldsymbol{A} : \lambda_{l,\tau} = 0$. To calculate $\boldsymbol{\Delta}$ matrix, then two cases are examined. The first is to provide the distance of the order of of its reference set, where the corresponding λ value is not 0. In this case the temporal order of the DMU under investigation minus the order of the DMU in its reference set is returned. In the second case, where the corresponding λ value is 0, then the corresponding $\boldsymbol{\Delta}$ value becomes a very large number M.

In GAMS formulation, $M = 10^7$. The construction procedure is shown in Fig. 2. For example, if the reference set of $DMU_{\tau=5}$ is formed by $\lambda_1 = 0.2$, $\lambda_2 = 0.5$ and $\lambda_4 = 0.3$ then corresponding δ values are computed upon the temporal distance of the $\lambda > 0$. For

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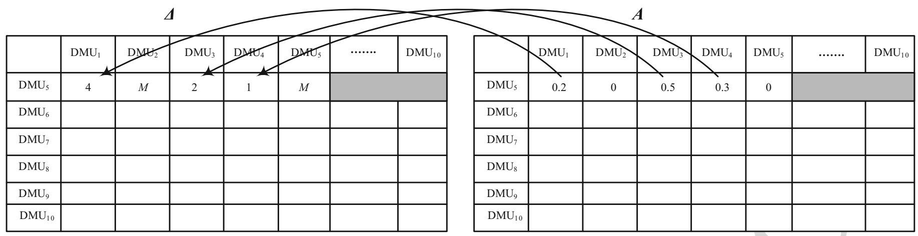

Fig. 2 Construction of $\boldsymbol{\Lambda}$ and $\boldsymbol{\Delta}$

instance, $\delta_{4,1} = 4$ because $\lambda_1 = 0.2 > 0$ and since the temporal point of DMU under investigation is 5 then the temporal distance is $5 - 1 = 4$. For the cases where $\lambda = 0$, then corresponding δ value equals a very large number M (for instance $\delta_{5,2}$).

Once $\boldsymbol{\Lambda}$ and $\boldsymbol{\Delta}$ matrices are calculated, spatial λ_τ^{max} and temporal δ_τ^{min} vectors are calculated as the maximum values of either each λ, δ values of the corresponding DMU under consideration.

```
lmax(kk)=smax(t,Alpha(kk,t));
dmin(kk)=smax(t$DD(kk,t),Delta(kk,t));
```

In case where $\delta_\tau^{min} = 0$ then $\delta_\tau^{min} = \epsilon$. The computation of Spatio-Temporal efficiency and construction of corresponding reference set are shown in GAMS code below.

Two variables are examined; namely `ST_EFF` which represents the weighted sum of spatial and temporal dimension and `BETA_HAT` which corresponds to Spatio-Temporal inefficiency variable $\hat{\beta}$. Binary variable `ST_ZETA(t)` corresponds to ζ_l. Since the model is Weighted Sum Model (WSM), then each weight assigned to spatial or temporal dimension is predefined. Each weight w_{sp}^s or w_t^s are complementary ($w_{sp}^s + w_t^s = 1$) and receive values in the range [0, 1]. On this instance each weight is given a specific value $w_{sp}^s = 0.1, ..., 1$ ($w_t^s = 1 - w_{sp}^s = 0.9, ..., 0$) with step equal to 0.1, since `weight(sc)=ORD(sc)/10`. The step can be reduced if the number of scenarios and corresponding denominator increase (for example `weight weight(sc)=ORD(sc)/100` for scenarios equal to 100).

```
VARIABLES
ST_EFF
BETA_hat;

Binary variables
Z(t);

Set sc /SC1*SC10/;

Parameter weight(sc), ww;
weight(sc)=ORD(sc)/10;
```

Similarly, constraints and objective function are constructed according to formulation of model 6. Objective function `ST_EFF` corresponds to $w_{sp}^s \cdot \sum_l^n \frac{\lambda_{l,\tau}}{\lambda_\tau^{max}} \cdot \zeta_l - w_t^s \cdot \sum_l^n \frac{\delta_{l,\tau}}{\delta_\tau^{min}} \cdot \zeta_l$. Constraints of model 6 resemble the one of initial model 3 with the exception of the introduction of binary variables ζ_l instead of λ values. Once the model is solved, for all weight representation regarding the spatial and temporal dimension, then results are stored

