Solving Portfolio Optimization Problems using AMPL

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Abstract

This work presents a new optimization software library which contains a number of financial optimization models. Roughly speaking, the majority of these portfolio allocation models aim to compute the optimal allocation investment weights, and thus they are particularly useful for supporting investment decisions in financial markets. Algebraic modeling languages are very well suited for prototyping and developing optimization models. All the financial optimization models have been implemented in AMPL mathematical programming modeling language and solved using either Gurobi Optimizer or Knitro (for those models having general nonlinear objectives). This proposed software library includes several well-known portfolio allocation models, such as the Markowitz mean-variance model, the Konno-Yamazaki absolute deviation model, the Black-Litterman model, Young's minimax model and others. These models aim either to minimize the variance of the portfolios, or maximize the expected returns subject to a number of constraints, or include portfolios with a risk-free asset, transaction costs, and others. Furthermore, we also present a literature review of financial optimization software packages and discuss the benefits and drawbacks of our proposed portfolio allocation model library. Since this is a work in progress, new models are still being added to the proposed library.

KEYWORDS

Financial Optimization, Mathematical Programming, AMPL.

1. INTRODUCTION

In today's complex environment of globalization, constantly increased competition, liberalization of markets and rapid changes in the international economic environment, the use of information technology in the field of finance is of vital importance for minimizing risks. Portfolio optimization mainly focus on asset allocation [Ibbotson, 2010], so the research field of financial optimization utilizes the capabilities of the information technology in order to ensure the investor's portfolio assets.

Due to inherent risks of economic-financial investments, various researchers tackle these problems by developing mathematical models for financial optimization. The novel work of Harry Markowitz on the well-known mean-variance model [Markowitz, 1952] [Markowitz, 1959] has set the basis for the development of this field. Moreover, six decades later several studies are still presented that derive from the Markowitz mean-variance model [Zopounidis, et al., 2014], [Kolm, et al., 2014], and [Adame-García et al., 2015]. A number of mathematical models based on the classic Markowitz model, have been also recently proposed in electricity investments and energy [Zhu and Fan, 2010]. More spe-

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cifically, the variance of the returns of the portfolio's stocks is a measure of risk so the majority of these mathematical models aim to minimize the variance (risk).

Despite the fact that there is a large number of efficient financial optimization software packages, more research efforts are still required in order to develop a more user-friendly non-commercial software package. This is the motivation of this research work; to develop a software library with a large number of the most well-known financial optimization models. The benefits of the proposed financial optimization software library are: first, it utilizes the rich features of the AMPL modelling language [Fourer et al., 2002], second the codes of the financial optimization models can be easily extended, and third that the majority of the operational research (OR) scientists are more familiar to general-purpose optimization modelling languages, rather than other programming languages. However, this is a work in progress so new models are still added to enrich the existing library.

The chapter is organised as follows. Next section presents a literature review on financial optimization software packages. Section 3 presents the financial optimization models that are currently implemented in the proposed optimization software library. Furthermore, some implementation details regarding AMPL and examples of two indicative financial optimization models with solution interpretations are also given in this section. Finally, the last Section concludes this work and discusses some future research directions.

2. LITERATURE REVIEW AND EXISTING PORTFOLIO OPTIMIZATION SOFTWARE PACKAGES

There are several optimization software packages for the solution of financial optimization problems. Apart from their price, all these financial optimization software packages differ in the number of the mathematical models they offer, in the variety of produced reports, in the solution time, in reliability, etc. An indicative list of some of the most well-known financial optimization software packages is presented in this section. Table 1 presents some brief information about these software packages, regarding their availability, required operating system, and whether they constitute add-in packages or not.

Table 1. Portfolio optimization packages – general information.

Package	Access	Add in package	Operating System
Hoadley portfolio optimizer	Commercial	Microsoft Excel	Microsoft Windows
StockPortfolio, Quadprog	Open-Source	R	Microsoft Windows, Mac OS
Financial Toolbox	Commercial	Matlab	Microsoft Windows, Linux, Mac OS X
Smartfolio	Commercial	Microsoft Excel	Microsoft Windows
MVO Plus	Commercial	-	Microsoft Windows
NUOPT	Commercial	S+	Microsoft Windows, Linux, Solaris
Mvport	Non-Commercial	Stata	Windows, Mac OS X, Unix, Linux
iOptima	Commercial	-	Microsoft Windows
Zephyr AllocationADVISOR	Commercial	-	Microsoft Windows

Following in Table 2, size limitations and data downloading / importing and reporting capabilities of these software packages, are also reported.

Table 2. Portfolio optimization packages – data management capabilities.

