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# In search for the most preferred solution in value efficiency analysis

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**Abstract**

Choosing the Most Preferred Solution (MPS), namely a real or artificial Decision Making Unit (DMU) reflecting the decision maker's preferences over the desirable structure of inputs and outputs, is of particular importance in Value Efficiency Analysis (VEA). In this paper, we review various MPS choices used in the VEA literature and propose some new, which rely respectively on the relative position of frontier DMUs, the Most Productive Scale Size (MPSS), the Average Production Unit (APU), and common vectors of weights. The suggested MPS choices reflect overall organizational goals such as the pursuit of scale economies and the maximization of structural efficiency, or the need to assess DMUs against common standards because of limited control over the resources allocated to them or autonomy in setting their own priorities. The potential implications of using different MPSs in VEA are illustrated by providing comparative empirical results using a dataset of 526 Greek cotton farms.

**Keywords** Value efficiency analysis · Most preferred solution · DMU frontier position · Most productive scale size · Average production unit · Common weights

**JEL classification** C14 · C44 · C61 · D24

## 1 Introduction

In several occasions where the performance of Decision Making Units (DMUs) is evaluated by means of Data Envelopment Analysis (DEA, Charnes et al. 1978), it is desired or necessary to consider the preferences of central management, supervising agencies or Decision Makers (DMs) that coordinate the operation of DMUs. This need might arise for purposes of performance monitoring, i.e., measuring the extent to which the performance of DMUs complies with overall behavioral or organizational objectives (monetary and non-monetary), as well as for performance control and future planning, namely designing mechanisms that redirect DMUs towards the achievement of managerial goals or normative performance standards.

Preferences in DEA studies are often elicited by means commonly used in Multiple Objective Linear Programming (MOLP), namely by incorporating expert information on the desirable input and output values for the evaluated DMUs (Korhonen et al. 2002). One form this might take is that of the Most Preferred Solution (MPS). The MPS is a non-dominated (i.e., strongly DEA-efficient) DMU or a combination of DMUs, which has the most desirable structure in DM's view, in the sense of maximizing his/her value (Korhonen 2002) or utility function (Yang et al. 2009). It may represent the structure according to which management in a firm wishes to reorganize its branches or it might be viewed as a mentor from which other DMUs can learn. The MPS was incorporated into DEA by Halme et al. (1999), in an approach coined Value Efficiency Analysis (VEA). In VEA, the DMUs are assessed against a frontier consisting of the extended DEA efficient facets intercepting at the MPS, which is chosen by the DM in a prior step. In essence, the marginal rates of substitution (MRSs) of inputs or transformation (MRTs) of outputs imposed on the

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evaluated DMUs are those observed on the DEA frontier for the MPS.

Choosing the MPS is an important issue in VEA, as it affects the resulting efficiency frontier and, consequently, the DMUs' efficiency scores (Korhonen et al. 2001). A suitably chosen MPS can yield valuable insights regarding the extent to which current DMUs' performance complies with managerial preferences or organizational goals, and provide the basis for a cost-saving or revenue-increasing restructuring. On the other hand, an inappropriate MPS choice might provide questionable efficiency scores, which may subsequently give rise to poor managerial decisions, such as an unnecessary and costly resource reallocation. Nevertheless, there seems to be no general rule for choosing the MPS in VEA. Instead, several suggestions have been made up to date. In many of these, the MPS is not chosen on the basis of some overall managerial objective and thus it is difficult to come up with an intuitive explanation for the DM's choice, while in others the chosen MPS may favor specialization in the production of a few outputs or in the use of a few inputs, which is often deemed unsatisfactory by managers (Epstein and Henderson 1989). Other MPS choices may compare DMUs with exceptionally performing benchmarks, assess them against a DMU operating with non-technically optimal scale, or zero and undefined values for MRSs and MRTs, i.e., using vectors of input/output weights including zero values. In addition, no empirical work has been done so far on how alternatively chosen MPSs may affect the VEA efficiency scores.

The objective of this paper is twofold: *First*, to expand the set of MPS choices. We first advocate that the DM could make a more informed choice of the MPS among the efficient DMUs, by paying attention on their position on the efficient frontier. In particular, we propose that prior to MPS choice the efficient DMUs are clustered based on whether they appear in the reference sets of other DMUs in DEA and whether they reside in frontier edges (Edvardsen et al. 2008). This clustering can provide additional information to the DM about the DMUs for which there is strong evidence of good relative performance, those that are potentially overspecialized, and those that may be associated with zero and/or undefined marginal rates. Alternatively, one may choose the MPS among those with Most Productive Scale Size (MPSS), i.e., those achieving maximal average productivity for their input/output bundle. This will ensure that the DMUs are assessed against a technically optimal scale, the achievement of which is a long-term organizational goal, which interests both individual DMUs as well as central management (Førsund and Hjalmarsson 1979). Our third proposed MPS is the combination of peers of the Average Production Unit (APU). The APU is an artificial DMU that operates with the group means quantities of inputs and outputs, and its technical efficiency score reflects the structural efficiency of the whole group of DMUs when resource allocation is centrally coordinated. Its structure reflects the one that each DMU should have in order for the group as a whole to realize its full potential output production, and the resulting VEA scores may be particularly useful for guiding future performance planning. Another proposal is to assess DMUs using on a common vector of strictly positive input/output weights in VEA, by choosing a combination of DMUs that generate a unique Fully Dimensional Efficient Facet (FDEF) as the MPS. This results in evaluating all DMUs against a common standard and well-defined MRSs and MRTs and could be useful in several cases where the assessed DMUs perform essentially the same task or have limited autonomy in setting their own priorities and objectives (i.e., choose individually the values of input/output weights).

The paper's second objective is to provide comparative empirical evidence on how alternative MPS choices may affect the estimated VEA efficiency scores. More specifically, using data for 526 Greek cotton farms, we compare the efficiency estimates obtained by the DEA model and those of VEA models with alternative MPS choices. The results of this analysis provide useful insights regarding the MPS choices that are more likely to result in excessive or negligible differences between the DEA and the VEA distributions of efficiency scores.

The rest of the paper unfolds as follows: In the second section, we present the VEA model while in the third section, we review the MPSs proposed previously in VEA and suggest four new. In the fourth and the fifth section, we illustrate how the choice of the MPS may affect VEA efficiency scores. Concluding remarks follow in the last section.

## 2 Materials and methods

In VEA, DM preferences are reflected through an implicitly known pseudo-concave value function (i.e., an indifference curve), that becomes tangent to the DEA efficient frontier at the point where the MPS is located.[1] This value function might reflect some organizational objective, i.e., be a cost or a profit function, but it might also reveal preferences other than those related with prices (Thanassoulis et al. 2008, p. 73). The empirical VEA frontier is then constructed as the lower envelope of the extended efficient facets intercepting at the MPS. As DEA facets are generated by extreme-efficient DMUs (i.e., those that reside at a point of the

[1] For a detailed treatment of VEA, see Joro and Korhonen (2015).

convex DEA frontier where more than one facets intercept, see Charnes et al. 1991), the MPS will be either a single extreme-efficient DMU or a combination of extreme-efficient DMUs that are *jointly* efficient, in the sense that they generate at least one common facet.[2] In the latter case, only those common efficient facets are extended to obtain the VEA frontier.

Introducing the MPS requires only slight modifications to the conventional DEA model. Let us consider a set of $K$ DMUs ($h = 1, \ldots, K$), that operate under the same technology and use $I$ ($i = 1, \ldots, I$) inputs to produce $J$ ($j = 1, \ldots, j$) outputs. The input and output vectors of each DMU are assumed to be semi-positive, that is, each DMU uses at least one input to produce at least one output. Further, we assume that the DM has select a set $R$ ($r = 1, \ldots, R$) of extreme-efficient DMUs as the MPS.[3] An output-oriented, variable-returns-to-scale (VRS) VEA model in its multiplier and envelopment form is given as (Halme and Korhonen 2015):

$$
\begin{array}{llll}
\min\limits_{u_j^k, v_i^k, u^k} & \sum\limits_{i=1}^{I} v_i^k x_i^k - u^k & & \\
s.t. & -\sum\limits_{j=1}^{J} u_j^k y_j^h + \sum\limits_{i=1}^{I} v_i^k x_i^h - u^k \geq 0 & h = 1, \ldots, K, h \neq r \\
& -\sum\limits_{j=1}^{J} u_j^k y_j^r + \sum\limits_{i=1}^{I} v_i^k x_i^r - u^k = 0 & r = 1, \ldots, R \\
& \sum_{j=1}^{J} u_j^k y_j^k = 1 & \\
& u_j^k \geq 0 & j = 1, \ldots, J \\
& v_i^k \geq 0 & i = 1, \ldots, I \\
& u^k \, free &
\end{array}
\qquad
\begin{array}{lll}
\max\limits_{\varphi_{VEA}^k, \lambda_h^k} & \varphi_{VEA}^k & \\
s.t. & \sum\limits_{h=1}^{K} \lambda_h^k y_j^h \geq \varphi_{VEA}^k y_j^k & j = 1, \ldots, J \\
& \sum_{h=1}^{K} \lambda_h^k x_i^h \geq x_i^k & i = 1, \ldots, I \\
& \sum\limits_{h=1}^{K} \lambda_h^k = 1 & \\
& \lambda_h^k \geq 0 & h = 1, \ldots, K, h \neq r \\
& \lambda_r^k \, free & r = 1, \ldots, R \\
& \varphi_{VEA}^k \, free &
\end{array}
\tag{1}
$$

where $x$ and $y$ are input and output quantities, $1/\varphi_{VEA} \in (0,1]$ is the efficiency score, $\lambda$ the intensity variables, and $v$, $u$ and $u^k$ are parameters to be estimated. The input-oriented counterpart of (1) is given as:

