Financial networks based on
Granger causality: A case study

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Abstract

Connectivity analysis is performed on a long financial record of 21 international stock indices
employing a linear and a nonlinear causality measure, the conditional Granger causality index
(CGCI) and the partial mutual information on mixed embedding (PMIME), respectively. Both
measures aim to specify the direction of the interrelationships among the international stock
indexes and portray the links of the resulting networks, by the presence of direct couplings
between variables exploiting all available information. However, their differences are assessed
due to the presence of nonlinearity. The weighted networks formed with respect to the causality
measures are transformed to binary ones using a significance test. The financial networks are
formed on sliding windows in order to examine the network characteristics and trace changes in
the connectivity structure. Subsequently, two statistical network quantities are calculated; the
average degree and the average shortest path length. The empirical findings reveal interesting
time-varying properties of the constructed network, which are clearly dependent on the nature of
the financial cycle.

Keywords: Granger causality, PMIME, financial network

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I. Introduction

The study of complex networks constitutes a relatively new field of research finding applications in different areas such as finance \cite{1,34}, neurophysiology \cite{31,10} and ecology \cite{12,35}. Exploiting information from graph theory, networks are formed based on the connectivity pattern of the involved variables. When significant interactions exist, a link connecting two nodes is present.

Four types of networks can be constructed taking into account the type of connections; undirected vs. directed and binary vs. weighted. The undirected connections imply that the connectivity is examined based on symmetric, i.e. correlation, measures, while directed ones entail the determination of the interrelationships based on causality measures.

In finance, correlation-based networks have been mainly developed, aiming at representing the complexity of interdependent systems, and market dynamics in general. A growing literature has emerged towards this direction, i.e. exploiting correlation measures to formulate (undirected) financial networks \cite{16,47,8}. The Minimum Spanning Tree has been introduced for the identification of an hierarchical arrangement for a portfolio of stocks preserving the most prominent relationships \cite{25}, and has been used to analyze stock markets, e.g. see \cite{24}. Further, the so-called Asset Graph decreases the complexity of the generated network, so that a network with \(n\) vertices (nodes) has \(n - 1\) edges (connections) \cite{27}. Both the aforementioned methods reduce the complete network to a basic minimum structure that contains only the most relevant information. Finally, the Planar Maximally Filtered Graph filters the full network at a chosen level, by varying the genus of the resulting filtered graph \cite{44}.

On the other hand, the standard Granger causality and its nonlinear extensions are adopted in order to construct Granger causality networks (which are directed) and to quantify the interrelationships among hedge funds, banks, broker/dealers, and insurance companies \cite{4,48}. Bivariate measures indicate both direct and indirect causal links, while multivariate ones capture only the direct ones by employing all available information. Multivariate methods have proved to be extremely useful, since pairwise analysis among multiple variables can give misleading results, e.g. pairwise connectivity analysis between three variables \(X, Y\) and \(Z\) could not distinguish between the cases a) \(X \rightarrow Y, X \rightarrow Z\) and \(Z \rightarrow Y\) and b) \(X \rightarrow Z\) and \(Z \rightarrow Y\) \cite{7}. In that spirit, transfer entropy is utilized to map connectivity among some of the largest international companies aiming at portraying determinant factors of the financial crisis \cite{33}, while the lead-lag effect in financial markets using information and network theories are studied in \cite{9}.

Our aim is to specify the direction of the interrelationships among 21 international stock indexes and depict the links of the resulting networks. Connectivity analysis is assessed based on a nonlinear, efficient causality measure, the partial mutual information on mixed embedding (PMIME) \cite{45,21}, while the standard linear Granger causality index (CGCI) \cite{13,11} is also employed for comparative reasons. We apply two direct causality measures in the effort to overcome the limitations of bivariate Granger causality tests and thus to adequately take into account the emerging complexity of the financial networks. Furthermore, we chose to use the PMIME for the identification of the connectivity patterns since it has been proved to be more effective than other well-known causality measures such as transfer entropy, especially when nonlinearities are present in the data \cite{21,29}, as well as many observed variables \cite{41,19}.

After identifying the causal effects among the variables based on the PMIME and the CGCI, the financial networks are derived and two statistical network quantities are calculated; the average degree (AD) and the average shortest path length (ASPL). We include in our analysis these measures because they quantify different characteristics of the network and describe adequately properties of real systems \cite{19}. In the resulting financial networks the nodes correspond to the international stock indexes and the connections represent the direct causal effects among them. We concentrate
on their time-varying characteristics, and via the detected dynamics we attempt to investigate the underlying mechanisms of systematic instability during financial drawbacks.