Package	Data downloading/importing	Reporting Capabilities	Size limitation
Hoadley portfolio optimizer	Historic stock prices from any web source, from external data	Excel spreadsheets, graphic visual charts	Up to 1,048,576 rows and 16,384 columns
StockPortfolio, Quadprog	Historic stock prices from Yahoo Finance	Text output	Up to 2^31-1 elements (2,137,483,647)
Financial Toolbox	Historic stock prices from numerous web sources, Importing external databases	Graphic-visual reports	Up to 2^48-1 elements and 8 TB maximum size of matrix (depending on the OS and software's version)
Smartfolio	Historic stock prices from Yahoo Finance, External databases	Excel spreadsheets, graphic-visual charts	Up to 1,048,576 rows and 16,384 columns, up to 64 portfolios for individual license
MVO Plus	Historic stock prices from external databases	Graphic-visual reports	Up to 20 total assets-stocks (5 in trial version)
NUOPT	Historic stock prices from external databases	Text output, graphic-visual reports	-
Mvport	Historic stock prices from Yahoo Finance	Text output	Up to 32767 variables
iOptima	Historic stock prices from external databases	Graphic-visual reports	Undisclosed
Zephyr AllocationADVISOR	Historic stock prices from external databases	Graphic-visual reports	Undisclosed

Microsoft Excel[1]: Microsoft Excel is one of the simplest software packages for financial optimization. The add-in package called Solver tool can be used for modeling and solving several financial optimization problems, such us a quadratic problem [Çetin and Göktas, 2009]. Using the Solver tool, one can easily define the cells that contain the optimal stock allocations, define the cell containing the objective function to be optimized, and also add the necessary constraints. The most classical example of a quadratic problem in portfolio optimization is Markowitz mean-variance model and a practical approach is discussed in [Myles and Mangram, 2013]. Consequently, data tables of the, e.g., stock returns or covariance matrices have to be read in order to make the above computations. Polak & Rogers [Polak et al., 2010] have also used Microsoft Excel Solver for solving minimax portfolio optimization problems. Additionally, Livingston [Livingston, 2013] showed that it is important to use Excel's built-in MMULT function for finding efficient portfolios. Recently, Lee [Lee, 2015] also reported the use of the MMULT function for solving risk loan portfolio optimization model based on CvaR risk measure.

[1] http://www.solver.com

Although, Microsoft Excel does not have any specific financial optimization package pre-installed, the commercial Hoadley Portfolio Optimizer[2] add-in for Microsoft Excel already exists in the market. Hoadley Portfolio Optimizer mostly uses Markowitz mean variance model in order to find the optimal portfolio. Additionally, it uses the Sharpe model and it has also indicative tutorials for both the Black-Litterman model and the general use of the package.

R[3]: R is an open-source software which is mostly used for statistical computing. However, it is also efficient on dealing with financial optimization problems [Pfaff, 2012]. A number of packages implemented in the R programming language assist the writing of the code for financial optimization models. Some of these packages are StockPortfolio and Quadprog. StockPortfolio can be used to download stock data from Yahoo Finance, build mathematical models and calculate the optimal asset allocations. By using the GetReturn function, the returns of the stocks can be downloaded. Also, by using the stockModel and the OptimalPort functions a model can be built and the optimal asset allocations can be calculated, respectively. The Quadprog package constitutes a solver for quadratic programming problems for computing the optimal portfolio using the Markowitz mean-variance model.

LINGO[4]: LINGO is a general-purpose mathematical optimization language and, thus, it can be used for financial optimization problems as well. Soleimani et al. recently reported an application of a genetic algorithm for Markowitz-based portfolio selection with minimum transaction lots, cardinality constraints and regarding sector capitalization and compared it with the LINGO exact solver [Soleimani et al., 2009].

MATLAB[5]: MATLAB (MATrix LABoratory) is one of the most well-known computational tools for scientific computing applications. Nevertheless, it is also very-well suited for solving financial optimization problems [Brandimarte, 2002]. MATLAB also contains the financial toolbox [Mathworks, 2000], which incorporates Portfolio Optimization and Asset Allocation, thus, providing useful tools for portfolio optimization theory, mean-variance portfolio optimization, Value at Risk analysis, and portfolio analysis. Also, Chen et al. in 2010 [Chen et al., 2010] demonstrated mean–variance spanning tests using MATLAB. Furthermore the validity of the maximum entropy method for the minimax portfolio selection problem with short sale restriction has been tested in MATLAB and illustrated in [Wu, 2009] using real stock data from Shanghai Stock Exchange.

SmartFolio 3[6]: SmartFolio is a user-friendly analytical tool and add-in to Microsoft Excel for assisting inventors regarding their portfolio. It enables users to manage data and historical returns of the stocks by using four optimization criteria (maximization of expected utility, minimization of target shortfall probability, maximization of Sharpe ratio, and benchmarking). Another interesting feature is that it supports robust portfolio optimization where, in scenario-based optimization, under the worst case scenario the resultant portfolios demonstrate optimal behavior.

GAMS[7] : The General Algebraic Modelling System is a mathematical modeling language for the solution of mathematical optimization problems. GAMS handles linear, non-linear and mixed integer optimization problems. A practical application of financial optimization problems and a library of a large number of financial optimization models are presented in [Zenios et al., 2009]. This library contains several models such as the Markowitz mean-variance model, the mean absolute deviation model of Konno and Yamazaki [Konno and Yamazaki, 1991], Sharpe's model and others. Moreover a construc-

[2] http://www.hoadley.net/options/develtoolsoptimize.htm
[3] https://www.r-project.org/
[4] http://www.lindo.com/
[5] http://www.mathworks.com/
[6] http://www.smartfolio.com/
[7] http://www.gams.com/

tion of a portfolio model in GAMS and an application on the Athens Stock Exchange is made in [Xidonas et al., 2010]

MVO Plus[8] : MVO (Mean-Variance Optimizer) Plus is a commercial tool for mean-variance optimization, based on the classical Markowitz model. Some of its benefits is that it effectively uses both geometric and algebraic means as input to solve Markowitz's model. Furthermore, MVO Plus simulates historic data of stocks in order to gauge their effectiveness (back testing).

