PRODUCTIVITY MEASUREMENT IN RADIAL DEA MODELS WITH A SINGLE CONSTANT INPUT

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ABSTRACT: We consider productivity measurement based on radial DEA models with a single constant input. We show that in this case the Malmquist and the Hicks-Moorsteen productivity indices coincide and are multiplicatively complete, the choice of orientation of the Malmquist index for the measurement of productivity change does not matter, and there is a unique decomposition of productivity change containing two independent sources, namely technical efficiency change and technical change. Technical change decomposes in an infinite number of ways into a radial magnitude effect and an output bias effect. We also show that the aggregate productivity index is given by the geometric mean between any two periods of the simple arithmetic averages of the individual contemporaneous and mixed period distance functions.

KEYWORDS: Malmquist and Hicks-Moorsteen productivity indices; measurement; decomposition; aggregation

JEL Classification Code: D21, D24

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September 15, 2015

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1 Introduction

Radial DEA models with a single constant input have gained increasing popularity in recent years for use in situations in which the relative performance of units is evaluated with reference to the outputs they produce or the services they provide and without reference to the resources they consume in the process. Applications of the single constant input model generally fit into four areas, each involving static performance evaluation.

One area that covers a wide range of applications, noted by Yang *et al.* (2014), occurs when ratio variables such as GDP per capita, output per hectare, value added per employee or a firm's revenue/cost ratio are used to evaluate performance, and the underlying data do not allow splitting ratio variables into numerators (outputs) and denominators (inputs). In this case the (desirable) ratio variables become outputs and there is a single constant input.

A second area is performance evaluation relative to best practice or to targets set by management. An early example was provided by Lovell and Pastor (1997), who analyzed target setting for bank branches. More recent examples include Halkos and Salamouris (2004) for evaluating the financial performance of Greek commercial banks; Wang, Lu and Lin (2012) on bank holding company performance; Odeck (2005, 2006) for road traffic safety units; Soares de Mello, Angulo-Meza and Branco da Silva (2009) for ranking the performance of countries in the Olympic Games; Lo (2010) for Kyoto Protocol target achievement; Liu *et al.* (2011) for the performance of Chinese research institutes; and Bezerra Neto *et al.* (2012) for agro-economic indices in polyculture.

A rapidly growing third area is the construction of composite indicators. Early examples include Thompson *et al.* (1986) and Takamura and Tone (2003) for comparative site evaluation. More recent examples include Cherchye, Moesen and Van Puyenbroeck (2004) for macroeconomic indicators of country performance; Mizobuchi (2014) for the OECD Better Life Index; Guardiola and Picazo-Tadeo

(2014) for life satisfaction indices; Zafra-Gomez and Muñiz Pérez (2010) and Lin, Lee and Ho (2011) for local government performance evaluation; Murias, deMiguel and Rodriguez (2008) for an educational quality indicator; Despotis (2005) for revising the Human Development Index; Lauer *et al.* (2004) on the performance of the world health system; and Zhou, Ang and Poh (2007), Bellenger and Herlihy (2009), Lo (2010), Sahoo, Luptacik and Mahlberg (2011), Rogge (2012) and Zanella, Camanho and Dias (2013) for environmental and ecological performance indicators.

A fourth area employs DEA as a multiple criteria decision analysis (MCDM) tool. Examples include Ramanathan (2006), Zhou and Fan (2007), Hadi-Vencheh (2010) and Chen (2011) for inventory classification, Seydel (2006) and Sevkli *et al.* (2007) for supplier selection, Lee and Kim (2014) and Charles and Kumar (2014) for service quality evaluation, and Yang et al. (2014) for the performance of Chinese cities and the performance of research institutes in the Chinese Academy of Sciences.

In this paper we take the use of the radial DEA models with a single constant input one step further by considering their potential use in inter-temporal performance evaluation by means of a pair of technology-based productivity indices, the Malmquist and the Hicks-Moorsteen indices.[1] In particular, (a) we compare the Malmquist and the Hicks-Moorsteen productivity indices for the single constant input model; (b) we develop a new decomposition of the sources of productivity change in this case and (c) we explore the aggregation of productivity changes from individual to group level. We show that in the single constant input case the Malmquist and the Hicks-Moorsteen productivity indices coincide, without having to impose restrictions on the structure of technology, and the orientation of the Malmquist productivity index does not matter. We also show that there is a unique decomposition of productivity change containing two independent components, technical efficiency change and technical change. Technical change decomposes in an infinite number of ways into a radial magnitude effect and an output bias effect. In addition, we show that the aggregate (group) productivity index equals the geometric mean between any two periods of the simple (un-weighted) arithmetic averages of the individual contemporaneous and mixed period distance functions.