$$
\begin{array}{llll}
\max\limits_{u_j^k, v_i^k, u^k} & \sum\limits_{j=1}^{J} u_j^k y_j^k + u^k & & \\
s.t. & \sum\limits_{j=1}^{J} u_j^k y_j^h - \sum\limits_{i=1}^{I} v_i^k x_i^h + u^k \leq 0 & h = 1, \ldots, K, h \neq r \\
& \sum\limits_{j=1}^{J} u_j^k y_j^r - \sum\limits_{i=1}^{I} v_i^k x_i^r + u^k = 0 & r = 1, \ldots, R \\
& \sum\limits_{i=1}^{I} v_i^k x_i^k = 1 & \\
& u_j^k \geq 0 & j = 1, \ldots, J \\
& v_i^k \geq 0 & i = 1, \ldots, I \\
& u^k \, free &
\end{array}
\qquad
\begin{array}{lll}
\min\limits_{\theta_{VEA}^k, \lambda_h^k} & \theta_{VEA}^k & \\
s.t. & \sum\limits_{h=1}^{K} \lambda_h^k y_j^h \geq y_j^k & j = 1, \ldots, J \\
& \sum\limits_{h=1}^{K} \lambda_h^k x_i^h \geq \theta_{VEA}^k x_i^k & i = 1, \ldots, I \\
& \sum\limits_{h=1}^{K} \lambda_h^k = 1 & \\
& \lambda_h^k \geq 0 & h = 1, \ldots, K, h \neq r \\
& \lambda_r^k \, free & r = 1, \ldots, R \\
& \theta_{VEA}^k \, free &
\end{array}
\tag{2}
$$

where $\theta_{VEA} \in (0,1]$ is the efficiency score. The constant-returns-to-scale (CRS) form of (1) and (2) are obtained by removing the free variable and the convexity constraint from their multiplier and envelopment forms, respectively. The CRS counterpart of (2), in its multiplier form, appears for the first time in Oral and Yolalan (1990) and Oral et al. (1992), where it is used to compare every DMU's performance to that of a particular efficient DMU, which is selected at a previous step.

The envelopment form of the models in (1) and (2) differs from those of conventional DEA in that the sign of the intensity variable corresponding to the MPS is free instead of restricted to be non-negative (Halme et al. 1999). In the multiplier form of the models, this corresponds to turning from inequality to equality the restriction referring to the MPS. This restricts the choice of input/output weights for the evaluated DMUs only to those that are optimal for the MPS.[4] In essence, the choice of the MPS results in evaluating every DMU using the MRSs and MRTs that are observed on the DEA frontier in the neighborhood of the MPS. The DM may view these marginal rates as adequate enough to apply globally as they reflect his/her own valuation of inputs and outputs. The evaluated DMUs for which at least one optimal vector of weights in DEA is also optimal for the MPS receive the most optimistic VEA score possible,

[2] If the DM selects a DEA-inefficient or weakly-efficient DMU (i.e., a dominated DMU that has at least one positive optimal value for an input or output slack), or a non-extreme-efficient DMU (one located on the interior of a facet) as the MPS, then the combination of the extreme-efficient DMUs that are identified as its peers in DEA can be used as the MPS instead (see e.g., Halme et al. 1999). The use of the peers of the DEA-inefficient DMU rather than its radial projection in the DEA frontier is advocated, as the latter might be associated with input and/or output slacks and thus may not be a non-dominated DMU. The same is the case for weakly efficient DMUs.

[3] Note that the number of extreme-efficient DMUs constituting the MPS cannot be more than $(I + J - 1)$ in DEA models with constant returns to scale (CRS) and $(I + J)$ in DEA models with variable returns to scale (VRS), as this is the maximum number of extreme-efficient DMUs that may generate an efficient facet of the DEA surface (see Olesen and Petersen 2003). These maximal facets are called fully Dimensional Efficient Facets (FDEFs) and are associated with a unique normal vector of input/output weights with strictly positive values (Olesen and Petersen, 2015).

[4] Such equality restrictions have been used for incorporating expert views in DEA in other studies as well, without referring explicitly to VEA. Zhu (2001) uses the CRS counterpart of (2) in its multiplier form to benchmark the quality of life of 20 cities against a set of peer DMUs that would necessarily contain three pre-selected cities identified by Fortune magazine as the top-three best cities in terms of quality of life (see his equation (8)). Furthermore, Cook et al. (2004) used input-oriented CRS and VRS VEA models under the name "fixed benchmark model" in order to measure the performance of out-of-sample DMUs (see their equations (9) and (10)). Also, Wang and Luo (2006) used a model that is equivalent to the input-oriented CRS VEA model, in which the frontier projection of an artificially constructed ideal DMU (IDMU), namely one that consumes the minimum sample quantities for each input while producing the maximum sample quantities for each output, corresponds to the MPS (see their equation (4)). The DEA frontier projection of the IDMU was obtained via a super-efficiency model.

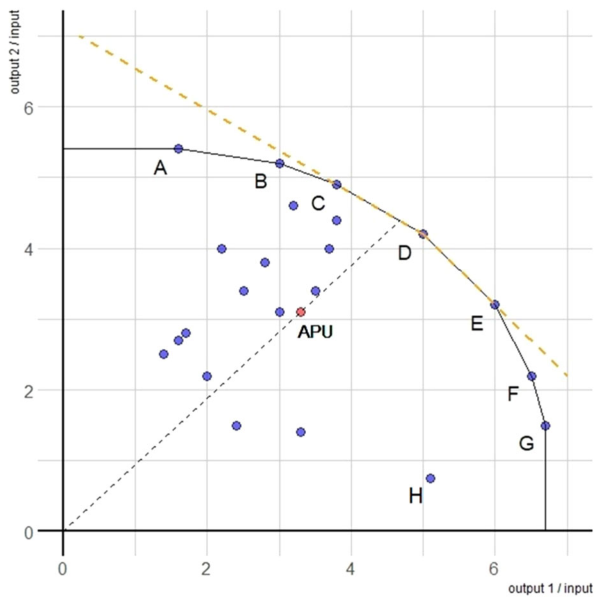


**Fig. 1** Extending efficient facets through VEA

namely one that is equal to their DEA efficiency score. The remaining DMUs, the input/output structure of which "deviates too much" from the one of the MPS (Korhonen et al. 2002, p. 59), are forced to accept less favorable weights in VEA compared to DEA, and their VEA scores are lower than the corresponding DEA ones.

The facet extensions in VEA are illustrated in Fig. 1 in the case of one-input-two-outputs technology. Choosing DMU D as the MPS implies the dashed line frontier by extending facets CD and DE. If the output price ratio ranges between the slopes of the two facets intercepting at D, the resulting VEA scores might be viewed as providing approximate estimates of overall (i.e., cost, revenue, or profit) efficiency (Joro and Korhonen 2015 p. 100). If the DM wishes to prioritize the production of the second output compared to that of the first one, he/she might choose DMU B as MPS, which would extend facets AB and BC. On the other hand, if the DMUs operate in a relatively uniform environment (e.g., are employees within the same organizational department), the DM may wish to evaluate them based on a common value system. This could be done, for instance, by choosing both C and D as MPS. Then, only the common facet between C and D is expanded, and DMUs are evaluated by using a common vector of weights (the one that is normal to facet CD). If, however, DMU G is chosen as the MPS, the VEA frontier will also include the vertical segment (weakly efficient facet) between DMU G and the horizontal axis, for which the MRT between the two outputs is undefined. This will allow the inefficient DMU H to assign a zero value to the weights of the first output in its evaluation by VEA.

## 3 MPS choice

This section is divided into a literature review subsection, where we present and evaluate previously suggested MPSs, and a subsection where we make four new suggestions for MPS choice.

### 3.1 Literature review

In this section we present nine suggestions used previously in the literature and discuss the rationale associated with each one and characteristics that may encourage or discourage its use by managers.

#### 3.1.1 DM personal judgment

In this case, DMs fully exert their judgments and obtain evaluation results that correspond to their legitimate priorities.[5] The DM may choose a single or several DMUs for this purpose. In the latter case, Korhonen et al. (2002) suggested to form a virtual DMU by averaging over the input and output quantities of the chosen DMUs. The so constructed DMU may be inefficient, indicating that the DM has conflicting preferences, and in this case its set of DEA peers should be used as MPS (see Joro and Korhonen 2015, p. 124).

#### 3.1.2 AHP importance weights

The Analytic Hierarchy Process (AHP) is suggested as another means to choose the MPS in VEA (Korhonen et al. 1998). It may be used to obtain the "best" combination among all the DEA-efficient DMUs, or among a subset of them. The chosen DMUs are used as alternatives in AHP and the DM performs pairwise comparisons among them. The MPS is then obtained as a combination of the chosen DMUs using the importance weights derived from AHP. This might be a time-consuming process if there is a large number of chosen DMUs. In addition, the resulting DMU might be inefficient, indicating poor judgment in the initial selection of DMUs. In this case, its set of DEA peers should be used as MPS.

#### 3.1.3 Interactive optimization

Halme et al. (1999) suggested the use of multi-criteria interactive optimization algorithms to choose the MPS. These algorithms enable the DM to search the efficient frontier and identify different non-dominated solutions. Halme et al. (1999) use the Pareto Race (see Korhonen and Wallenius 1988), in which a MOLP problem is iteratively solved to

[5] We are indebted to an anonymous reviewer for suggesting this interpretation.

obtain an efficient input/output combination, which has the maximum (minimum) possible value for each output (input). In each iteration, the DM reviews the resulting combination and can prioritize which input (output) should be further decreased (increased) at the expense of others, i.e., determine the direction on which the next (and possibly more preferred) input/output combination will be searched for. The algorithm stops when the DM decides that the last identified input/output combination is the MPS. In most of the cases, this is a combination of efficient DMUs.

An alternative proposed by Korhonen et al. (2002) is the Visual Interactive Method for Discrete Alternatives (VIMDA) (see Korhonen 1988). It is similar to Pareto Race but in each iteration it identifies an input/output combination corresponding to an existing DMU rather than a combination of DMUs. Such algorithms can be time-consuming and require a DM that is willing to participate and direct the algorithm according to his/her preferences (Thiele et al. 2009). This may increase management workload and the risk of providing a poor judgment. Also, in practical applications DMs usually view the existing DMUs as more reliable benchmarks compared to combinations of DMUs (Korhonen et al. 2002).