S+[9] : S+ is a commercial programming environment of Solution Metrics Company for the solution of large-scale optimization problems. It is used for a wide range of applications such as circuit optimization, linear and non-linear problems, statistical analysis and last but not least for financial optimization. S+ includes a tool called NUOPT (Numerical OPTimization) which combines statistical and graphical environments. NUOPT provides user with solvers that efficiently solve linear, mixed integer, quadratic and non-linear optimization problems. As far as its applications in financial optimization are concerned, it mostly makes use of the Markowitz mean-variance model finding the optimal asset allocation even for large-scale problems. Another library of financial optimization models is also presented in [Scherer and Martin, 2005].

Stata[10] : Stata is a commercial, general purpose software for data management, statistical analysis, and simulation and regression analysis. It is being used as a research tool in several fields like sociology, political science, biomedicine and economics. Due to the non-commercial add-in package mvport [Dosamantes, 2013], it is also used in financial optimization too. Mvport consists a Stata package for mean-variance portfolio optimization. Also, it has a number of functions that collect online updated data for stocks and analyze them, in order to find the minimum variance of the portfolio and thus the optimal asset allocations.

iOptima[11] : iOptima is a commercial software of Finvent company for portfolio optimization. It offers a user friendly environment and a number of efficient solvers for several financial problems. As far as the portfolio optimization is concerned, it uses the classical Markowitz mean-variance approach.

Zephyr AllocationADVISOR[12] : Zephyr AllocationADVISOR is a commercial optimization software, which is specialized in portfolio simulation and optimization. Some of its key benefits are the creation of efficient portfolios custom to the investor's view and his profile, detailed comparison of different portfolios, portfolio projection, and graphical presentation of the results. The portfolio optimization and asset allocation utilities of Zephyr AllocationADVISOR, use either the classic Markowitz mean-variance model or the Black Litterman model.

3. FINANCIAL OPTIMIZATION MATHEMATICAL MODELS

The proposed software optimization library consists of ten models as shown in Table 3. All these models are either linear [Mansini et al., 2014], non-linear, or quadratic; thus, a suitable solver is

[8] http://www.effisols.com/
[9] http://www.solutionmetrics.com.au/
[10] http://www.stata.com/
[11] http://www.finvent.com/
[12] http://www.styleadvisor.com/

called in each case. Furthermore each one model has a specific aim and objective. More specifically this software optimization library consists of the most well-known Markowitz's mean-variance model [Markowitz, 1952], [Markowitz, 1991], [Steinbach, 2001], which aims to minimize the risk of the portfolio subject to a number of constraints. In this case, the variance between the stocks is the measure of risk that has to be minimized.

Furthermore, there are some variations of the classic Markowitz model such as the Markowitz's mean-variance model with upper bound, where the optimal allocations do not exceed an upper bound. Other variants include the Markowitz's mean-variance model with risk free asset, where a risk free asset [Pang, 1980] is added to the portfolio to examine its effectiveness in relation with the other risky assets. Moreover, the Markowitz's mean-variance model with transaction costs where the model, apart from seeking the optimal asset allocations, offers also the optimal amounts which should be additionally bought and sold. All the variations of Markowitz's models and the classical model are examples of quadratic programming models.

Additionally, the Sharpe model [Sharpe, 1989], [Sharpe, 1992], and [Sharpe, 1994] aims to maximize the portfolio's Sharpe ratio subject to budget constraint, and the Factor model which minimizes the risk given the fact that a factor has affected the portfolio.

Furthermore, the formulation of the Black Litermans' model [Black and Litterman, 1992] is almost the same as that of Markowitz classic mean-variance model, however, it also considers the market equilibrium in combination with investor's view. Roughly speaking, the difference lies in the expected returns of the stocks where in classical Markowitz's model these are just the arithmetic means of the individual stocks and in Black Littermans' model they are affected also by the market equilibrium and the investor's view as mentioned before.

Moreover, the Konno-Yamazaki mean absolute deviation (MAD) model [Konno and Yamazaki, 1991] aims to minimize the mean absolute deviation subject to a number of constraints.

The Conditional Value at Risk model (CVaR) [Rockafellar and Uryasev, 2000] aims to minimize the conditional value which is at risk or correspondingly the probability of large losses of the portfolio. Last but not least, the Young's minimax model [Young, 1998] uses games theory methodology and aims to maximize the minimum returns of the portfolio subject to some constraints.