The structure of the paper is as follows. In Sec. II details on the financial data set are provided. In Sec. III the direct causality measures are briefly presented. Further, we deploy the methodology for the formulation of the binary networks and present the average degree and the average shortest path length. The results are reported and discussed in Sec. IV while Sec. V concludes the paper.

II. Data

The data set comprises daily closing prices of 21 stock indexes from America (Argentina, Brazil, Mexico, US), Europe (Austria, Belgium, Denmark, France, Germany, Greece, Italy, Netherlands, Prague, UK, Spain, Sweden, Switzerland), Asia (China, Hong Kong, Japan), and Australia, and spans from December 12, 1997 to January 23, 2015. The data have been collected from the Luxembourg Stock Exchange, the Central Bank of Brazil Statistical Database, the NASDAQ OMX Global Index Data and Yahoo Finance. The time series length is 4461. The periods of market-wide decline in stock prices correspond to 1) the dot.com bubble and the massive loss in the market value of companies from March 2000 to October 2002 and 2) the 2007-2009 financial crisis.

Because stock prices are recorded on different stock exchanges, time mismatches may arise. Since in our application causality measures are used instead of correlation to define connectivity, the exact informational content of differences in world time zones is taken into account without the need to intervene in the structure of time series by imposing lags. In this way, we are in line with the global analysis we want to perform including many international stock exchanges. The misalignment takes place also due to national holidays, unexpected events etc. The problem can be dealt by using lower frequency data, removing observations from stock indexes that correspond to a non-trading day in other markets, replacing the missing value by the value of the previous day or by the mean of the previous and next observation or even applying linear interpolation. There are other more involved approaches to adjust the connectivity measures in time series with gaps/missing observations, e.g. [28], but this is not implemented here. As in [23], the data are synchronized by filling the gaps with the last available information, so that the autocorrelation structure of the resulting returns series is little affected. As emphasized by [32], extending [39, 40] findings, the power of the augmented Dickey–Fuller unit root test is superior when missing points are replaced by the previous ones.

Since both methodologies require stationarity, prices are transformed into logarithmic returns. If $X_i(t)$ denotes the closing price of stock index $i$ at time $t$, the logarithmic returns are calculated as: $Y_i(t) = \log X_i(t) - \log X_i(t - 1)$, for $i = 1, \ldots, 21$ and $t = 2, \ldots, 4461$. No co-integration relationship is present in the data as confirmed by the Johansen co-integration test [15].

To address the dynamic nature of financial networks the connectivity measures are estimated over rolling windows. To do so, the data are separated into consecutive overlapping segments of length $n = 250, 500, 750, 1500$ and $3000$, respectively, assuming 250 trading days per year, while the sliding step is set to 250 points.

III. Methodology

i. Conditional Granger causality index

The linear conditional Granger causality index (CGCI) has been vastly used in real-world applications for the identification of the connectivity pattern of the examined variables. Further, its notion has been recently extended for the formulation of networks in different fields such as functional genomics [2].
A significance test is applied within the estimation procedure of PMIME, which determines the VAR in \( K \) variables and of order \( P \) is fitted to the presumed response time series \( Y_1 \):

\[
y_1(t + 1) = \sum_{j=1}^{P} a_{1j} y_1(t - j + 1) + \sum_{j=1}^{P} a_{2j} y_2(t - j + 1) + \cdots + \sum_{j=1}^{P} a_{Kj} y_K(t - j + 1) + \epsilon_1(t),
\]

where \( a_{ij}, i = 1, \ldots, K, j = 1, \ldots, P \), are the coefficients of the unrestricted model and \( \epsilon_1 \) the residuals with variance \( s^2_{1U} \). Similarly is defined the restricted VAR, but keeping aside \( Y_2 \), i.e. omitting the second sum term (in the right hand side) of Eq. 1. If the sum of the squared residuals of the unrestricted model is significantly larger than those of the restricted one, then \( Y_2 \) Granger causes \( Y_1 \). The CGCI for \( Y_2 \to Y_1 \) conditioning on the remaining variables is then given by the following formula

\[
\text{CGCI}_{Y_2 \to Y_1 | Y_3, \ldots, Y_K} = \ln(s^2_{1R}/s^2_{1U}),
\]

where \( s^2_{1U}, s^2_{1R} \) are the variances of the unrestricted and restricted model residuals, respectively.