#### 3.1.4 Prior or external information

Prior or external information, regarding previous evaluation results or achievements, may be used by DMs to choose a single or a set of MPS. Marshall and Shortle (2005) made this suggestion but its usefulness depends on the accuracy of the relevant information. As these might refer to a different sample of DMUs, another set of inputs and outputs, and different "environmental" conditions, they might not be representative for the evaluated DMUs at hand.

#### 3.1.5 Best-in-input or best-in-output DMUs

Korhonen et al. (1998) suggested choosing as the MPS a best-in-input DMU, namely one that uses the smallest quantity of a particular input, or a best-in-output DMU, i.e., one that produces the largest quantity for a given output. In a multiple-input-multiple-output setting, there will be more than one best-in-input and best-in-output DMUs, in which case one of them should be chosen as the MPS. The choice may be facilitated if the DM views a particular input or output as overwhelmingly more important than all other inputs or outputs (as e.g., is the case with employee salaries in public services, see Joro and Viitala 2004). Such views could however be reflected directly in the specification of inputs and outputs by excluding all other inputs or outputs from the analysis.

The use of a best-in-input or a best-in-output DMU may result in assessing the DMUs against a technically non-optimal scale. This is because a best-in-input DMU is usually of very small size and possibly of sub-optimal scale, and a best-in-output DMU is often large-sized and has supra-optimal scale. Also, the choice of a best-in-output MPS might imply a management directive towards increasing production disregarding the costs this may incur, while a best-in-input MPS might reflect the need for urgent budget cuts, without considering whether the resulting decreased production will be able to meet demand in the future.

#### 3.1.6 IDMU

The IDMU uses the sample minimum quantities of each input to produce the sample maximum quantities of each output. It is thus "best" in all inputs and outputs. If it is not among the evaluated DMUs, it cannot be used as the MPS, but its DEA frontier projection could be. For this purpose, one may estimate its efficiency score by means of a super-efficiency DEA model and use its efficient projection of inputs and outputs as MPS (Wang and Luo 2006). Since the frontier projection of the IDMU may contain slacks, the set of IDMU peers may instead be used as MPS, to ensure that it is a non-dominated DMU. In several occasions, the IDMU may look as a suitable MPS choice but its input/output bundle is likely to differ from most of the evaluated DMUs. This in turn will result in VEA efficiency scores that differ significantly from the DEA efficiency scores.

#### 3.1.7 Most frequent peer

In this case, the MPS is the efficient DMU appearing the most times as a peer in the DEA model. This DMU is an example-to-follow for most of the DMUs, and it may be viewed as reasonable benchmark or "global leader" (Oral and Yolalan 1990); Oral et al. 1992) for them. Then, the VEA efficiency scores for most of the DMUs will be equivalent to their DEA ones, and thus the use of VEA will not provide additional insights to central management compared to the results of the DEA model. In addition, a DMU acting as a peer for a large number of DMUs could be a potential outlier if it performs extremely better in relative terms compared to the DMUs it influences (Bogetoft and Otto 2011, p. 147), in which case it should be excluded from the sample rather than being used as MPS.

#### 3.1.8 Maximum (or infinite) super-efficiency

Halme and Korhonen (2015) suggested choosing as the MPS the DMU with the maximum super-efficiency score. In a CRS setting, the DEA super-efficiency model always results in *finite* scores, in which case it is rather straightforward to choose the MPS. On the other hand, the VRS super-efficiency DEA model may result in an infeasible solution for some DMUs. One may then choose as the MPS either the DMU with the maximum *finite* super-efficiency score or one among the DMUs for which the super-efficiency model has an infeasible solution. The

DMU with the maximum super-efficiency score will most probably be among those that exert the most influence on the other DMUs' efficiency scores (Wilson 1995), in the sense that it already appears as a peer for quite many DMUs. Then, the VEA model is not likely to provide additional insights to management. Also, DMUs with very large super-efficiency scores are often regarded as outliers (Wilson 1995; Banker and Chang 2006), in which case such a DMU should not be used as MPS. On the other hand, DMUs for which the VRS super-efficiency model has an infeasible solution are usually located at some "end-point" of the DEA frontier (Seiford and Zhu 1999), i.e., are likely overly specialized and are associated with MRSs and MRTs that are not well-defined (as DMUs A and G in Fig. 1). If they are used as MPS, the VEA efficiency scores are likely to differ significantly from those of the DEA model and one or more of the inputs and the outputs will likely be assigned zero weights.

#### 3.1.9 Minimum average Coefficient-of-Variation of optimal weight vectors

According to Gonzalez et al. (2010), the efficient DMU with the minimum variability across its different optimal weight vectors is chosen as the MPS. To identify it, one needs first to estimate a VEA model using in sequence every efficient DMU as the MPS. For each of these models, one should calculate the Coefficient-of-Variation (CV) for the optimal values of every input and output weight and then take their average value. The MPS is chosen as the efficient DMU for which the average CV in the corresponding VEA model is the minimum. This might be appealing for DMs that want to avoid highly dissimilar optimal weight vectors among the evaluated DMUs in the VEA model but it can be relatively time-consuming. Furthermore, a common vector of weights across DMUs, which would reflect the greatest possible congruence (Gonzalez et al. 2010), i.e., the minimum variability, among DMUs in selecting their optimal weights, is not guaranteed.

### 3.2 Some new suggestions

In this section we expand the set of MPS choices in VEA by suggesting four new, each of which may be useful to managers for certain reasons.

#### 3.2.1 Informed personal judgment

In the first of our suggestions the DM exerts his/her personal judgments by explicitly considering the position of DMUs on the DEA efficient frontier. Some of the efficient DMUs reside closer to most of the sample DMUs while others use a somewhat more extreme input/output bundle. In addition, some efficient DMUs are associated with zero or undefined marginal rates while others are not, some can remain efficient even if their input/output bundle changes, and for some there do not exist DMUs with similar input/output structure in the sample. Classifying the DMUs based on such features may aid the DM in making a more informed personal judgment when choosing the MPS.

We consider two main classifications of the efficient DMUs based on their position on the frontier. In the first one, the DMUs are classified as either active or self-evaluators (Edvardsen et al. 2008). The former are efficient DMUs that appear as peers for at least one inefficient DMU, while the latter appear as peers only for themselves, in the sense that the maximum optimal values of the corresponding intensity variables are equal to zero for every inefficient DMU. Each of the active and self-evaluator DMUs can be further classified as an interior or an exterior. For an exterior DMU, at least one among its adjacent facets is weakly efficient, while for an interior this is not true.[6] An interior active DMU resides closer to most of the sample DMUs and its use as MPS might result in moderate (and even insignificant) differences between the DEA and the VEA efficiency scores. An exterior active DMU may use a more extreme input/output bundle, and if used as MPS, a zero weight will be assigned to one or more inputs and/or outputs for some of the evaluated DMUs. The interior self-evaluators are "alone in the crowd", while the exterior self-evaluators are "far out", located at an "end-point", i.e., use an extreme input/output bundle and may be very small- or large-scaled (Edvardsen et al. 2008). In both cases, significant differences should be expected between the DEA and VEA efficiency scores. In addition, some inputs and/or outputs are more likely to have a zero weight if an exterior self-evaluator is used as the MPS.

The second classification partitions the efficient DMUS into terminal and non-terminal ones (Krivonozhko et al. 2015). A terminal DMU will remain efficient even if the quantity of one of its inputs (outputs) is increased (decreased), while for a non-terminal one this is not true.[7] Each terminal DMU may be further classified as being either interior or exterior, but all non-terminal DMUs are interior.[8] An exterior terminal DMU

[6] The classification of efficient DMUs into exteriors or interiors is obtained by enveloping the efficient DMUs "from below" (Edvardsen et al. 2008) through a modified version of the Additive DEA model in which inputs are treated as outputs and vice versa. A DMU with a zero (positive) optimal value is classified as an exterior (interior).

[7] Terminal DMUs are adjacent to at least one-dimensional facet (Krivonozhko et al. 2015). They are identified by estimating a series of linear programs, one for each different input and output, each of which aims at maximizing the value of the intensity variable of a given extreme-efficient DMU while allowing for the particular input (output) of the DMU to increase (decrease) along a one-dimensional ray. A DMU is classified as terminal if the optimal value of its intensity variable equals one in at least one of those linear programs. Otherwise, it is non-terminal.

[8] Krivonozhko et al. (2015) show that the set of terminal DMUs contains that of exterior DMUs as a subset, i.e., each exterior DMU is also a terminal DMU, but a terminal DMU may be either an interior or an exterior.

is more likely to be located on "end-points" of the frontier compared to an interior terminal DMU, but Krivonozhko et al. (2015) note that both classes may contain quite normal efficient units. Thus, the use of an exterior terminal DMU may result in significant or insignificant differences between the DEA and VEA efficiency scores, and the same may be the case when an interior terminal DMU is the MPS. On the other hand, the use of a non-terminal DMU as MPS is less likely to result in assessing the DMUs against unacceptable marginal rates, while when using a terminal DMU this is expected to occur.

#### 3.2.2 Most productive scale size

Our second suggestion is to choose a DMU with MPSS as MPS. Such DMUs operate with technically optimal scale, namely maximize average productivity for their input/output mix. Each such DMU is efficient under both a CRS and a VRS DEA model, i.e., resides on a frontier segment in which CRS prevails and scale elasticity equals one (Banker 1984). The use of an MPSS DMU as the MPS in VEA ensures that DMUs are assessed against a technically optimal scale. The resulting VEA scores could yield useful insights for central management. They might be used for reorganizing or incentivizing the DMUs so that they adjust to the optimal scale, the pursuit of which constitutes a long-term organizational goal.

In several cases, there are multiple MPSS DMUs, each of which operates with the technically optimal scale for its own input/output bundle (Banker and Thrall 1992). In this case Banker (1984) noted that obtaining the overall optimal scale for the underlying technology requires the use of additional knowledge or information. This can be provided by the DM by means of choosing one DMU or a combination of DMUs among those with MPSS as the MPS. The chosen input/output bundle might be close to that of most DMUs in the sample, in which case the VEA efficiency scores may differ only moderately from their DEA counterparts. Alternatively, there might be significant differences between the DEA and VEA efficiency scores if the DM chooses an MPSS DMU with somewhat extreme mix of inputs and outputs.