Table 3 Library of portfolio optimization models implemented in AMPL

Name	Aim	Objective	Type of problem	Solver
Markowitz (MVO)	Minimize	Variance	Quadratic	Gurobi
Markowitz with upper bound	Minimize	Variance	Quadratic	Gurobi
Markowitz with Risk free Asset	Minimize	Variance	Quadratic	Gurobi
Markowitz with Transaction costs	Minimize	Variance	Quadratic	Gurobi
Sharpe	Maximize	Sharpe Ratio	Non-Linear	KNITRO
Factor	Minimize	Variance	Non-Linear	KNITRO
Black-Litterman	Minimize	Variance	Quadratic	KNITRO
Konno-Yamazaki(MAD)	Minimize	Mean absolute deviation	Linear	Gurobi
Conditional Value at Risk (VaR)	Minimize	Conditional value at risk	Linear	Gurobi
Young	Maximize	Minimum Returns	Linear	Gurobi

Despite the fact that Black-Litterman's model is a quadratic problem, a non-linear solver has been used in this case, due to the fact that non-linear computations had to be preprocessed in order to obtain the expected returns of the stocks. This is due to the fact that, a self-contained / native AMPL code was used for the formulation of the library models, without any link to other callable software to make extra computations.

3.1 AMPL implementation details

AMPL (A Mathematical Programming Language) is a mathematical modelling language for the solution of large optimization problems. Financial optimization constitutes an important optimization problem, and thus AMPL is able not only to efficiently model such problems but also to solve them by calling appropriate state-of-the-art solvers. The architecture of the proposed library is illustrated with analytical flow charts for each step. Initially, the general idea of the library and the way AMPL works is depicted in Figure 1.

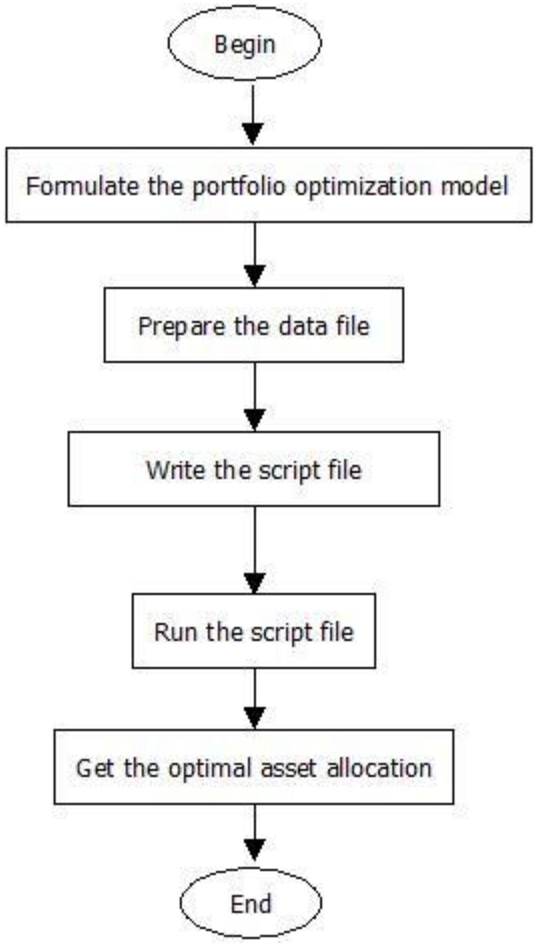

Fig 1. General architecture of the library.

The user has to use three files; the model file which contains the portfolio optimization model, the data file, and finally the script file which has to be run in order to get the optimal asset allocation.

The architecture of these files is shown below. More specifically in Figures 2 and 3 the architectures of the model file and the script file are illustrated, respectively. To begin with the model file as shown in Figure 2, the formulation of the model file consists of some simple steps as defining the parameters of the model, the variables which denote the asset allocations, the formulation of the objective function, and the constraints.

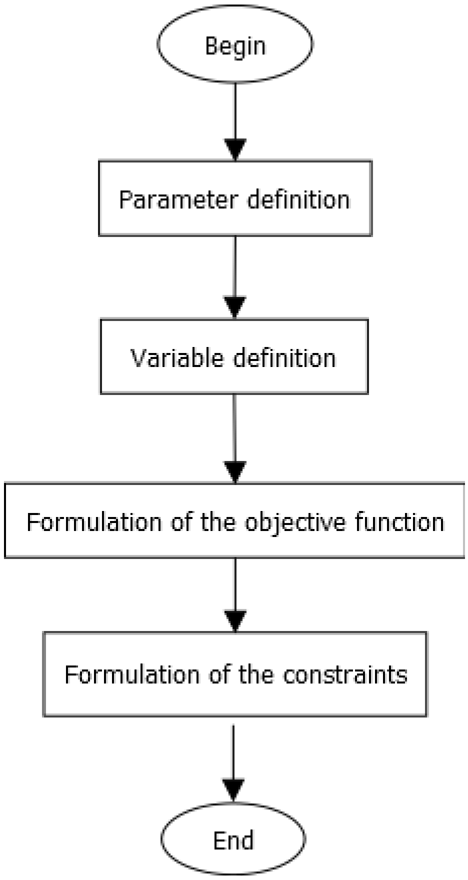

Fig. 2 Architecture of the model file.

The next step is the preparation of the data file which contains all the data required by the model. Firstly the returns of the stocks have to be downloaded and included in this file, then the portfolio's required return and the available budget have to be declared which vary according to the decision maker (investors) and requirements. Finally, an upper bound of investment may also be declared. Afterwards, the script file loads the model, reads the data, and sets various options (e.g., selection of solver, time limit, format of the results) and returns the optimal asset allocations computed by the solver.