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in tables or vectors. The resulting efficiency is subjected to either the spatial or temporal dimension based on the weight on each term of the objective function. Also, based on the weight on each dimension, the Spatio-Temporal efficiency is constructed upon a single DMU of its reference set based on each of the two dimensions. After each MSW-MILP model is solved, the model solution status is returned using the modelstat function.

```
EQUATIONS OBJ1
          CON1_ST(ji)
          CON2_ST(ds)
          CON3_ST(und)
          CON4_ST
          CON5_BETA_HAT;

OBJ1..          ST_EFF=E=ww*(1/lmax_c)*SUM(t$(ORD(t) LE
Counter),alp(t)*Z(t))-
                              (1-ww)*(1/dmin_c)*SUM(t$(ORD(t) LE
Counter),delt(t)*Z(t));
CON1_ST(ji)..   SUM(t$(ORD(t) LE Counter),Z(t)*DATA(t,ji))
=L=Y(ji);
CON2_ST(ds)..   SUM(t$(ORD(t) LE Counter),Z(t)*DATA(t,ds))
-Y(ds)*(1+BETA_hat)=G=0;
CON3_ST(und)..  SUM(t$(ORD(t) LE Counter),Z(t)*DATA(t,und))
+Y(und)*(BETA_hat-1)=L=0;
CON4_ST..       SUM(t$(ORD(t) LE Counter),Z(t))=E=1;
CON5_BETA_HAT..  BETA_hat=G=0;

Model ST_DEA /OBJ1, CON1_ST, CON2_ST, CON3_ST, CON4_ST,CON5
_BETA_HAT/;

loop(kkk$(ORD(kkk) LE Counter),
   Y(j)=DATA(kkk,j);
   Counter=Counter+1;
   lmax_c=lmax(kkk);
   dmin_c=dmin(kkk);
   alp(t)=Alpha(kkk,t);
   delt(t)=Delta(kkk,t);
   loop(sc,
     ww=weight(sc);
     SOLVE ST_DEA MAX ST_EFF USING MIP;
     loop(kkk1$(ORD(kkk1) LE Counter),
     REP1(kkk,'modelstat') = ST_DEA.modelstat;
     res_BETA_hat(sc,kkk)=1/(1-BETA_hat.l);
     res_z(sc,kkk,kkk1)=Z.l(kkk1);
      );
   );
);
```

For the solution of the LP and the WSP - MILP DEA models, CPLEX solver has been used. (GAMS CPLEX 1996).

Table 5 $\boldsymbol{A}$ matrix for LP model 7

	DMU_1	DMU_4	DMU_5	DMU_6	DMU_8	DMU_10
DMU_5			1			
DMU_6				1		
DMU_7	0.74	0.18		0.08		
DMU_8					1	
DMU_9	0.008	0.32		0.67		
DMU_10						1

4 Results

The results of the proposed model are presented in this section. These results are associated with the parameters and the variables of the LP and WSM MILP DEA models.

Based on the description of the model presented above, firstly Model 4 model is solved. For two DMUs (e.g. $DMU_{\tau=7}$, the analytical form of Model 4 is shown below. Based on Table 4, $DMU_{\tau=7}$ consumes 7 units to produce 6 and 12 desirable outputs respectively. Also, through the assumed production procedure, 11 units of undesirable outputs are produced.