#### 3.2.3 Average production unit

Our next suggestion is to use the combination of DMUs that are the peers of the Average Production Unit (APU), namely an artificially constructed DMU that operates with the sample means quantities of inputs and outputs, as the MPS. This reflects the objective of maximizing the structural efficiency of an overall entity that coordinates a set of DMUs.[9] This entity might be either a firm operating through a network of multiple branches or plants, or an industry of similar firms. Structural efficiency is a normative rather than a positive measure (Karagiannis 2015), in the sense that it assesses the extent of potential improvement of the entity (firm or industry) as a whole, as if it were a single DMU utilizing and coordinating (through centralized resource allocation) the total quantities of inputs and outputs. The maximum potential output for the entity could be realized if each of the coordinated DMUs had the input/output structure of the APU and then removed its technical inefficiencies (Kittelsen and Førsund, 1992; Karagianis 2015) as well as input and/or output slacks.

When the APU peers are used as the MPS, the VEA efficiency scores reflect the relative performance of DMUs from the perspective of fully centralized management and can provide useful insights to managers that coordinate a firms' branches or to authorities planning a sectoral reorganization. The APU input/output bundle is relatively close to that of many DMUs, and thus one might expect moderate changes in the efficiency scores in VEA compared to DEA. However, the efficiency scores of DMUs using extreme input/output bundles may decrease considerably. For example, in Fig. 1 where the APU is radially projected on the efficient facet CD and thus its DEA-efficient peers are DMUs C and D, VEA evaluates all DMUs compared to the extended facet CD, and the DMUs A, G and H exhibit large decreases in efficiency compared to their corresponding DEA scores.

#### 3.2.4 Common weights

Our fourth suggestion concerns evaluating all DMUs using a common vector of strictly positive input/output weights. This results in evaluating all DMUs based on a common standard (Kao and Hung 2005) and might thus be preferred when DMs wish to prevent individual DMUs from setting and pursuing their own priorities. This could be the case if the DMUs are homogeneous enough, operate under a common policy framework (Cook et al. 2019), and/or in the same environment (e.g., professors engaging in teaching and research activities within the same university faculty). Potential discrepancies between the results using common weights and conventional DEA should then indicate the effect of special circumstances under which a DMU operates (Roll et al. 1991), or a DMU that may be prioritizing its own objectives over those of the organization. This suggestion for choosing the MPS reflects the greatest possible congruence among DMUs in selecting their optimal weights and is, to the best of our knowledge, the only one securing the assessment of DMUs against well-defined MRSs and MRTs.

Common and strictly positive weights across DMUs are guaranteed in VEA when a single FDEF of the DEA

[9] Using the sample average DMU as the MPS generalizes in a sense the suggestion made by Korhonen et al. (2002) to obtain the MPS by averaging across a pre-selected subset of efficient DMUs.

frontier is extended. This will occur if the unique combination of $(I+J-1)$ extreme efficient DMUs that supports an FDEF when CRS is assumed (or $(I+J)$ DMUs in VRS models) (Olesen and Petersen 2003; 2015) is chosen as the MPS, provided that at least one FDEF exists. In most cases, the DEA frontier is generated by multiple FDEFs. The DM should then choose one among those FDEFs to be extended in the VEA model. The choice can be facilitated if one identifies all the FDEFs of the DEA efficient surface and the combinations of DMUs spanning each, which is frequently done using mixed-integer linear programs (see Olesen and Petersen 2003; Fukuyama and Sekitani 2012; Davtalab-Olyaie et al. 2014). The DM can then review these results and choose the FDEF against which DMUs will be assessed. The use of common weights in VEA will more likely result in efficiency scores that differ, in a statistically significant sense, from those of the DEA model. More specifically, only DMUs which are already projected by DEA in the chosen FDEF, i.e., those for which the combination of efficient DMUs generating the FDEF coincides with their set of peers, will retain the same efficiency score, while the remaining ones will exhibit at least slight decreases in efficiency. The differences can be on average large if a DMU with a relatively extreme input/output bundle is among those generating the chosen FDEF.

## 4 Data, variables and modeling choices

For our empirical application we use data for 526 Greek cotton farms obtained from the Farm Accounting Data Network (FADN). The FADN covers large entrepreneurial farms as defined in the farms structure survey of the EU, in which each farm is classified by commodity according to its main source of revenue. That is, a farm is classified as a cotton producer if at least two thirds of its revenue come from the production of cotton.

Output orientation is usually considered as the more appropriate choice when measuring efficiency in agriculture, in which input choices are made well in advance of output realization. (Karagiannis 2014). We also assume that input and output prices are uniform across DMUs, since the agricultural sector is widely considered as a rather competitive one, where there is usually a large number of farmers specializing on the production of a particular commodity and facing similar prices for the resources used and their final product. In this case, input and output data expressed both in terms of quantities and in terms of values (i.e., costs and revenues) can be used to assess technical efficiency (see Portela 2014). We use four inputs, namely land measured in ha, labor (including family and hired workers) measured in annual working hours, intermediate inputs (i.e., fertilizer, pesticides, etc.) measured in euros, and capital stock (including machinery and buildings) measured in terms of the end-of-the-year book values (in euros) and a single output, measured in terms of total gross revenue (in euros).

Average values of the model variables are given in Table 1. In that, we also include information on additional farm characteristics. These are farm size, the farmer's age, the geographic region in which each farm is located, the percentages of own and irrigated land, the percentage of family labor employed, as well each farm's degree of specialization in the production of cotton.[10] Such variables account for important factors which affect the operating conditions of farms and consequently, their input/output structure and can provide insights regarding the closeness of the MPS's structure compared to that of the majority of the sample farms.

Most of our sample farms are located in Central Greece (i.e., Thessaly, 55.9% of the sample), while the rest are almost equally divided between Northern (Macedonia and Thrace) and Southeastern (namely Sterea Ellada and Aegean Islands) Greece (23.4% and 19.4% respectively). Only a small fraction (1.3%) of farms is located at Western (Epirus and Peloponnesus) Greece. On average, the sample farms are relatively specialized in the production of cotton, rent about 44% of their land, while most of them are of large size according to FADN standards and are operated by middle-aged farmers (see Table 2).

## 5 Empirical results

This section provides the first thorough comparative empirical analysis of the variability in VEA efficiency scores for alternative MPS choices. For these purposes several models were estimated. More specifically, technical and scale efficiency scores for the sample DMUs (including the APU) were obtained by estimating conventional CRS and VRS DEA models. We find 12 farms to be both technical and scale efficient (i.e., have MPSS), while there were 21 technically efficient farms, most of which (17) operating with increasing returns-to-scale (RTS). The complete set of the efficient farms is given in Table 2. On average, inefficiency is more due to producing below the frontier rather than operating at non-optimal scale (average technical and scale efficiency equal 0.598 and 0.947, respectively), while

[10] In FADN, farm size is defined in terms of gross value added. FADN defines nine size classes, which are grouped here into three categories, namely small, medium, and large farms. We also define three different age bands, namely younger (less than 40 years old), middle-aged (between 40 and 60 years) and older farmers (over 60 years old). The degree of specialization is measured by the Herfindhal concentration index (defined as $H_k = \sum_j s_{jk}^2$, where $s_{jk}$ is the share of the jth output in total production of the kth farm). A value of $H$ equal to unity indicates complete specialization, whereas smaller values reflect increased diversification.

**Table 1** Sample average estimates of model variables

| | |
|---|---|
| Revenue (in euros) | 6434.103 |
| Land (in ha) | 1188.139 |
| Labor (in annual working hours) | 2045.654 |
| Intermediate inputs (in euros) | 2369.776 |
| Capital (in euros) | 5703.852 |
| Number of farms in the sample | 526 |
| Farms from Northern Greece | 123 |
| Farms from Western Greece | 7 |
| Farms from Central Greece | 294 |
| Farms from South-Eastern Greece | 102 |
| Small size farms | 45 |
| Medium size farms | 207 |
| Large size farms | 274 |
| Farms owned by younger farmers | 67 |
| Farms owned by middle-aged farmers | 386 |
| Farms owned by older farmers | 73 |
| Own land (%) | 0.662 |
| Irrigated land (%) | 0.829 |
| Family labor (%) | 0.870 |
| Specialization index | 0.736 |

supra-optimal scale farms appear to operate closer to optimal scale compared to sub-optimal scale farms.

In addition, we estimated super-efficiency CRS and VRS DEA models. For eight farms the VRS model resulted in an infeasible solution. Separate super-efficiency DEA models were estimated for the IDMU by including it in the sample, among which the one assuming VRS resulted in an infeasible solution. We also estimated CRS and VRS VEA models using each of the efficient farms as the MPS, to identify the farm for which the variability across the optimal input and output weight vectors is minimum, as suggested in Gonzalez et al. (2010). The FDEFs generating the CRS and VRS DEA frontiers (14 FDEFs in the CRS frontier and 68 FDEFs in the VRS one) were identified using the mixed integer binary optimization algorithm of Davtalab-Olyaie et al. (2014).

## 5.1 Choice of MPS

A two-step procedure was used to choose the MPS for the CRS and VRS VEA models. In that, we considered all the MPS choices discussed in the third section apart from external information, interactive optimization, personal judgments and the AHP. This is because external information is not available while the other three suggestions require the presence of a DM. In the first step, we identified the DMUs that could be the MPS in each case assuming CRS and VRS. These are indicated by a rectangle in their corresponding cell in Table 2, the last column of which shows the number of different MPSs indicated by each choice.

More specifically, the farm appearing the most times as peer in the CRS and VRS DEA models, the best-in-output farm and the four best-in-input farms were identified. With CRS, only the best-in-output farm is efficient, while with VRS the best-in-input farms are efficient as well. We also identified the farms with the minimum average CV, those having the maximum finite super-efficiency score with CRS and VRS, as well as those for which the VRS DEA super-efficiency model resulted in an infeasible solution. CRS super-efficiency models do not have infeasible solutions, and thus there is a dash in the respective cell in the last column of Table 2.