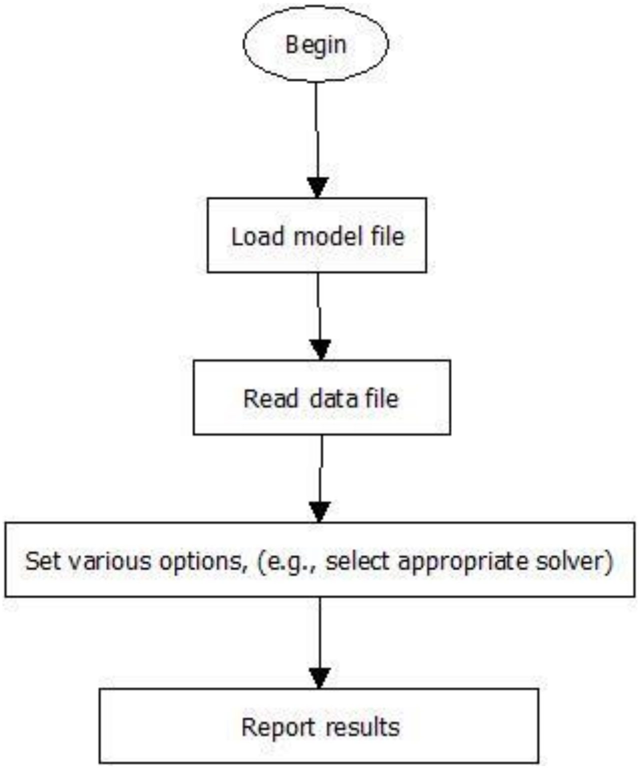

Fig. 3 Architecture of the script file.

Gurobi or CPLEX solvers have been used, since the majority of these models have linear objectives. However some more complex models due to their non-linear objectives, force us to use other solvers such as KNITRO or Minos. Following in the next subsection, two indicative financial optimization models of the proposed library will be presented.

3.2 Konno-Yamazaki Mean Absolute Deviation model

In 1991 Hiroshi Konno and Hiroaki Yamazaki introduced the Mean Absolute Deviation (MAD) model [Konno and Yamazaki, 1991]. The central idea of this model is that since the measure of risk in this case is considered the mean absolute deviation, there is no need to compute the covariance matrix as in classic Markowitz mean variance model. Therefore the objective function seeks to minimize the mean absolute deviation of the portfolio subject to some constraints. The mathematical formulation of the Konno Yamazaki MAD model is as follows:

$$\min \sum_{t=1}^{T} \left| \sum_{j=1}^{n} a_{jt} x_j \right| / \mathrm{T}$$

subject to

$$\sum_{j=1}^{n} r_j x_j \geq \rho M_0$$

$$\sum_{j=1}^{n} x_j = M_0$$

$$0 \leq x_j \leq u_j \quad j = 1 \ldots n$$

Which is also equivalent to its linear form:

$$\min \sum_{t=1}^{T} y_t / T$$

$$s.t.$$

$$y_t + \sum_{j=1}^{n} \alpha_{jt} \, \mathrm{x}_i \geq 0 \qquad t = 1, \ldots T,$$

$$y_t - \sum_{j=1}^{n} \alpha_{jt} \, \mathrm{x}_i \geq 0 \qquad t = 1, \ldots T,$$

$$\sum_{j=1}^{n} r_j x_j \geq \rho \mathrm{M}_0$$

$$\sum_{j=1}^{n} x_j = M_0$$

$$0 \leq x_j \leq u_i \qquad \mathrm{j} = 1, \ldots n$$

- The target of the objective function is the minimization of the portfolio's mean absolute deviation (MAD)

  n : number of the assets

  x_j : optimal allocation

T : time period e.g. years

a_{jt} : Mean absolute deviation (MAD) of the stocks

r_j : expected returns of the assets

M_0 : total budget

ρ : target return

u_j : upper bound of optimal allocations

Example: Konno and Yamazaki's MAD model aims to minimize the mean absolute deviation of the portfolio, given the historical annual returns retrieved from Yahoo Finance. For each one of the 5 years (2010-2015) for eighteen (18) stocks as shown in Table 4, the minimum target return, that a moderate investor requires from the portfolio (0.095 per annum), and the total budget he holds (150,000$). More specifically the expected returns of each one of the three stocks are the arithmetic average of the historic returns for all 5 years. The mean absolute deviation is the absolute value of the subtraction of the returns of the stocks[i,j] from the expected return of stock[j]. Therefore according to Konno and Yamazaki, this quantity has to be minimized, provided that:

- The sum of the expected returns of the assets multiplied by the optimal allocations is no less than the goal value.
- The sum of invested capital should be equal to a pre-specified investor's budget (M_0).
- The optimal allocations should be non-negative and should not exceed the upper bound of investment (if there is one).

3.2.1 Results interpretation

The optimal asset allocations and the set of stocks are analytically shown in Table 4:

Table 4 Konno and Yamazaki MAD model optimal asset allocation.