In relating the above results with those previously presented in the literature note the following: *first*, the Malmquist and the Hicks-Moorsteen productivity indices coincide not only when restrictions are imposed on the structure of technology, such as a single input or a single output and constant returns to scale, as was claimed by

Bjurek (1996), or constant returns to scale and inverse homotheticity, as was shown by Färe, Grosskopf and Roos (1996)[2], or constant returns to scale and technological stagnation, as was shown by O'Donnell (2012; 258), or constant returns to scale and Hicks-neutral technical change, as was shown by Mizobuchi (2015), but also in the case of a single constant input. *Second*, it is not only the case of a global constant-returns-to-scale technology that there is a unique decomposition of productivity change but also the case of a single constant input. *Third*, it seems that the single constant input model is the only known case that the geometric mean between any two periods of the simple arithmetic averages of the individual contemporaneous and mixed period distance functions provides a consistent measure of aggregate productivity change by means of the Malmquist productivity index.

Although we explicitly consider an output-oriented model with a single constant input, the results can easily be extended to an input-oriented model with a single constant output. And although we motivated the exercise with empirical examples in which a single constant input is plausible, an extension to the case of a single constant output is also easily motivated.[3]

2 The Main Results

The Malmquist and the Hicks-Moorsteen indices are the two technology-based productivity indices that complement the set of price-based productivity indices (e.g., Fisher and Törnqvist), and their main advantage is that their measurement does not require price data. The former is expressed in terms of distance functions defined on the benchmark technology and the latter in terms of distance functions defined on the best practice technology. The two characterizations of technology differ in their returns to scale properties, and also in their technical change properties.

The output- and input-oriented Malmquist productivity indices are defined as:

$$M_O = \left[\frac{\tilde{D}_O^t(x^{t+1},y^{t+1})}{\tilde{D}_O^t(x^t,y^t)}\frac{\tilde{D}_O^{t+1}(x^{t+1},y^{t+1})}{\tilde{D}_O^{t+1}(x^t,y^t)}\right]^{\frac{1}{2}} \quad (1\mathbf{a})$$

$$M_I = \left[\frac{\tilde{D}_I^t(x^t,y^t)}{\tilde{D}_I^t(x^{t+1},y^{t+1})}\frac{\tilde{D}_I^{t+1}(x^t,y^t)}{\tilde{D}_I^{t+1}(x^{t+1},y^{t+1})}\right]^{\frac{1}{2}}, \quad (1\mathbf{b})$$

in which $x \in R_+^N$ and $y \in R_+^M$ are respectively input and output quantity vectors, and $\tilde{D}_O(x,y) = \min\{\lambda: (x,y/\lambda) \in \tilde{\mathbf{T}}\}$ and $\tilde{D}_I(y,x) = \max\{\delta: (y,x/\delta) \in \tilde{\mathbf{T}}\}$ are respectively output and input distance functions defined on a *benchmark* technology $\tilde{\mathbf{T}} = \{(y,x): x$ can produce $y\}$ that exhibits (global) constant returns to scale. These distance functions are defined on data and technology from the same time period, in which case $\tilde{\mathbf{D}}_o^t(x^t,y^t) \leq 1$ and $\tilde{\mathbf{D}}_i^t(y^t,x^t) \geq 1$, and also data and technology from adjacent time periods, in which case $\tilde{\mathbf{D}}_o^t(x^{t+1},y^{t+1}) \gtreqless 1$ and $\tilde{\mathbf{D}}_i^t(y^{t+1},x^{t+1}) \gtreqless 1$ (i.e., data from one period may not be feasible with technology from the other period). The Malmquist productivity indices were introduced and named by Caves, Christensen and Diewert (1982) but with distance functions defined on a *best practice* technology allowing for variable returns to scale. Grifell-Tatjé and Lovell (1995) showed however that this formulation prevents economies of size and diversification from contributing to productivity change, and it is now standard practice to define Malmquist productivity indices as in (1a) and (1b), because this formulation allows economies of size and diversification to contribute to productivity change. It does so by distinguishing the benchmark technology satisfying constant returns to scale from the best practice technology allowing for variable returns to scale, with deviations between the two reflecting the presence of economies of size and diversification and nothing else.