The peers of the APU and the IDMU were identified, albeit for the latter only with CRS. Each farm was also classified based on its position on the DEA frontier, following the two classification schemes presented in the third section. From Table 2 we see that at least one farm is included in every group with VRS, while with CRS there are no self-evaluators. Also, a farm may be classified in a different group with CRS and with VRS. In addition, we identified the combinations of efficient farms generating the 14 FDEFs of the CRS frontier and the 68 FDEFs of the VRS one. With CRS, each efficient farm generates at least one FDEF, while this is not the case with VRS.

The second step involved choosing *one* DMU or a combination of DMUs to use in the empirical application when more than one DMUs or combination of DMUs could be the MPS. This is more likely to be the case when the DM chooses the MPS among (i) interior active, (ii) exterior active, (iii) self-evaluators, (iv) interior terminal, (v) exterior terminal, (vi) non-terminal, (vii) MPSS, (vii) best-in-input and best-in-output DMUs, (viii) the DMUs for which the VRS super-efficiency model has an infeasible solution and (ix) the combinations of DMUs generating an FDEF. See the last column in Table 2, where for most of these choices there are multiple alternatives for the MPS. For each of these choices, we chose as the MPS the farm for which land quantity was closest to the average quantity of land among those farms indicated as potential MPSs by the choice. A similar procedure was followed in the case of common weights, namely to choose one combination of farms generating an FDEF. We ranked the efficient farms in terms of their deviation from the sample average land quantity and chose the farm with the minimum deviation. If the farm ranked second shared a common facet with the one ranked first, we considered it for the combination. Otherwise, we bypassed it and moved to the next farm in the ranking. This process ended when a combination of farms generating an FDEF was obtained. After the MPS choice, a VEA model was estimated for each of the alternative MPSs with CRS and with VRS.

**Table 2** Extreme-efficient DMUs chosen as the MPS by alternative choices

| MPS choice | Farm (in coded numbers) 32 | 37 | 69 | 91 | 119 | 130 | 142 | 143 | 145 | 147 | 154 | 160 | 178 | 183 | 197 | 201 | 203 | 216 | 226 | 235 | 241 | 252 | 267 | 273 | 275 | 293 | 314 | 368 | 404 | 410 | 411 | 415 | 524 | sum[a] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CRS | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Most times as peer in DEA | | | | | | | | | | | | | | | | | | | | | | ■[b] | | | | | | | | | | | | 1 |
| Maximum finite superefficiency | | | | | | | | | | | | | ■ | | | | | | | | | | | | | | | | | | | | | 1 |
| Infeasible superefficiency | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | - |
| Best-in-output | | | | | | | | | | | | | | | | | | | | | | ■ | | | | | | | | | | | | 1 |
| Best-in-input | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0 |
| Minimum average CV | | | | | | | | | | | | | | ■ | | | | | | | | | | | | | | | | | | | | 1 |
| IDMU | | ■ | | | | | | | | | | | ■ | | | | | | | | | | | | | | | ■ | | | | | | 1 |
| Interior active | | | | | | | | | | | □ | | | □ | | | | | | | | □ | | | | ■ | | □ | | | | | | 5 |
| Interior self-evaluator | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | - |
| Exterior active | | □ | | | | ■ | | | | | | □ | □ | | | □ | | | | | | | | | | | □ | | | | | □ | | 7 |
| Exterior self-evaluator | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | - |
| Interior terminal | | | | | | | | | | | ■ | | | □ | | | | | | | | □ | | | | | | □ | | | | | | 4 |
| Exterior terminal | | □ | | | | ■ | | | | | | □ | □ | | | □ | | | | | | | | | | | □ | | | | | □ | | 7 |
| Non-terminal | | | | | | | | | | | | | | | | | | | | | | | | | | ■ | | | | | | | | 1 |
| MPSS | | □ | | | | ■ | | | | | □ | □ | □ | □ | | □ | | | | | | □ | | | | □ | □ | □ | | | | □ | | 12 |
| APU | | | | | | | | | | | | | | | | | | | | | | ■ | | | | ■ | ■ | | | | | ■ | | 1 |
| FDEF | | □ | | | | ■ | | | | | □ | □ | ■ | □ | | □ | | | | | | ■ | | | | □ | □ | □ | | | | ■ | | 14 |
| Times suggested as MPS | | 5 | | | | 4 | | | | | 4 | 4 | 6 | 5 | | 4 | | | | | | 7 | | | | 5 | 5 | 5 | | | | 5 | | |
| VRS | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Most times as peer in DEA | | | | | | | | | | | | | | | | | | | | | | ■ | | | | | | | | | | | | 1 |
| Largest finite superefficiency | | | | | | | | | | | | | ■ | | | | | | | | | | | | | | | | | | | | | 1 |
| Infeasible superefficiency | | | ■ | | □ | | | | □ | | | | | | □ | | | | □ | □ | | | □ | | | | | | □ | | | | | 8 |
| Best-in-output | | | | | | | | | | | | | | | | | | | | | | ■ | | | | | | | | | | | | 1 |
| Best-in-input | | | ■ | | □ | | | | | | | | | | | | | | | □ | | | | | | | | | □ | | | | | 4 |
| Minimum average CV | | | | | | | ■ | | | | | | | | | | | | | | | | | | | | | | | | | | | 1 |
| IDMU | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | - |
| Interior active | | | | | | ■ | | □ | □ | □ | □ | | □ | □ | | | □ | | □ | | | | □ | □ | □ | □ | □ | □ | □ | □ | □ | □ | | 19 |
| Interior self-evaluator | | | | | | | □ | | | | | | | | | | | ■ | | | | | | | | | | | | | | | | 2 |
| Exterior active | ■ | □ | □ | □ | | | | | | | | □ | | | □ | □ | | | | □ | □ | □ | | | | | | | | | | | □ | 11 |
| Exterior self-evaluator | | | | | ■ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1 |
| Interior terminal | | | | | | ■ | | □ | □ | | □ | | □ | □ | | | □ | □ | □ | | | | □ | □ | □ | □ | □ | □ | □ | □ | □ | □ | | 19 |
| Exterior terminal | □ | □ | □ | □ | □ | | | | | | | □ | | | □ | □ | | | | □ | ■ | □ | | | | | | | | | | | □ | 12 |
| Non-terminal | | | | | | | □ | | | ■ | | | | | | | | | | | | | | | | | | | | | | | | 2 |
| MPSS | | □ | | | | ■ | | | | | □ | □ | □ | □ | | □ | | | | | | □ | | | | □ | □ | □ | | | | □ | | 12 |
| APU | | | | | | ■ | | | | | | | | | | | | | | | | ■ | | | | ■ | ■ | | | | | | | 1 |
| FDEF | | □ | | | | ■ | □ | □ | □ | □ | □ | □ | □ | □ | □ | □ | □ | | □ | □ | □ | ■ | □ | □ | □ | ■ | ■ | □ | □ | □ | □ | ■ | □ | 68 |
| Times suggested as MPS | 2 | 4 | 4 | 2 | 4 | 5 | 4 | 3 | 4 | 4 | 4 | 4 | 5 | 4 | 4 | 4 | 3 | 2 | 4 | 5 | 3 | 7 | 4 | 3 | 3 | 5 | 5 | 4 | 5 | 3 | 3 | 4 | 3 | |

[a]In the case of the IDMU, the APU, and the FDEF, the last column refers to the number of combinations of DMUs that serve as peers for the IDMU and the APU (one) and the number of FDEFs identified in the CRS and VRS DEA models (14 with CRS and 68 with VRS)

[b]Each rectangle highlights that the corresponding farm is identified as MPS by the respective choice. The filled rectangles refer to the DMUs or combinations of DMUs chosen as the MPS in the application of this paper

The farms or the combination of farms chosen as the MPS are indicated by a filled rectangle in the respective cell of Table 2. From that we see that a particular MPS choice may result in choosing a different MPS with CRS and with VRS (see the average CV). In addition, some farms are frequently suggested as the MPS: with CRS two farms are suggested as the MPS (either solely or within a combination of farms) six and seven times respectively, while with VRS case one farm is suggested as the MPS seven times while six farms are suggested five times each. This can be attributed to the fact that for many of the MPS choices multiple DMUs could be the MPS.

The economic and socio-demographic characteristics of the chosen MPS are given in Table 3. Most of these are medium or large in size, are located in Central Greece and operated by middle-aged farmers (ages 40 to 60). On the other hand, only a few chosen MPSs are located in Northern Greece. More specifically, farm #32 is a medium-sized farm located in Northern Greece that is chosen as an exterior active MPS with VRS. It operates with a sub-optimal scale and it owns and irrigates very low percentages of its land compared to the average. Farm #69 is located in Northern Greece, operates with sub-optimal scale and uses the lowest quantity of land in the sample (best-in-input), while it is also chosen as MPS among the farms with an infeasible VRS super-efficiency model. It is thus possibly located at an “end point” of the frontier. The same is likely the case for farm #119, which is a sub-optimal scale farm located in Northern Greece and chosen as an exterior self-evaluator MPS. On the other hand, the selected interior self-evaluator farm #216 is of sub-optimal scale but large in size and located in Central Greece, as most of the sample DMUs. It is thus more likely to be “alone in the crowd”.

Farm #130 is a large size, optimal-scale farm located in Western Greece and operated by a middle aged-farmer that is chosen as the MPS multiple times (as exterior active and exterior terminal with VRS, as interior active and interior terminal with VRS, and as MPSS). Its input-output bundle is somewhat similar to the average, suggesting that it is located close to most of the farms in the sample. The same is likely for farm #293, which has a similar structure, operates with technically optimal scale and is chosen as interior active and non-terminal MPS with CRS. On the other hand, the non-terminal MPS with VRS (farm #147) is a medium-sized one, although it has similar socio-demographic characteristics with farm #293 and is also of optimal scale. The other farms chosen based on their frontier location (farm #154 as interior terminal with CRS and farm #241 as exterior terminal with VRS) are both large-sized and located in Central Greece. Farm #241 is however of supra-optimal scale and is operated by a young farmer, while farm #154 operates with technically optimal scale.