Stock Name	Full Stock Name	Stock Exchange	Optimal Asset Allocation ($)	Optimal Asset Allocation (%)
CAT	Caterpillar Inc.	NYSE	3389.47	2.26%
ORCL	Oracle Corporation	NYSE	6435.8	4.291%
RIG	Transocean Ltd.	NYSE	1602.7	1.068%
VDSI	VASCO Data Security International Inc.	NasdaqCM	18735.6	12.49%
XOM	Exxon Mobil Corporation	NYSE	4798.37	3.199%
FCEL	FuelCell Energy Inc.	NasdaqGM	3163.84	2.109%
MSFT	Microsoft Corporation	NasdaqGS	18401.2	12.267%
INL.DE	Intel Corporation	XETRA	14827.2	9.885%

SNE	Sony Corporation	NYSE	3024.13	2.016%
BTX	BioTime, Inc.	NYSE MKT	2191.36	1.461%
QCOM	QUALCOMM Incorporated	NasdaqGS	4190.68	2.794%
RIG	Transocean Ltd.	NYSE	1596.79	1.065%
KNDI	Kandi Technologies Group, Inc.	NasdaqGS	9651.89	6.435%
HOT	Starwood Hotels & Resorts Worldwide Inc.	NYSE	5281.46	3.521%
AIZ	Assurant Inc.	NYSE	44244.4	29.496%
VNO	Vornado Realty Trust	NYSE	4860.33	3.24%
TDW	Tidewater Inc.	NYSE	1548.4	1.032%
AAPL	Apple Inc.	NasdaqGS	2056.33	1.371%
Total			**150,000**	**100%**

3.3 Young's minimax model

During the last 15 years game theory had a large impact in portfolio optimization. Many studies have been reported that combined these two fields, resulting to some important portfolio optimization models based on the minimax solution. Young's Minimax model [Young, 1998] was the novel work in the combination of game theory with portfolio optimization, and a few years later other similar models such as Cai's model [Cai et al., 2000], Teo's model [Teo and Yang, 2001] and Deng's model [Deng et al., 2005] followed. Also a comparison test has been implemented to prove that the optimal allocations of Konno-Yamazaki MAD model are very similar to the classic Markowitz's Mean-Variance model whereas Young's Minimax model and Markowitz's MVO differ somehow [Hoe et al., 2010]. In this subsection Young's Minimax model is presented.

In 1998 Martin Young introduced the minimax model in portfolio optimization [Young, 1998]. The essence of this model lies in the minimax formulation of game theory, so the objective function is to maximize the minimum returns of the portfolio subject to some constraints. The mathematical formulation of Young's minimax model is:

$$\max_{M_p, w} M_p$$

subject to

$$\sum_{j=1}^{N} w_j y_{jt} - M_p \geq 0, \quad t = 1...T$$

$$\sum_{j=1}^{N} w_j \bar{y}_j \geq G$$

$$\sum_{i=1}^{N} w_j \leq W$$

$$0 \leq w_j \leq u, \quad j = 1...N$$

- The target of the objective function is the maximization of the portfolio's minimum returns (M_p)

  w_j : Optimal allocation

  M_p : Portfolio's minimum returns

  y_{jt} : Historic monthly returns of the shares

  $\bar{y}_j$: expected returns of the assets

  W : Investor's budget

  G : Target return

  N : Number of the assets

  u : Upper bound of optimal allocations

  T : Time period e.g. months

The AMPL model file as also the corresponding script file are as follows:

Table 5 AMPL model and script files, for the Young's Minimax model.

AMPL model file

```
# parameter definition
param n > 0;                  # number of shares
param T > 0;                  # number of months
param W;                      # budget
param RetMat{1..T, 1..n};   # historic monthly returns for selected shares
param u;                      # Upper limit for investing in a single share
param G;                      # target return of the portfolio
param Mp{1..n};               # minimum portfolio
param ExpRet{1..n};           # expected returns of shares
param stdv{1..n};             # standard deviation of shares

# variable definition
```

```
var w{1..n} >=0;

# objective function
maximize MinimumReturn:
sum {j in 1..n} w[j]*Mp[j];

# constraints
subject to TargetReturn:
sum {j in 1..n} ExpRet[j]*w[j] >= G;

subject to Budget:
sum {j in 1..n} w[j] <= W;

subject to bounds {j in 1..n}:
0 <= w[j] <= u;
```

AMPL script file

```
# script file for the Young Minimax model

model Young.mod;
data Young.dat;

let {j in 1..n} ExpRet[j] := sum{i in 1..T} RetMat[i,j]/T;
let {j in 1..n} stdv[j] := sqrt((sum{i in 1..T} (RetMat[i,j] -ExpRet[j])^2)/T);
let {j in 1..n} Mp[j] := ExpRet[j]-stdv[j];

option solver gurobi_ampl;
solve;
display w;
```

- **Example**: Young's minimax model aims to maximize the minimum returns of the portfolio given the same historical annual returns for the stocks used in the previous example, the minimum target return that the same investor requires from the portfolio (0.095), the total budget he holds (150,000$) and the upper bound of investment in a single stock (120,000$). More specifically the expected returns of each one of the three stocks are the arithmetic average of the historic returns for all 12 months. The minimum returns are the result of subtraction of standard deviation of stock[j] from the expected return of stock[j].

3.3.1 Results interpretation

The optimal asset allocations and the set of stocks are analytically shown in Table 6:

Table 6. Young's minimax model optimal asset allocation.