The non-oriented Hicks-Moorsteen productivity index is defined as the ratio of Malmquist output and input quantity indices, viz.:

$$HM = Q_I y/Q_I x = [(D_O{}^t(x^t, y^{(t+1)}))/(D_O{}^t(x^t, y^t))\ (D_O{}^{(t+1)}(x^{(t+1)}, y^{(t+1$$

in which $D_O(x,y)$ and $D_I(y,x)$ are respectively output and input distance functions defined as above but on the *best practice* technology $\mathrm{T} = \{(y,x): x$ can produce $y\}$ that allows for variable returns to scale. These distance functions also are defined on data and technology from the same time period, and data and technology from adjacent time periods. This index was introduced by Bjurek (1994, 1996), who did not however give it its popular name. Because HM is expressed as the ratio of an output quantity index to an input quantity index, Bjurek (1994, 1996) called it, prophetically, "The Malmquist Total Factor Productivity Index". O'Donnell (2012; 257) characterizes HM as a "multiplicatively complete" productivity index because it is expressed as the ratio of an output quantity index to an input quantity index, with both indices being non-negative, non-decreasing and linearly homogeneous, and notes that the Malmquist productivity indices M_O and M_I do not share this desirable property, which implies that they cannot always be interpreted as measures of productivity change.

2.1 *Measurement*

We assume now that $x \in R^1_+$, and that the single input is constant, both across producers, the context Lovell and Pastor (1999) examined, and through time, a new dimension we introduce here in order to extend efficiency analysis to productivity analysis. Without loss of generality we scale the single constant input, up or down, such that $x^t = x^{t+1} = 1$. To establish a relationship between the Malmquist and the Hicks-Moorsteen productivity indices in the single constant input case notice first that $Q_x = 1$ when $x^t = x^{t+1} = 1$. Then, extending Proposition 2 of Lovell and Pastor (1999) that an output-oriented constant returns to scale DEA model with a single constant input is equivalent to an output-oriented variable returns to scale DEA model with a single constant input, we have

$$\tilde{D}_O^t(1, y^t) = D_O^t(1, y^t) \qquad \text{(3a)}$$

$$\tilde{D}_O^s(1, y^r) = D_O^s(1, y^r) \text{ for } s \neq r, \qquad \text{(3b)}$$

which equates the benchmark technology with the best practice technology, not globally but over the restricted domain $x^t = x^{t+1} = 1$. Substituting (3a) and (3b) into (1a) and (2) and comparing the result with (2) yields our first result:

$$M_O = HM = \left[\frac{\tilde{D}_O^t(1, y^{t+1})}{\tilde{D}_O^t(1, y^t)} \frac{\tilde{D}_O^{t+1}(1, y^{t+1})}{\tilde{D}_O^{t+1}(1, y^t)}\right]^{\frac{1}{2}} = \left[\frac{D_O^t(1, y^{t+1})}{D_O^t(1, y^t)} \frac{D_O^{t+1}(1, y^{t+1})}{D_O^{t+1}(1, y^t)}\right]^{\frac{1}{2}}. \quad \text{(4)}$$

In the case of a single constant input, the output-oriented Malmquist and the Hicks-Moorsteen productivity indices coincide, and are equivalently defined on the benchmark and best practice technologies over the restricted domain. This provides one situation in which M_o is a multiplicatively complete productivity index. Moreover, since the common technology satisfies constant returns to scale,

$$\tilde{D}_O^t(1, y^t) = \frac{1^t}{\bar{\tilde{D}}_I}(1, y^t) \tag{5a}$$

$$\tilde{D}_O^s(1, y^r) = \frac{1}{\tilde{D}_I^s(1, y^r)} \text{ for } s \neq r, \tag{5b}$$

and thus we can show using (4) and (1b) that in the case of a single constant input $M_O = M_I$, implying that the choice of orientation for the measurement of productivity change does not matter. In this case, M_I is also a multiplicatively complete productivity index.

The above results are summarized in the following proposition:

Proposition 1: In the single constant input case, the Malmquist and the Hicks-Moorsteen productivity indices coincide, they are multiplicatively complete and the choice of orientation for the measurement of productivity change does not matter for the Malmquist productivity index.