The best-in-output farm #252 is a large-sized, relatively capital-intensive farm located in Central Greece which appears the most times as a peer with CRS and with VRS. It is thus a very influential peer, as is likely the case for farm #178, which is the one with the maximum finite super-efficiency score for both model specifications. It is located in Central Greece but is of medium size and relatively more labor-intensive. On the other hand, the two farms suggested as the MPS with the minimum variability in their optimal weights with CRS (farm #183) and with VRS (farm #142) have a very small scale compared to the average. Both are located in Central Greece but the latter operates with a technically sub-optimal scale and appears as a peer only for itself, suggesting that it is located at an “end-point” of the frontier.

In the case of the APU, a combination of four farms is the MPS either with CRS or with VRS. Each farm in these combinations is MPSS, while most of these are large-sized farms located Central Greece and operated from middle-aged farmers. The same is the case for the combinations of farms (four with CRS and five in VRS) selected as MPS in the case of common weights. Most of the farms in these combinations have an input/output bundle relatively close to the average. For the case for common weights this is a result of the process we followed to select the associated FDEF. On the other hand, the IDMU peers are three MPSS farms, which utilize very low capital quantities compared to the average. One of these is located in Northern Greece, while two of them own excessively low proportions of their cultivated land.

## 5.2 Comparative results between DEA and VEA models

The VEA efficiency scores are always less than or equal to the corresponding DEA scores. This implies a decrease in average efficiency compared to the DEA model (see Table 4), and a left ward shift of the VEA distribution of efficiency scores compared to that of DEA (see Fig. 2). For some of the MPS choices these shifts are large, for others there are only moderate, while for some MPS choices the VEA distribution of efficiency scores is not statistically different from that of the DEA scores (see Table 5) based on average shifts in rank (given as $R = \frac{1}{k}\left(\sum_{k=1}^{K}\left|rank_A\left(y^k\right) - rank_B\left(y^k\right)\right|\right)$, see Saisana et al. 2005) and distribution equality tests (Banker and Natarajan 2011).

More specifically, the use as the MPS of (i) the farm that appears the most times as a peer, (ii) the one with the maximum finite super-efficiency score, and (iii) the best-in-output farm results in efficiency distributions that do not differ, in a statistically significant way, between DEA and VEA, irrespective of the RTS assumption (see Table 5). The same is essentially true with CRS for the non-terminal and the interior-active MPS, which are the same farm. In these cases, the results from the VEA model do not offer some

**Table 3** Economic and socio-demographic characteristics of the MPSs

| Farm (in coded number) | Revenue (in euros) | Land (in ha) | Labor (in annual working hours) | Intermediate inputs (in euros) | Capital (in euros) | Region | Farm size | Farmer age | Own land (%) | Irrigated land (%) | Family labor (%) | Specialization index[c] | RTS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 32 | 3160 | 1050 | 554 | 717 | 6384 | Northern | medium | 41 | 0.457 | 0.267 | 1.000 | 0.541 | irs |
| 37 | 6090 | 1620 | 1673 | 1975 | 458 | Northern | large | 59 | 0.133 | 0.620 | 0.727 | 0.762 | crs |
| 69 | 2029 | 360 | 321 | 689 | 3091 | Northern | small | 56 | 1.000 | 1.000 | 1.000 | 0.500 | irs |
| 119 | 1067 | 255 | 1267 | 279 | 1499 | Northern | small | 48 | 1.000 | 0.686 | 1.000 | 0.446 | irs |
| 130 | 7760 | 1240 | 3070 | 1070 | 3510 | Western | large | 46 | 0.290 | 1.000 | 0.961 | 0.607 | crs |
| 142 | 1670 | 210 | 1323 | 386 | 487 | Central | small | 64 | 1.000 | 1.000 | 1.000 | 0.693 | irs |
| 147 | 6741 | 670 | 955 | 1717 | 1527 | Central | medium | 55 | 0.701 | 1.000 | 1.000 | 1.000 | crs |
| 154 | 13578 | 1480 | 1029 | 3663 | 3337 | Central | large | 45 | 0.243 | 0.946 | 0.979 | 0.976 | crs |
| 178 | 4050 | 300 | 2222 | 776 | 162 | Central | medium | 33 | 1.000 | 1.000 | 0.734 | 0.546 | crs |
| 183 | 3393 | 420 | 820 | 1344 | 320 | Central | medium | 43 | 0.833 | 1.000 | 1.000 | 1.000 | crs |
| 216 | 9839 | 1360 | 640 | 2726 | 3971 | Central | large | 51 | 0.279 | 0.735 | 1.000 | 0.784 | irs |
| 241 | 13470 | 970 | 3679 | 5872 | 8257 | Central | large | 39 | 0.381 | 1.000 | 0.952 | 0.280 | drs |
| 252 | 31726 | 3130 | 1494 | 5808 | 10037 | Central | large | 60 | 0.284 | 0.831 | 0.871 | 0.907 | crs |
| 293 | 11799 | 1000 | 1800 | 2146 | 4703 | Central | large | 54 | 1.000 | 1.000 | 1.000 | 1.000 | crs |
| 314 | 13123 | 1060 | 1700 | 2406 | 8300 | Central | large | 57 | 1.000 | 1.000 | 1.000 | 1.000 | crs |
| 368 | 4902 | 695 | 917 | 1367 | 723 | Central | medium | 27 | 0.115 | 0.806 | 1.000 | 0.648 | crs |
| 415 | 3661 | 410 | 1664 | 531 | 1750 | Central | medium | 55 | 0.756 | 1.000 | 0.784 | 0.856 | crs |
| Average | 6434.103 | 1188.139 | 2045.654 | 2369.776 | 5703.852 | | | | 0.662 | 0.829 | 0.870 | 0.736 | |

**Table 4** Efficiency scores for alternative MPS choices

| Model | | Average | | Minimum | | Median | | Standard deviation | | Efficient farms | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | CRS | VRS | CRS | VRS | CRS | VRS | CRS | VRS | CRS | VRS |
| DEA | | 0.562 | 0.598 | 0.091 | 0.091 | 0.575 | 0.607 | 0.207 | 0.225 | 12 | 33 |
| VEA | MPS choice | | | | | | | | | | |
| 1 | Most times as peer in DEA | 0.555 | 0.586 | 0.090 | 0.091 | 0.565 | 0.593 | 0.206 | 0.219 | 11 | 23 |
| 2 | Maximum finite superefficiency | 0.551 | 0.576 | 0.091 | 0.091 | 0.559 | 0.580 | 0.205 | 0.219 | 12 | 26 |
| 3 | Infeasible superefficiency | – | 0.363 | – | 0.043 | – | 0.346 | – | 0.188 | – | 5 |
| 4 | Best-in-output | 0.555 | 0.586 | 0.090 | 0.091 | 0.565 | 0.593 | 0.206 | 0.219 | 11 | 23 |
| 5 | Best-in-input | – | 0.363 | – | 0.043 | – | 0.346 | – | 0.188 | – | 5 |
| 6 | Minimum average CV | 0.373 | 0.288 | 0.038 | 0.044 | 0.346 | 0.282 | 0.194 | 0.145 | 5 | 6 |
| 7 | IDMU | 0.373 | – | 0.039 | – | 0.351 | – | 0.192 | – | 4 | – |
| 8 | Interior active | 0.540 | 0.524 | 0.090 | 0.078 | 0.557 | 0.537 | 0.200 | 0.195 | 6 | 7 |
| 9 | Interior self-evaluator | – | 0.208 | – | 0.016 | – | 0.168 | – | 0.159 | – | 5 |
| 10 | Exterior active | 0.508 | 0.417 | 0.077 | 0.060 | 0.520 | 0.416 | 0.190 | 0.185 | 4 | 4 |
| 11 | Exterior self-evaluator | – | 0.201 | – | 0.027 | – | 0.194 | – | 0.118 | – | 4 |
| 12 | Interior terminal | 0.415 | 0.524 | 0.046 | 0.078 | 0.391 | 0.537 | 0.202 | 0.195 | 6 | 7 |
| 13 | Exterior terminal | 0.508 | 0.498 | 0.077 | 0.087 | 0.520 | 0.515 | 0.190 | 0.196 | 4 | 6 |
| 14 | Non-terminal | 0.540 | 0.477 | 0.090 | 0.061 | 0.557 | 0.463 | 0.200 | 0.210 | 6 | 11 |
| 15 | MPSS | 0.508 | 0.524 | 0.077 | 0.078 | 0.520 | 0.537 | 0.190 | 0.195 | 4 | 7 |
| 16 | APU | 0.498 | 0.499 | 0.079 | 0.078 | 0.513 | 0.514 | 0.184 | 0.186 | 4 | 5 |
| 17 | FDEF | 0.482 | 0.497 | 0.071 | 0.078 | 0.494 | 0.514 | 0.184 | 0.185 | 4 | 5 |

additional insights to managers compared to those of the DEA model. This should be expected for the first two MPS choices, as they are based on influential DMUs appearing as peers for a large proportion of farms. For the other choices, it is explained by the fact that the farms chosen as MPSs have an input/output bundle that is close to that of most of the sample farms. Note also that when the non-terminal DMU is the MPS, all inputs are important for the estimation of efficiency, in the sense that all farms assign a positive value to the weights attached to each input.

On the other hand, when the MPS is an (interior or exterior) self-evaluator there are statistically significant (see Table 5) differences between the VEA and the DEA distributions of efficiency scores, and the same holds with VRS for the minimum "average CV" choice. In these cases, we observe the largest left ward shifts in the VEA distribution of efficiency scores compared to that of DEA. This is expected to occur in most cases where the MPS is either an interior self-evaluator that is located "alone in the crowd", or an exterior self-evaluator located on an "end-point" of the frontier, as these DMUs appear as peers only for themselves. It also occurs in our case for the VRS VEA model with the minimum "average CV" MPS choice as well, as the chosen farm is also an interior self-evaluator (see Table 2). In all three cases, the correlation between the VEA and the DEA efficiency scores is particularly low.[11] In addition, when an exterior self-evaluator farm is the MPS, some of the inputs are irrelevant for the estimation of efficiency. More specifically, a zero value is assigned to the weights attached to land and capital by all farms.