Stock Name	Full Stock Name	Stock Exchange	Optimal Asset Allocation ($)	Optimal Asset Allocation (%)
CAT	Caterpillar Inc.	NYSE	0	0.00%
ORCL	Oracle Corporation	NYSE	30,000	20.00%
RIG	Transocean Ltd.	NYSE	0	0.00%
VDSI	VASCO Data Security International Inc.	NasdaqCM	0	0.00%
XOM	Exxon Mobil Corpora-	NYSE	0	0.00%

	tion			
FCEL	FuelCell Energy Inc.	NasdaqGM	0	0.00%
MSFT	Microsoft Corporation	NasdaqGS	120,000	80.00%
INL.DE	Intel Corporation	XETRA	0	0.00%
SNE	Sony Corporation	NYSE	0	0.00%
BTX	BioTime, Inc.	NYSE MKT	0	0.00%
QCOM	QUALCOMM Incorporated	NasdaqGS	0	0.00%
RIG	Transocean Ltd.	NYSE	0	0.00%
KNDI	Kandi Technologies Group, Inc.	NasdaqGS	0	0.00%
HOT	Starwood Hotels & Resorts Worldwide Inc.	NYSE	0	0.00%
AIZ	Assurant Inc.	NYSE	0	0.00%
VNO	Vornado Realty Trust	NYSE	0	0.00%
TDW	Tidewater Inc.	NYSE	0	0.00%
AAPL	Apple Inc.	NasdaqGS	0	0.00%
Total			**150,000**	**100%**

4. CONCLUSIONS AND FUTURE WORK

Finance and decision making theory are among the most useful research fields in today’s market. Thus, the focus of this work was to show that the combination of these two research fields, i.e., financial optimization has a plethora of interesting real-world applications. The proposed software library that contains these financial optimization models assists the investor to make the best decision for asset allocations in finance in order to both ensure his investments and have a satisfactory return of his portfolio by giving him a plethora of models which use different methodologies.

It is well-known that algebraic modelling languages are ideal tools for rapid prototyping and optimization model development. Thus, our proposed portfolio optimization models software package utilizes the flexibility and convenience of the AMPL modelling language. A strong point of the proposed work is the variety of the state-of-the-art portfolio optimization models which aim is to advice the investor about the optimal asset allocation.

Generally speaking, only a few financial optimization packages use the mean absolute deviation model of Konno and Yamazaki and/or the game theoretic Young’s minimax model. Furthermore this software library of portfolio optimization models is non-commercial (apart of course from AMPL) and every researcher, or investor may use these models. Furthermore, the code of these models can be easily extended or modified, since the majority of the operational and financial researchers are rather more familiar with mathematical modelling languages than with common programming languages.

This work is in progress and more portfolio optimization models (especially game theoretic portfolio optimization models) are still being added to our library. Moreover, as a future research direction we plan to implement all these models, using the open-source GNU MathProg modeling language. The open-source GNU MathProg modeling language features a syntax similar to the one of AMPL. Finally it is worth exploring how multicriteria decision analysis (MCDA) has vitally aided portfolio optimization in the past decades [Zopounidis and Doumpos, 2013] and [Zopounidis et al., 2015], leading us to further enrich our library.

REFERENCES

Adame-García M.V., Rodríguez F. F., Sosvilla Rivero, S, 2015. Portfolios in the Ibex 35 index: Alternative methods to the traditional framework, a comparative with the naive diversification in a pre- and post- crisis context .*Documentos de Trabajo del Instituto Complutense de Análisis Económico (ICAE)*. Vol 1, No.7, pp. 1-33.

Black F. and Litterman R., 1992. Global portfolio optimization. *Financial Analysts Journal*, Vol 48, No 5, pp. 28–43.

Brandimarte P. 2002. *Numerical Methods in Finance A MATLAB-Based Introduction*. New York, USA: John Wiley & Sons. Inc.

Cai X.Q., Teo K.L, Yang X.Q., Zhou X.Y, (2000). Portfolio optimization under a minimax rule. *Management Science*. Vol. 46, pp. 957–972.

Çetin N, Göktas Ü 2009. An Excel Solver Model for a Blending Type Optimization Problem in Mining with Quadratic Programming, Proceedings of the first International Syposium on Sustainable Development, pp. 364-368.

Chen W.P., Chung H., Ho K.Y., Hsu T.L., 2010. Portfolio Optimization Models and Mean–Variance Spanning Tests. In: Cheng-Few Lee, Alice C. Lee, John Lee *Handbook of Quantitative Finance and Risk Management*. USA: Springer US. 165-184.

Deng X.T., Li Z.F., Wang S.Y., (2005). A minimax portfolio selection strategy with equilibrium, European Journal of Operational Research Vol. 166, pp. 278–292.

Dosamantes, C.A.D., 2013. Commands for financial data management and portfolio optimization. Presented in Mexican Stata Users Group meeting in Mexico City, México.

Fourer R., Gay D. M., Kernighan B.W., 2002. *AMPL: A Modeling Language for Mathematical Programming*. Duxbury Press, 2nd edition.

Hoe L.W., Hafizah J.S., Zaidi I., 2010. An empirical comparison of different risk measures in portfolio optimization. *BEH - Business and Economic Horizons*. Vol. 1, No. 1, pp. 39-45.