$$
\begin{array}{ll}
max & \beta \\
 & s.t. \\
 & 5 \cdot \lambda_1 + 6 \cdot \lambda_2 + 7 \cdot \lambda_3 + 1 \cdot \lambda_4 + 7 \cdot \lambda_5 \leq 2 \\
 & 6 \cdot \lambda_1 + 13 \cdot \lambda_2 + 3 \cdot \lambda_3 + 4 \cdot \lambda_4 + 11 \cdot \lambda_5 \leq 6 \cdot (1+\beta) \\
 & 14 \cdot \lambda_1 + 7 \cdot \lambda_2 + 9 \cdot \lambda_3 + 11 \cdot \lambda_4 + 12 \cdot \lambda_5 \leq 12 \cdot (1+\beta) \\
 & 12 \cdot \lambda_1 + 2 \cdot \lambda_2 + 3 \cdot \lambda_3 + 4 \cdot \lambda_4 + 6\lambda_5 = 11 \cdot (1-\beta) \\
 & \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 = 1 \\
 & \lambda_j \geq 0, \; j = 1, ..., 5 \\
 & \beta \, free
\end{array}
\tag{7}
$$

It can be seen in LP model 7, that the summation is done over the DMUs, the order f which is less than or equal to the DMU under investigation. The results of LP model 7 are optimal $\beta^{\star}$ values indicating the inefficiency of each DMU and $\boldsymbol{A}$ table where the λ values of the reference set of the initial model are shown in Table 5.

A case where Spatial and Temporal dimension is illustrated is $DMU_{\tau=7}$ where its reference set consists of $DMU_{\tau=1}$ ($\lambda_1 = 0.74$), $DMU_{\tau=4}$ ($\lambda_4 = 0.18$) and $DMU_{\tau=6}$ ($\lambda_6 = 0.08$). Based on the spatial dimension (proximity in space to $DMU_{\tau=7}$), the highest value is for $\lambda_1 = 0.74$. In terms of temporal dimension, the DMU which is closer to $DMU_{\tau=7}$ from its reference set is $DMU_{\tau=6}$. Also, the efficiency (or inefficiency) values are shown in Table 6 where it can be seen that $DMU_{\tau=7}$ and $DMU_{\tau=9}$ are not efficient.

Based on $\boldsymbol{A}$ matrix, the corresponding $\boldsymbol{\Delta}$ matrix which measures the temporal distances is constructed. To illustrate the functionality of $\boldsymbol{\Delta}$ matrix, the reference set of $DMU_{\tau=7}$ is examined. Since three DMUs belong to its reference set ($DMU_{\tau=1}$, $DMU_{\tau=4}$ and $DMU_{\tau=6}$), then its temporal distance is 6, 3 and 1 respectively. For DMUs which do not belong to its reference set (i.e. λ =0) the temporal distance equals to a very large number ($M \approx 10^7$). The results of $\boldsymbol{\Delta}$ matrix are shown in Table 7.

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Table 6 Results of efficiency scores based on Model 4

	$1-\beta$
DMU_5	1
DMU_6	1
DMU_7	0.95
DMU_8	1
DMU_9	0.79
DMU_{10}	1

Having computed matrices $\boldsymbol{A}$ matrix and $\boldsymbol{\Delta}$, the next step is to solve the WSM-MILP model 6. Vectors λ_τ^{max} and δ_τ^{min} are calculated based on the corresponding matrices for spatial and temporal dimension. The calculation of λ_τ^{max} is straightforward and is the maximum λ value of the reference set of the DMU under investigation, where, $\delta_\tau^{min} = \max_j \delta_{l,\tau} : \lambda_{l,\tau} \neq 0$. In case where the temporal distance is 0, which can be found if the DMU under investigation is efficient reference set consist of the same DMU, then a small number is assigned (i.e. 0.001) since this vector will be in the denominator. For example, $\lambda_{\tau=7}^{max} = 0.74$ where $\delta_\tau^{min} = 6$. Both vectors are used for normalization and shown in Table 8.

For the solution of Model 6, parameters derived either from optimal solutions of LP Model 4 (for example $\boldsymbol{A}$) or parameters which are constructed upon the latter information (for example $\boldsymbol{\Delta}$) are required.