Large leftward shifts in the VEA distribution of efficiency scores compared to DEA are observed for a series of other MPS choices. These are (i) the best-in-input farm and the farm for which the VRS super-efficiency model results in an infeasible solution (which in this case is the same farm), (ii) the farm with the minimum average CV with CRS, (iii) the IDMU peers, (iv) an interior terminal farm when CRS is assumed, and (iv) an exterior active farm with VRS. In all these cases, the VEA efficiency scores decrease, on average, by more than 30% compared to DEA (see Table 4). This suggests that the input/output bundle used by the MPS in each case is quite dissimilar from the bundles used by most of the farms. For MPSs with infeasible super-efficiency scores or the IDMU peers, this may often be expected, as the former are usually located at an "end-point" of the frontier, while the latter uses a rather extreme input/output bundle. In these two cases some of the inputs are irrelevant for the estimation of efficiency. This is true for capital in the case of the MPS with infeasible super-efficiency score and for land in the case of the IDMU peers, indicating that the farms are assessed by means of non-well defined marginal rates. For the remaining choices in this group, large differences between the DEA and the VEA efficiency scores may or may not be the case. For example, in our case there are large differences between the VEA and DEA distributions

[11] Simple and Spearman rank correlations between the DEA model and VEA models with alternative MPS specifications are given in Table A1 in the online supplementary material of this paper.

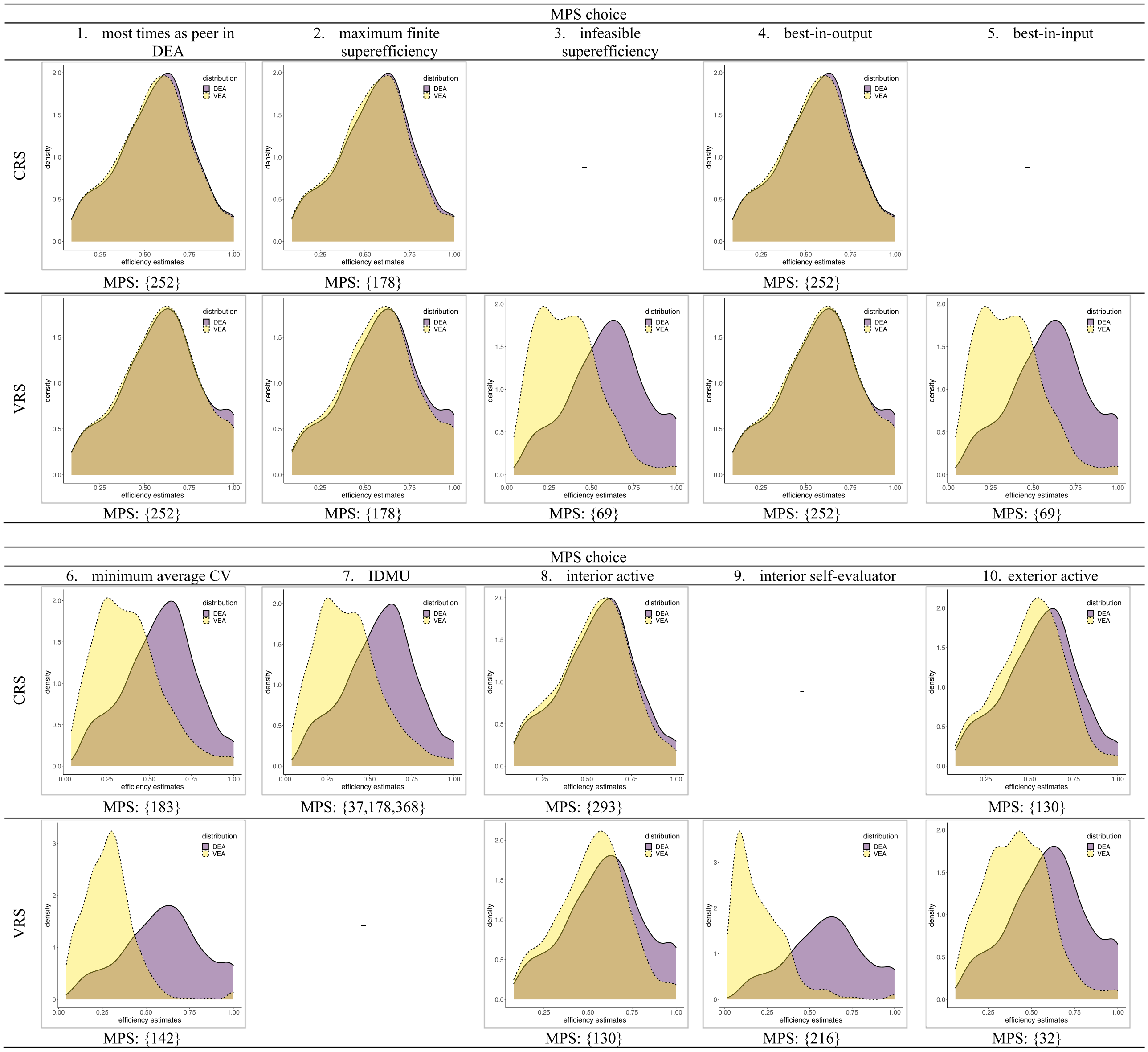


**Fig. 2** DEA and VEA distributions of efficiency scores

of efficiency scores not only when the chosen best-input farm is the MPS, but also if some of the other three best-in-input farms (which have similar economic and socio-demographic characteristics to the chosen farm) are used as the MPS instead. This however may not occur in a different sample and/or model specifications.

Lastly, the VEA distribution of efficiency scores differs only moderately from DEA for the following MPS choices: (i) an exterior active farm in the CRS model, (ii) either an interior active or an interior terminal farm with VRS, (iii) the MPSS choice for both model specifications, (iv) an exterior terminal and a non-terminal farm with VRS, (v) common weights and (vi) the APU peers with CRS and VRS. In the first three of these cases, the same farm #130 is used as the MPS, while in all of them the differences between the DEA and the VEA distributions of efficiency scores are significant in a statistical sense (see Table 5). This indicates that, even though the changes in efficiency are moderate, the use of VEA does result in additional insights to management with respect to the results obtained from the DEA model. Among those cases, significant differences between the DEA and the VEA efficiency scores may be expected when the APU's peers are used as the MPS and in common weights VEA, but not necessarily for the other cases. In common weights VEA, only 21 (with CRS) and seven farms (with VRS) have

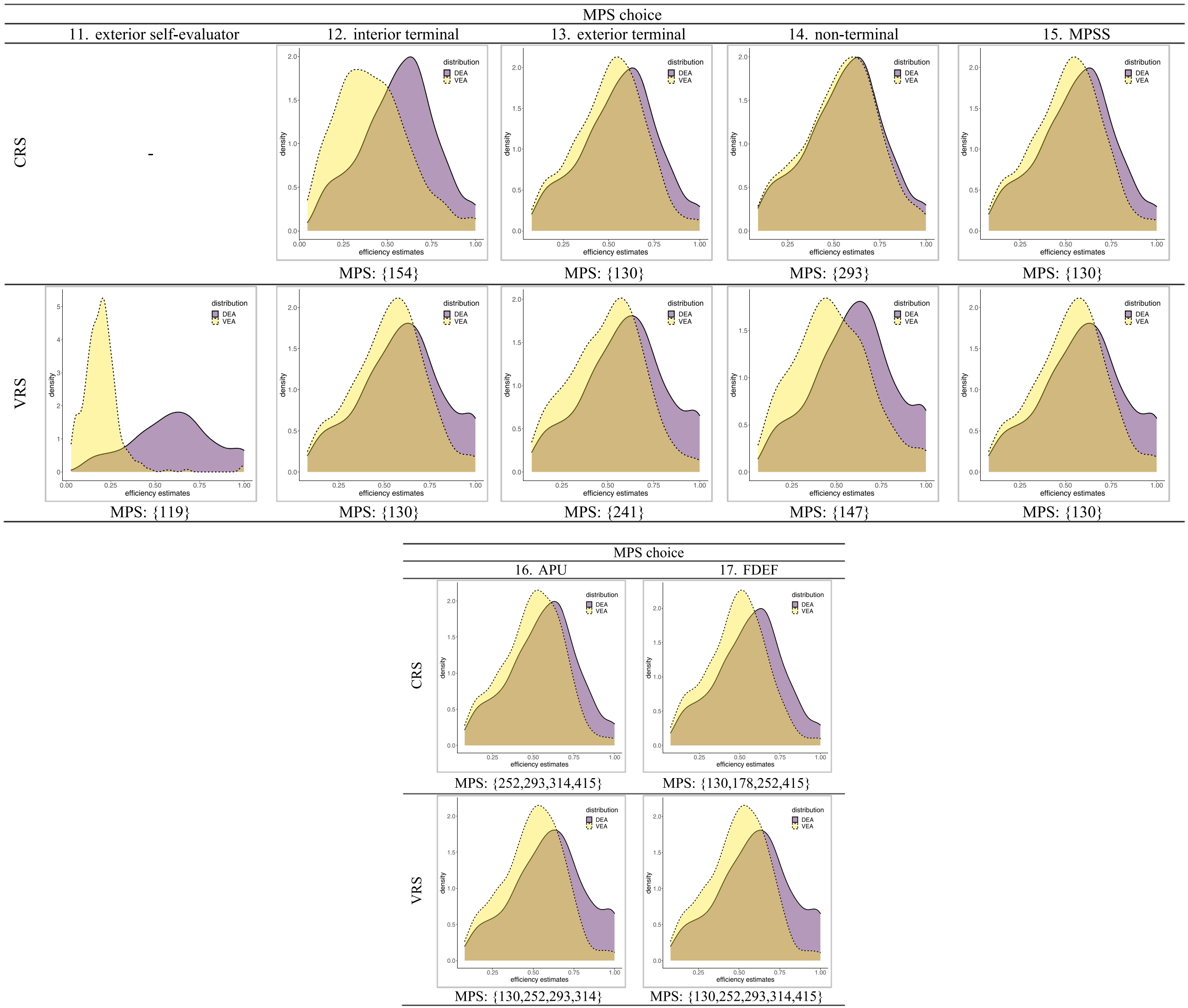


**Fig. 2** (continued)

efficiency scores equal to their corresponding DEA scores, while for the remaining farms VEA efficiency scores decrease at least slightly compared to their DEA counterparts. The differences are moderate as the farms forming the chosen combination have input/output bundles that are similar to those of most DMUs in the sample but could be larger if a farm with a somewhat extreme bundle was chosen in the combination. When the APU's peers are the MPS, farms with an input/output structure that is close to the average, i.e., that of the APU, exhibit slight or no decreases in efficiency in the VEA model, while the scores of farms with extreme bundles decrease considerably. In addition, in the case of a non-terminal MPS, no inputs are irrelevant for the for the estimation of efficiency, while when an (interior or exterior) terminal farm is the MPS (either with CRS or with VRS) some farms assign a zero weight to one or more of the land, labor, and/or capital inputs.