Ibbotson R.G., 2010. The Importance of Asset Allocation. *Financial Analysts Journal*. Vol. 66 No. 2, pp. 18-20.

K.L. Teo, X.Q. Yang (2001). Portfolio selection problem with minimax type risk function, *Annals of Operations Research*. Vol. 101, pp. 333–349.

Kolm P.N., Tütüncü R., Fabozzi F.J, 2014. 60 Years of portfolio optimization: Practical challenges and current trends. *European Journal of Operational Research*. Vol 234, No. 2, pp.356-371.

Konno H. and Yamazaki H., 1991. Mean-absolute deviation portfolio optimization model and its applications to Tokyo stock market. *Management Science*, Vol. 37, No 5, pp. 519–531.

Lee M.C., 2015. Risk loan portfolio optimization model based on CvaR risk measure. *Ecoforum*. Vol. 4 No.2, pp. 56-64.

Livingston L.S., 2013. Adding Markowitz and Sharpe to portfolio investment projects. *Business Education & Accreditation*. Vol. 5 No. 2, pp. 79-91.

Mansini R., Ogryczak W., Speranza G. (2014). Twenty years of linear programming based portfolio optimization. *European Journal of Operational Research*. Vol. 234, No. 1, pp. 518-535.

Markowitz H., 1952. Portfolio selection. *Journal of Finance*, Vol 7, No 1, pp. 77–91.

Markowitz H., 1959. *Portfolio Selection.* John Wiley & Sons, Inc., New York.

Markowitz H., 1991. Foundations of Portfolio Theory. *Journal of Finance,* Vol 46, No 2, pp. 469-477.

Mathworks 2000. *Financial Toolbox: For use with Matlab: User's Guide*. New York, MathWorks Inc.

Myles E. Mangram 2013. A simplified perspective of the Markowitz portfolio theory, *Global journal of business research*, Vol. 7, No. 1, pp. 59-70.

Pang J. 1980. A parametric linear complementary technique for optimal portfolio selection with a risk-free asset, *Operations Research* Vol. 28, No. 4, pp. 927-941.

Pfaff B. 2012. *Financial Risk Modelling and Portfolio Optimization with R*, Germany, A John Willey & Sons Ltd.

Polak G.G., Rogers D.F., D.J., 2010. Risk management strategies via minimax portfolio optimization. *European Journal of Operational Research*. Vol. 207 No. 1, pp. 409-419.

Rockafellar R. T. and Uryasev S., 2000. Optimization of conditional value-at-risk. *The Journal of Risk*, Vol 2, pp. 21–41.

Scherer B. and Martin D. 2005. *Introduction to modern portfolio Optimization with NuOpt, S-Plus and S+Bayes*. Chicago, Springer.

Sharpe W.F., 1989. Mean-Variance Analysis in Portfolio Choice and Capital Markets by Harry M. Markowitz. *Journal of Finance,* Vol 44, No 2, pp. 531–535.

Sharpe W. F., 1992. Asset allocation: Management style and performance measurement. *Journal of Portfolio Management*, Vol 18, No 2, pp. 7–19.

Sharpe W. F., 1994. The Sharpe ratio. *Journal of Portfolio Management*, Vol 21, No 1, pp. 49–58.

Soleimani H., Golmakani H.R., Salimi M.H., 2009. Markowitz-based portfolio selection with minimum transaction lots, cardinality constraints and regarding sector capitalization using genetic algorithm, *Expert Systems with Applications*, Vol. 36, No. 1, pp. 5058-5063.

Steinbach M. C., 2001. Markowitz revisited: mean-variance models in financial portfolio analysis. SIAM Review, Vol 43 No 1, pp.31–85.

Wu Z.W., Song X.F., Xu Y.Y., Liu K., (2009). A note on a minimax rule for portfolio selection and equilibrium price system. *Applied Mathematics and Computation.* Vol. 208, No. 1, pp. 49-57.

Xidonas P., Mavrotas G., Psarras J. 2010. Portfolio construction on the Athens Stock Exchange: a multiobjective optimization approach. *Optimization*. Vol. 59, No. 8, pp. 1211-1229.

Young M. R., 1998. A Minimax Portfolio Selection Rule with Linear Programming Solution, *Management Science*, Vol. 44, No. 5, pp. 673–683.

Zenios. S., Nielson S. and Consiglio A. 2009. *Practical Financial Optimization: A library of GAMS Models*. New York, NY John Wiley & Sons.

Zhu L, Fan Y., 2010. Optimization of China's generating portfolio and policy implications based on portfolio theory. *Energy*. Vol. 35 No. 1, pp. 1391-1402.

Zopounidis C., and Doumpos M., 2013. Multicriteria decision systems for financial problems. *TOP*. Vol.21, No. 2, pp. 241-261

Zopounidis C., Doumpos M., Fabozzi F.J., 2014. Preface to the Special Issue: 60 years following Harry Markowitz's contributions in portfolio theory and operations research. *European Journal of Operational Research,* Vol. 234 No. 2, pp. 343-345.

Zopounidis C., Galariotis E., Doumpos, M., Sarri S., Andrisopoulos K., 2015. Multiplecriteria decision aiding for finance: An updated bibliographic survey. *European Journal of Operational Research.* Vol.247, No.1, pp. 339-348