The WSM-MILP ST DEA model for $DMU_{\tau=7}$ assuming that spatial dimension is weighted by 70%, subsequently temporal dimension is weighted by 30% is provided below:

$$\begin{aligned}
&max\ 0.7 \cdot \frac{0.74 \cdot \zeta_1 + 0.18 \cdot \zeta_4 + 0.08 \cdot \zeta_6}{0.74} \\
&\quad -0.3 \cdot \frac{6 \cdot \zeta_1 + M \cdot \zeta_2 + M \cdot \zeta_3 + 3 \cdot \zeta_4 + M \cdot \zeta_5 + M \cdot \zeta_6 + M \cdot \zeta_7}{5} \\
&\quad s.t. \\
&\quad 5 \cdot \zeta_1 + 6 \cdot \zeta_2 + 7 \cdot \zeta_3 + 1 \cdot \zeta_4 + 5 \cdot \zeta_5 + 9 \cdot \zeta_6 + 7 \cdot \zeta_7 \leq 7 \\
&\quad 6 \cdot \zeta_1 + 1 \cdot \zeta_2 + 3 \cdot \zeta_3 + 4 \cdot \zeta_4 + 10 \cdot \zeta_5 + 15 \cdot \zeta_6 + 10 \cdot \zeta_7 \leq 6 \cdot (1+\beta) \\
&\quad 14 \cdot \zeta_1 + 7 \cdot \zeta_2 + 9 \cdot \zeta_3 + 11 \cdot \zeta_4 + 6 \cdot \zeta_5 + 4 \cdot \zeta_6 + 12 \cdot \zeta_5 \leq 12 \cdot (1+\beta) \\
&\quad 12 \cdot \zeta_1 + 2 \cdot \zeta_2 + 3 \cdot \zeta_3 + 4 \cdot \zeta_4 + 11\zeta_5 + 10 \cdot \zeta_6 + 11 \cdot \zeta_7 \leq 11 \cdot (1-\beta) \\
&\quad \zeta_1 + \zeta_2 + \zeta_3 + \zeta_4 + \zeta_5 + \zeta_6 + \zeta_7 = 1 \\
&\quad 1 - \hat{\beta} \geq 1 \\
&\quad \zeta_j \in \{0, 1\},\ j = 1, ..., 7
\end{aligned} \tag{8}$$

From WSM-MILP formulation 8, Spatio-Temporal efficiency is computed and shown in Fig. 3 for$DMU_{\tau=7}$. It can be seen that Spatio-Temporal efficiency $\frac{1}{1-\hat{\beta}}$, is lower for low values of weight to spatial dimension and higher for higher values on the spatial dimension. This step wise figure is explained since in the region of $0.1 \leq w_{sp} < 0.5$ or $0.5 \leq w_t \leq 0.9$ to temporal weight, $DMU_{\tau=6}$ is selected since is temporally closer to $DMU_{\tau=7}$. Therefore the selection of this single DMU provides a value for Spatio-Temporal efficiency equals to 0.6. For spatial weight values in the region of $0.6 \leq w_{sp} \leq 1$ or $0.1 \leq w_t \leq 0.4$ to temporal weight, then $DMU_{\tau=1}$ is selected which is temporally more distant to the DMU

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Table 7 Temporal distances matrix ($\boldsymbol{\Delta}$)

	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10
DMU5	10^7	10^7	10^7	10^7	0	10^7	10^7	10^7	10^7	10^7
DMU6	10^7	10^7	10^7	10^7	10^7	0	10^7	10^7	10^7	10^7
DMU7	6	10^7	10^7	3	10^7	1	10^7	107	10^7	10^7
DMU8	10^7	10^7	10^7	10^7	10^7	10^7	10^7	0	10^7	10^7
DMU9	8	10^7	10^7	5	10^7	10^7	10^7	1	10^7	10^7
DMU10	10^7	10^7	10^7	10^7	10^7	10^7	10^7	10^7	10^7	0

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Table 8 Results for vectors λ_τ^{max} and δ_τ^{min}

	λ_τ^{max}	δ_τ^{min}
DMU5	1	0.01
DMU6	1	0.01
DMU7	0.74	6
DMU8	1	0.01
DMU9	0.67	8
DMU10	1	0.01