This result complements previous findings regarding the relation between the Malmquist and Hicks-Moorsteen productivity indices. Bjurek (1996) has claimed that they coincide if the best practice technology satisfies constant returns to scale and either a single output is produced with multiple inputs or multiple outputs are produced with a single input. Färe, Grosskopf and Roos (1996) have shown that for multi-output, multi-input technologies they coincide if the common (benchmark and best practice) technology satisfies constant returns to scale and inverse homotheticity. O'Donnell (2012) has shown that they coincide if technology satisfies constant returns to scale and there is no technological change. Mizobuchi (2015) has generalized O'Donnell's result by showing that they coincide under constant returns to scale and Hicks-neutral technical change. Finally Peyrache (2014) has introduced a radial productivity index to provide the "missing link" between the Malmquist and Hicks-Moorsteen productivity indices, the nature of the link depending on the scale properties of the underlying technology.

2.2 *Decomposition*

The single constant input assumption greatly simplifies the decomposition of the Malmquist and the Hicks-Moorsteen productivity index. First, as Lovell and Pastor (1999) noted, with a single constant input scale is not an issue. Consequently the "scale effects," as alternatively defined by Färe *et al.* (1994), Ray and Desli (1997), Balk (2001) and Lovell (2003), are all equal to one and thus have no impact on productivity change. In addition, the input mix and the output mix effects in Balk (2001) and Lovell (2003) are also equal to one and have no impact on productivity change. Finally, the input bias of technical change identified by Färe *et al.* (1997) and the scale bias of technical change identified by Zofio (2007) are both unity and have no impact on productivity change.[4]

By taking all the above into consideration we can show that in the single constant input case there is a unique decomposition of the Malmquist and the Hicks-

Moorsteen productivity index into a technical efficiency change effect and a technical change effect ΔT. The decomposition is

$$M_O = M_I = HM = \Delta TE \cdot \Delta T \tag{6}$$

$$= \frac{D_O^{t+1}(1,y^{t+1})}{D_O^t(1,y^t)}\left[\frac{D_O^t(1,y^t)}{D_O^{t+1}(1,y^t)}\frac{D_O^t(1,y^{t+1})}{D_O^{t+1}(1,y^{t+1})}\right]^{\frac{1}{2}}.$$

Other than the scale-restricted technology cases identified by Bjurek (1996), Färe, Grosskopf and Roos (1996) and Peyrache (2014), this is the second case known in the literature in which the Malmquist productivity index decomposes uniquely into a technical efficiency change component and a technical change component.

The technical change effect decomposes into a radial magnitude effect TM and an output bias effect OB. However, there being an infinite number of rays in output space along which to measure the magnitude effect, this decomposition is not unique. If we measure the radial magnitude effect along a ray through y^t the decomposition becomes

$$\Delta T = TM \cdot OB \tag{7}$$

$$= \frac{D_O^t(1,y^t)}{D_O^{t+1}(1,y^t)} \left[\frac{D_O^t(1,y^{t+1})}{D_O^{t+1}(1,y^{t+1})} \left(\frac{D_O^t(1,y^t)}{D_O^{t+1}(1,y^t)} \right. \right]^{\frac{1}{2}} .$$

If the magnitude of technical change is the same along rays through y^t and y^{t+1}, OB = 1 and the output bias effect makes no contribution to productivity change.[5]

Combining (6) and (7) yields one complete decomposition of productivity change as

$$M_O = M_I = HM = \Delta TE \cdot TM \cdot OB \qquad (8)$$

$$= \frac{D_O^{t+1}(1,y^{t+1})}{D_O^t(1,y^t)} \frac{D_O^t(1,y^t)}{D_O^{t+1}(1,y^t)} \left[\frac{D_O^t(1,y^{t+1})}{D_O^{t+1}(1,y^{t+1})} \left(\frac{D_O^t(1,y^t)}{D_O^{t+1}(1,y^t)} \right. \right]^{\frac{1}{2}} .$$

From (8) we can see that there are three independent sources of productivity change: change in technical efficiency, radial technical change and output biased technical change. Values greater (less) than one for each of the components in (8) indicate a positive (negative) impact and thus result in productivity improvements (deteriorations), while values equal to one denote no contribution to productivity change.

The above results may be summarized in the following proposition:

Proposition 2: In the single constant input case, there is a unique decomposition of productivity change into technical efficiency change and technical change. The contribution of technical change decomposes in a non-unique way into neutral technical change and output biased technical change.

2.3 Aggregation

We turn next to the aggregation of individual productivity indices into an aggregate productivity index. Zelenyuk (2006) has shown that the aggregate output-oriented Malmquist productivity index equals the harmonic mean of individual output-oriented Malmquist productivity indices with common price individual revenue shares (defined over all outputs) used as weights. Accordingly, individual cost shares (defined over all inputs) should be used in aggregating input-oriented Malmquist productivity indices.