## 5.3 Comparative results among VEA models

Comparing the efficiency distributions among VEA models with alternative MPSs can provide additional insights.[12] No doubt, the VEA distributions of efficiency scores are the same among those MPS choices for which the same farm is used as the MPS. In our case, these are (i) a farm with an infeasible super-efficiency score and a best-in-input farm, (ii) an exterior active, an exterior terminal and an MPSS farm with CRS, (iii) an interior active, an interior terminal and an MPSS farm with VRS, (iv), an interior active and a non-terminal farm with CRS, and (v) the farm appearing the most times as a peer in DEA and the best-in-output farm, for both model specifications (see Table 2).

[12] We thank an anonymous reviewer for suggesting this part of the discussion.

**Table 5** Statistical tests between DEA and VEA

| VEA | MPS choice | Average rank shift | | Mann–Whitney | | Banker F1[a] | | Banker F2 | |
|---|---|---|---|---|---|---|---|---|---|
| | | CRS | VRS | CRS | VRS | CRS | VRS | CRS | VRS |
| 1 | Most times as peer in DEA | 9.167 | 12.522 | 0.639 | 0.814 | 1.021 | 1.03 | 1.031 | 1.032 |
| 2 | Maximum finite superefficiency | 11.579 | 17.512 | 0.959 | 1.600 | 1.032 | 1.062 | 1.046 | 1.081 |
| 3 | Infeasible superefficiency | – | 78.899 | – | 16.262***[b] | – | 1.915*** | – | 2.847*** |
| 4 | Best-in-output | 9.167 | 12.522 | 0.639 | 0.814 | 1.021 | 1.03 | 1.031 | 1.032 |
| 5 | Best-in-input | – | 78.899 | – | 16.262*** | – | 1.915*** | – | 2.847*** |
| 6 | Minimum average CV | 75.662 | 55.558 | 13.865*** | 20.985*** | 1.714*** | 2.264*** | 2.507*** | 3.686*** |
| 7 | IDMU | 75.579 | – | 14.283*** | – | 1.709*** | – | 2.488*** | – |
| 8 | Interior active | 16.924 | 49.307 | 1.675* | 5.512*** | 1.061 | 1.212*** | 1.086 | 1.279*** |
| 9 | Interior self-evaluator | – | 99.032 | – | 23.529*** | – | 3.027*** | – | 6.655*** |
| 10 | Exterior active | 40.820 | 68.930 | 4.455*** | 13.209*** | 1.153** | 1.631*** | 1.223** | 2.107*** |
| 11 | Exterior self-evaluator | – | 62.406 | – | 24.320*** | – | 2.864*** | – | 5.589*** |
| 12 | Interior terminal | 68.685 | 49.307 | 11.418*** | 5.512*** | 1.528*** | 1.212*** | 2.051*** | 1.279*** |
| 13 | Exterior terminal | 40.820 | 60.338 | 4.455*** | 7.359*** | 1.153** | 1.314*** | 1.223** | 1.472*** |
| 14 | Non-terminal | 16.924 | 58.622 | 1.675* | 8.909*** | 1.061 | 1.415*** | 1.086 | 1.706*** |
| 15 | MPSS | 40.820 | 49.307 | 4.455*** | 5.512*** | 1.153** | 1.212*** | 1.223** | 1.279*** |
| 16 | APU | 39.169 | 55.605 | 5.329*** | 7.452*** | 1.183*** | 1.294*** | 1.265*** | 1.401*** |
| 17 | FDEF | 41.169 | 55.442 | 6.647*** | 7.564*** | 1.236*** | 1.298*** | 1.365*** | 1.409*** |

[a]F1 (F2) test compares the DEA and VEA distributions of efficiency scores, assuming an exponential (half-normal) distribution of the efficiency scores (see Banker and Natarajan 2011)

[b]Three, two and one stars denote statistical significance at 1%, 5% and 10% respectively

In addition, based on Banker's test (see Supplementary Tables A2 and A3 in the online Supplementary Material),[13,14] we can infer that there are no significant differences among the efficiency scores of VEA models when the MPS is either (i) the farm appearing the most times as a peer in DEA, (ii) the farm with the maximum finite super-efficiency score, (iii) the best-in-output farm (both with CRS and with VRS), (iv) an interior active or (v) a non-terminal farm with CRS. This is to be expected in our case, as the VEA distributions of efficiency scores for these MPS choices do not differ significantly from that of DEA. In our case, the best-in-output farm appears also as a peer for most farms, and the interior active and the non-terminal farms used as MPS are the in fact same farm. Also, the VEA efficiency scores do not differ in a statistically significant sense when the chosen MPS is based either on (i) the APU, (ii) an MPSS farm, or (iii) common weights. This is explained by the fact that the combinations of farms in the common weights and the APU choices in our case are similar to each other and include in most cases the MPSS farm #130, which however may not be the case with other datasets.

On the other hand, the VRS VEA distributions of efficiency scores differ in a statistically significant way from one another when the MPS is either (i) an interior self-evaluator farm, (ii) an exterior self-evaluator farm or (iii) a farm for which the super-efficiency score is infeasible. This indicates that statistically significant differences between VEA distributions of efficiency scores were found when the MPS choices reflect DMUs with a rather extreme input/output bundles. In these cases, as we have seen before, the VEA efficiency scores for each of these MPS choices are significantly different from those of DEA.

## 6 Concluding remarks

VEA can be a very useful tool for performance evaluation, providing guidance towards informed decision-making. The efficient frontier against which the DMUs are assessed in VEA depends on the chosen MPS. In this paper, we first reviewed several MPS choices previously used in the literature. For some of these, there is a difficulty to intuitively

[13] In the case of pairwise comparisons among different VEA models, the Banker test statistics are calculated by placing in the numerator the VEA model for which the sum of the logarithms of its inefficiency scores is the largest. This guarantees that the test statistic is always greater than or equal to one.

[14] Results from correlation analysis, shifts in rank and Man-Whitney tests of equality among the VEA efficiency scores with alternative MPS choices are given in Supplementary Tables A4–A7 in the online supplementary material of this paper.

explain the DMs' choice, as they do not explicitly consider some overall organizational objective, while others may compare DMUs against exceptionally performing benchmarks or inappropriate MRSs and MRTs. We then made four new suggestions for choosing the MPS: *First*, to make a more informed personal choice by explicitly considering the relative position of efficient DMUs on the DEA frontier. *Second*, choose a DMU with MPSS as the MPS, which results in assessing the DMUs against the technically optimal scale in DM's view. *Third*, choose the set of APU's peers as the MPS. In this case the resulting VEA scores resemble the extent of efficiency from the perspective of fully centralized management, and can be useful for DMs who coordinate resource allocation and pursue the objective of structural efficiency maximization. *Fourth*, to evaluate all DMUs based on common and strictly positive (i.e., well defined) weights, by choosing as the MPS a unique combination of DMUs generating an FDEF. This results in evaluating the DMUs against a common standard, which may be a prerequisite when management wishes to fully limit the assessed DMUs' autonomy in setting their own objectives.

The empirical comparative analysis using data on Greek cotton farmers provides useful results on how MPS choice may affect the VEA efficiency scores: *First*, the use of an influential peer as the MPS (the DMU appearing the most times as a peer and the one with the maximum finite super-efficiency score) does not offer additional insights to managers compared to the results obtain from the DEA model. *Second*, MPSs that are frequently located on "end-points" of the DEA frontier (those with an infeasible super-efficiency score and interior- or exterior-self-evaluators) appear to result in large differences on efficiency scores between the DEA and VEA models and in some of the inputs being irrelevant for the estimation of the VEA efficiency scores. *Third*, the use of both an (interior or exterior) terminal as well as a non-terminal DMU as MPS may result in significant differences between the DEA and VEA efficiency scores, but in the latter case all inputs were important for the estimation of efficiency while in the former case, zero optimal weights were assigned to some inputs. *Fourth*, both MPS choices pursuing minimum variability among the DMUs' optimal weights (minimum average CV and common weights) resulted in significant differences between the DEA and VEA efficiency scores. This may often be the case for the common weights choice. *Fifth*, the VEA scores when the MPS is either the APU or an MPSS DMU differ significantly, in a statistical sense, from that of the DEA model, which may often be the case for the APU.

On the other hand, the same VEA efficiency scores were obtained from different MPS choices for which the same DMU was used as the MPS, while similar scores were obtained from alternative MPS choices in which an influential peer is the MPS, namely the DMU appearing the most times as a peer and the DMU with the maximum finite super-efficiency score. Similarly, the VEA efficiency scores when the MPS was chosen based either on the APU, an MPSS farm, or common weights were not statistically different from each other. However, choices in which the MPS may often be a DMU with a rather extreme input/output bundle, namely self-evaluators and DMUs with infeasible super-efficiency scores, resulted in significantly different VEA scores with one another.

The empirical analysis conducted in this study provided the first thorough overview on the effect of MPS choice on the VEA scores. As our empirical findings may be data specific, a promising task for future research would to empirically assess the effect of MPS choice on VEA scores using data from other sectors and countries. Such studies could provide valuable insights that would complement those of the present study. Furthermore, as the incorporation of the MPS in VEA models restricts the assessed DMUs' choice of optimal values of input/output weights in a manner similar to that of introducing weight restrictions in DEA models, another avenue for future research would be to explore the relationship between VEA and weight-restricted DEA models in more detail.

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## Compliance with ethical standards

**Conflict of interest** The authors declare no competing interests.

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