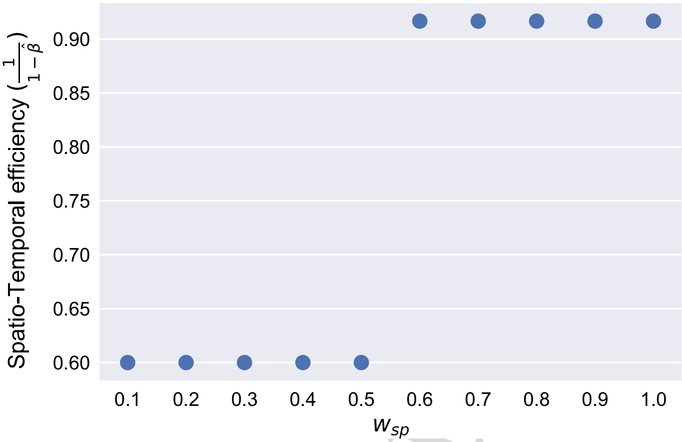

Fig. 3 Spatio-Temporal efficiency of $DMU_{\tau=7}$ for w_{sp}

under investigation but is spatially closer to that. In this case, the Spatio-Temporal efficiency is higher and equals to 0.916.

5 Conclusions

Assessing the efficiency of units that evolve over time is crucial as one has to take into account both the spatial and temporal dimensions. Measuring the efficiency of evolving units or of units in an evolving environment, has raised great attention in DEA literature. The major advantage of measuring evolving units is that Decision Makers can identify and actually measure how well units perform on a temporal basis. If for example the units are consecutive versions of software or the units are innovative products that change over time, then the change in the rate can be easily assessed with a wide selection of DEA models. Nevertheless, these models that are applied for dealing with time series data examine only the spatial dimension and leave out the temporal. In this paper ST-DEA model is extended to incorporate the undesirable outputs which considers time and space together as the trade-off factors to provide the decision support for DMs. The advantage of the new formulation is the construction of Spatio-Temporal reference set based on desirable and undesirable outputs. Applications of this new formulation can be found in almost all research areas and relevant fields (from Electricity production to E-waste management and waste management in general). Aim of the model is to provide a new efficiency measure based on the Spatio-

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Temporal reference set which is formulated based on a Multi-Objective Weighted Sum Model Mixed Integer Linear Programming model (WSM MILP). To stress the applicability of the model, the proposed extension of ST-DEA model has been applied to a toy example through GAMS code. The proposed ST-DEA model can be extended to almost any DEA formulation, especially on the Network DEA models which deal with DMUs representing time points.

6 Appendix: GAMS formulation

```
SETS t DMUs /DMU1*DMU10/
     kk(t) /DMU5*DMU10/
     j Inputs and Outputs /Dummy, Output1, Output2, UndOutput/
     ji(j) Inputs /Dummy/
     ds(j) Outputs /Output1, Output2/
     und(j) Undesirable output /UndOutput/
     headers /DMU, modelstat, solvestat, objval,temporal,data/;

TABLE DATA(t,j) outputs of each DMU
      Dummy  Output1   Output2  UndOutput
DMU1    5      6          14        12
DMU2    6      1           7         2
DMU3    7      3           9         3
DMU4    4      4          11         4
DMU5    5     10           6        11
DMU6    9     15           4        10
DMU7    7      6          12        11
DMU8    4     16           9         5
DMU9    5     10           8         6
DMU10  10      20          3         2;

VARIABLES
EFFICIENCY
PHI;
POSITIVE VARIABLES
LL(t);

PARAMETERS Y(j) slice of data
           eff_k(t) efficiency report;

PARAMETER Counter;

Counter=4;

EQUATIONS OBJ
          CON1(ji)
          CON2(ds)
          CON3(und)
          CON4;
```


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```
OBJ..       EFFICIENCY=E=Phi;
CON1(ji)..  SUM(t$(ORD(t) LE Counter),LL(t)*DATA(t,ji))=L
=Y(ji);
CON2(ds)..  SUM(t$(ORD(t) LE Counter),LL(t)*DATA(t,ds))=G
=Y(ds)*(1+Phi);
CON3(und).. SUM(t$(ORD(t) LE Counter),LL(t)*DATA(t,und))=L
=Y(und)*(1-Phi);
CON4..      SUM(t$(ORD(t) LE Counter),LL(t))=E=1;

alias(kk,kkk);
alias(t,kkk1);