Using these results and given (5) it can be argued that inputs can also be used to aggregate individual output-oriented Malmquist productivity indices in the single constant input case. However, since the input is constant across individual units, and can be normalized to unity, this simplifies the aggregation process: since all individual units use the same amount of input, a common weight is attached to all units, resulting eventually in an equally-weighted mean with the shares being given by the inverse of the total number of units under consideration.

To show this we start by adapting Zelenyuk's (2006) definition of the aggregate output-oriented Malmquist productivity index to the single constant input model to generate

$$M_o^A = \left[\frac{\tilde{D}_o^t\left(1, \sum_k y_k^{t+1}\right)}{\tilde{D}_o^t\left(1, \sum_k y_k^t\right)} \frac{\tilde{D}_o^{t+1}\left(1, \sum_k y_k^{t+1}\right)}{\tilde{D}_o^{t+1}\left(1, \sum_k y_k^t\right)}\right]^{\frac{1}{2}}, \tag{9}$$

where k is used to index firms. It is convenient to re-state the above relation in terms of efficiency measures in order to be able to incorporate previous results in the literature regarding aggregation of efficiency indices, and so

$$M_o^A = \left[\frac{F_o^t\left(1, \sum_k y_k^{t+1}\right)}{F_o^t\left(1, \sum_k y_k^t\right)} \frac{F_o^{t+1}\left(1, \sum_k y_k^{t+1}\right)}{F_o^{t+1}\left(1, \sum_k y_k^t\right)}\right]^{\frac{1}{2}}, \tag{10}$$

where $F_O(x, y) = D_O(x, y) \leq 1$.[6] Bjurek, Kjulin and Gustafsson (1992) considered the aggregation of F_O in the single output case. Manipulating their equation (5) yields

$$F_O = \left[\sum_k \tilde{s}_k \left(F_{Ok}\right)^{-1}\right]^{-1}, \tag{11}$$

where $\tilde{s}_k$ refers to the (potential) output share of the kth firm. In the case of multi-output technologies, Färe and Zelenyuk (2003) have shown that the aggregation weights become output price dependent and under the assumptions that all firms face the same price for each output and all firms are equally allocatively efficient they reflect revenue shares. Substituting this in the above relation yields

$$F_O = \left[\sum_k \frac{\Sigma_j p_j y_{jk} (F_{Ok})^{-1}}{\Sigma_k \Sigma_j p_j y_{jk}}\right]^{-1} = \frac{\Sigma_k \Sigma_j p_j y_{jk}}{\Sigma_k \Sigma_j p_j \tilde{y}_{jk}} = \frac{\Sigma_k \Sigma_j p_j \tilde{y}_{jk} F_{Ok}}{\Sigma_k \Sigma_j p_j \tilde{y}_{jk}} =$$

$$\sum_k \left(\frac{\Sigma_j p_j \tilde{y}_{jk}}{\Sigma_k \Sigma_j p_j \tilde{y}_{jk}}\right) F_{Ok} = \sum_k \tilde{s}_k F_{Ok}, \qquad (12)$$

where j is used to index outputs, $\tilde{y} = \frac{y}{F_O}$, and the second and third equalities reflect the radial nature of F_O.

From the first-order conditions for profit maximization subject to an output distance function, and assuming constant returns to scale, yields $\sum w_i x_i = \sum p_j \tilde{y}_j$ (a proof is relegated to the Appendix), where w refers to input prices that are assumed to be the same for all firms and i is used to index inputs. In the case of a single constant input the above relation becomes $w = \sum_j p_j \tilde{y}_j$. Summing over firms, $Kw = \sum_k \sum_j p_j \tilde{y}_j$, and dividing through we get

$$= \frac{\Sigma_j p_j \tilde{y}_j}{\Sigma_k \Sigma_j p_j \tilde{y}_j} = \tilde{s}_k,$$

(13)

where K is the total number of firms in the group or industry. Then (12) is written as:

$$F_O = \frac{\Sigma_k F_{Ok}}{K},$$

(14)

and combining (14) with (9) and (10) yields

$$M_O^A = \left[\frac{\sum_k \tilde{D}_{Ok}^t(1, y_k^{t+1})}{\sum_k \tilde{D}_{Ok}^t(1, y_k^t)} \frac{\sum_k \tilde{D}_{Ok}^{t+1}(1, y_k^{t+1})}{\sum_k \tilde{D}_{Ok}^{t+1}(1, y_k^t)}\right]^{\frac{1}{2}}.$$

Thus the aggregation rule for the Malmquist productivity index in the single constant input case is summarized in the following proposition:

Proposition 3: In the single constant input case, the aggregate output-oriented Malmquist productivity index is given by the geometric average between any two periods of the simple (un-weighted) arithmetic average of the individual contemporaneous and mixed period output distance functions.

3. Concluding Remarks

In this paper we examine the three main features of productivity analysis, namely measurement, decomposition and aggregation, for the case of radial DEA models with a single constant input. We show that in this case the use of either technology-based productivity index, i.e., Malmquist or Hicks-Moorsteen, yields the same measurement result and in addition, the choice of orientation of the former does not matter. Moreover, there are only three independent sources of productivity growth, i.e., the effect of technical efficiency change, the magnitude effect of technical change, and the effect of output biased technical change, although the attribution of technical change to radial and bias components is not unique. Consequently, the long debate about the choice of the most appropriate scale related effect surrounding the literature on the decomposition of the technology-based productivity indices does not carry over to this "helmsman" type of model. In addition, we have shown that consistent aggregation of firm-level productivities to industry level is possible using the geometric mean between any two periods of the simple arithmetic average of the individual contemporaneous and mixed periods output distance functions. We hope these theoretical findings prove to be helpful to applied researchers who will extend the use of this "helmsman" type of model into other areas of efficiency and productivity analysis. Even though in recent years this model has received increased recognition in several fields of performance evaluation, more applications are to come.

Acknowledgements

We are grateful to three perceptive reviewers, whose comments and criticisms have greatly improved this paper. We also have received helpful comments from seminar audiences in Spain and Taiwan.

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Appendix

To prove that $\sum w_i x_i = \sum p_j \tilde{y}_j$ under constant returns to scale consider the optimization problem

$$\pi(p, w) = \max_{x,y}\{p'y - w'x : D_O(x, y,) \le 1\}$$

with first-order conditions

$$-w_i - \lambda \frac{\partial D_O}{\partial x_i} = 0$$

$$p_j - \lambda \frac{\partial D_O}{\partial y_j} = 0$$

$$1 - D_O(x, y) = 0\,.$$

In the presence of technical inefficiency the last relation of the first-order conditions holds with equality only for $\tilde{y}$, i.e. the frontier projected output quantities, and for all first-order conditions to hold simultaneously, the derivatives should be evaluated at $\tilde{y}$. Then multiply the first relation of the first-order conditions by x_i and sum over all inputs to obtain

$$-\sum_i w_i x_i =$$

and since under constant returns to scale D_o is homogeneous of degree -1 in x, the right side collapses to $-\lambda$. Similarly, multiply the second relation of the first-order conditions by $\tilde{y}_j$ and sum over all outputs to obtain

$$\sum_j \frac{\partial D_o}{\partial y_j} y_j$$

and since regardless of the nature of scale economies D_o is homogeneous of degree +1 in y, the right side collapses to λ. It follows directly that $\sum_i w_i x_i = \sum_j p_j \tilde{y}_j$. Balk (2001) follows a similar approach in discussing scale and productivity change.

Footnotes

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[1] Both Odeck (2005, 2006) and Lin, Lee and Ho (2011) used the single constant input model in an intertemporal context. However neither study provides any reasoning behind the standard decomposition of the Malmquist productivity index that involves no scale-related effect. As we show below, however, this is an inherent part of the radial single constant input model.

[2] Førsund (1997) has shown that these restrictions on the structure of technology coincide with distance functions introduced in Section 2 below satisfying homogeneity, identity, separability, proportionality and monotonicity properties.

[3] In the original version of this paper we extended the single constant input framework to a multiple constant input framework. However two reviewers have persuaded us that the extension is economically difficult to motivate and mathematically trivial.

[4] Proofs of these claims are available from the authors.

[5] The non-uniqueness of the decomposition of ΔT is independent of the number of inputs and the number of outputs (provided $M \geq 2$). If we measure the radial magnitude effect along a ray through y^{t+1} the decomposition of ΔT is

$$[T = TM\ (OB = (D_1 O^{\uparrow} t\ (1, y^{\uparrow}(t+1)))/(D_1 O^{\uparrow}(t+1)\ (1, y^{\uparrow}(t+1)))\ [(D_1 O^{\uparrow} t\ (1, y^{\uparrow} t))/(D_1 O^{\uparrow}(t +$$

[6] If efficiency is measured by $\frac{1}{D_o(x,y)} \geq 1$ then the resulting aggregation rule would be in terms of harmonic rather than arithmetic averages.