PARAMETER REP(kk,headers), REP1(kk,headers) solution report
summary, Counter1(kkk);
Parameter Alpha(kk,t);

MODEL DEA1/OBJ,CON1,CON2, CON3,CON4/;

Parameter sm1, sm2;

loop(kkk$(ORD(kkk) LE Counter),
   Y(j) = DATA(kkk,j);
   Counter=Counter+1;
   Counter1(kkk)= Counter;
   SOLVE DEA1 MAX PHI USING LP;
   REP(kkk,’DMU’) = Counter;
   REP(kkk,’objval’) = 1-PHI.l;
   REP(kkk,’solvestat’) = DEA1.solvestat;
   REP(kkk,’modelstat’) = DEA1.modelstat;
   loop(kkk1$(ORD(kkk) LE Counter),
      Alpha(kkk,kkk1)=LL.l(kkk1);
   );
);

Parameter Var(kk)
loop(kk,
     Var(kk)=CARD(t)-CARD(kk)+ORD(kk);
    );
Parameter Delta(kk,t);

set DD(kk,t), DD1(kk,t);

DD(kk,t)$(Alpha(kk,t) NE 0)=YES;
DD1(kk,t)$(Alpha(kk,t) EQ 0)=YES;

Delta(kk,t)$DD(kk,t)=var(kk)$DD(kk,t)-ORD(t)$DD(kk,t);
Delta(kk,t)$DD1(kk,t)= 1E7;
```

```
Variables
st_eff
phi_hat;

Binary variables
Z(t);

Set sc /SC1*SC10/;

Parameter nn;
nn = CARD(sc);

Parameter weight(sc), ww;
weight(sc)=ORD(sc)/nn;
Parameters lmax(kk), dmin(kk), lmax_c, dmin_c, alp(t), delt(t),
res_z(sc,kk,t),res_phi_hat(sc,kk);

lmax(kk)=smax(t,Alpha(kk,t));
dmin(kk)=smax(t$DD(kk,t),Delta(kk,t));

loop(kk,
 if(dmin(kk)=0,
    dmin(kk)=1e-3;
  );
);

Parameter Counter2;

Counter2 = 4;

EQUATIONS OBJ1
          CON1_ST(ji)
          CON2_ST(ds)
          CON3_ST(und)
          CON4_ST
          CON5_PHI_HAT;

OBJ1..          ST_EFF=E=ww*(1/lmax_c)*SUM(t$(ORD(t) LE
Counter2),alp(t)*Z(t))-(1-ww)*(1/dmin_c)*SUM(t$(ORD(t) LE
Counter2),delt(t)*Z(t));
CON1_ST(ji)..   SUM(t$(ORD(t) LE Counter2),Z(t)*DATA(t,ji))=L
=Y(ji);
CON2_ST(ds)..   SUM(t$(ORD(t) LE Counter2),Z(t)*DATA(t,ds))=G
=Y(ds)*(1+Phi_hat);
CON3_ST(und)..  SUM(t$(ORD(t) LE Counter2),Z(t)*DATA(t,und))=L
=Y(und)*(1-Phi_hat);
CON4_ST..       SUM(t$(ORD(t) LE Counter2),Z(t))=E=1;
CON5_PHI_HAT..  1-Phi_hat=G=1;
```


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```
Model ST_DEA /OBJ1, CON2_ST, CON3_ST, CON4_ST, CON5_PHI_HAT/;

loop(kkk$(ORD(kkk) LE Counter2),
   Y(j)=DATA(kkk,j);
   Counter2=Counter2+1;
   lmax_c=lmax(kkk);
   dmin_c=dmin(kkk);
   alp(t)=Alpha(kkk,t);
   delt(t)=Delta(kkk,t);
   loop(sc,
     ww=weight(sc);
     SOLVE ST_DEA MAX ST_EFF USING MIP;
     loop(kkk1$(ORD(kkk1) LE Counter),
     REP1(kkk,'modelstat') = ST_DEA.modelstat;
     res_phi_hat(sc,kkk)=1-Phi_hat.l;
     res_z(sc,kkk,kkk1)=Z.l(kkk1);
      );
   );
);
```

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