The statistical significance of the CGCI is assessed by the \( F \)-test. The considered null hypothesis \( H_0 \) is that the additional coefficients of the unrestricted VAR model are all zero \([5]\), i.e. the variable \( Y_2 \) is not driving \( Y_1 \).

ii. Partial mutual information on mixed embedding

The partial mutual information on mixed embedding (PMIME) \([21]\) is an information causality measure similar to transfer entropy \([20]\), but designed to overcome the problem of the curse of dimensionality by selecting only relevant delayed variables for explaining the response, and therefore it is reliably implemented for time series of many variables (e.g. see \([22]\)).

Let us consider the variables \( Y_1, \ldots, Y_K \) and a common maximum lag value \( L_{max} \). To estimate PMIME, we formulate the optimal mixed embedding vector \( w_t = [w^{Y_1}_t, \ldots, w^{Y_K}_t] \) of selected delays from all variables, i.e. by finding the components from the set \( B = \{y_{1,t}, y_{1,t-1}, \ldots, y_{1,t-L_{max}+1}, y_{K,t}, \ldots, y_{K,t-L_{max}+1}\} \) that best explains the future \( y_{1,t+1} \) of the response variable \( Y_1 \). The estimation procedure of PMIME starts with an empty vector \( w^0_t \). At each step \( j \), a new vector \( w^j_t \) is formed by adding the component \( w^j_t \in B \) to the existing vector \( w^{j-1}_t \) (from step \( j - 1 \)), so that \( w^j_t \) gives the most information about the future of \( Y_1 \) conditioning on the components in \( w^{j-1}_t \)

\[
w^j_t = \arg \max_{w^j_t \in B \setminus w^{j-1}_t} I(y_{1t+1}; w^j_t | w^{j-1}_t),
\]

where \( I(\cdot) \) is the mutual information estimated effectively using the nearest neighbors method \([20]\). The procedure terminates when the conditional mutual information found in Eq. 3 at some step is found not to be significantly larger than zero, implementing an appropriate randomization significance test. Given the obtained \( w_t = [w^{Y_1}_t, \ldots, w^{Y_K}_t] \), the PMIME measure is defined as the fraction of two mutual information terms

\[
\text{PMIME}_{Y_2 \to Y_1 | Y_3, \ldots, Y_K} = \frac{I(y_{1t+1}; w^Y_1, \ldots, w^Y_K)}{I(y_{1t+1}; y_t)}.
\]

quantifying the fraction of information about the response \( Y_1 \) that can be explained only by \( Y_2 \). A significance test is applied within the estimation procedure of PMIME, which determines the optimal number of lagged terms in the mixed embedding vector that best explains the future of the respective response variable. No other significance test is required to determine the existence of causality after estimating the PMIME. It takes values \( \geq 0 \), where 0 indicates the absence of causal links, while there is causality when positive values are obtained. Regarding the type I and II error
rates for the PMIME, simulation results indicated its proper size and high power, especially when nonlinear couplings exist (e.g. see [21,29]). We notice that the free parameter \( L_{\text{max}} \) can take a sufficiently large value without affecting the performance of the measure.

iii. Networks measures

The mathematical representation of a network is the adjacency matrix \( A = a_{ij} \), with \( i, j = 1, \ldots, N \) is a \( N \times N \) matrix for a network of \( N \) nodes, where the component \( a_{ij} \) indicates a directed connection from node \( i \) to \( j \). In weighted networks, the estimated connectivity measure is utilized, while in the binary networks, ones and zeros are entered in the cells of \( A \), based on the presence or absence of an interrelationship.

Depending on the network type, a number of measures have been suggested in order to reveal fundamental connectivity properties [12]. Most measures are typically defined at the node level, e.g. node degree and the clustering coefficient, and their averages over all nodes can be considered as global measures, characterizing the network structure. Moreover, global measures are also specified without reference to each node, such as the betweenness centrality.

Our results are extracted from binary directed networks. The connectivity adjacency matrices are not symmetric, as the estimated causality measures indicate the direction of the causal effects. The entries of the adjacency matrix are equal to one (connection exists), when the significance test for the CGCI gives rejection of \( H_0 \) (\( p\)-value < 0.05) or PMIME > 0, and zero (no connection) otherwise.

In particular for the CGCI, correction for multiple testing is considered and the False Discovery Rate (FDR) criterion for a given significance level \( \alpha \) (we use \( \alpha = 0.05 \)) is applied [3]. First, the \( p \)-values of the \( m \) Fisher tests for all pairs \( Y_i \to Y_j \) are set in ascending order \( p(1) \leq p(2) \leq \ldots \leq p(m) \), where \( m = K(K - 1) \). The rejection of the null hypothesis of no-causality at the significance level \( \alpha \) is decided for all variable pairs for which the \( p \)-value of the corresponding test is less than \( p(k) \), where \( p(k) \) is the largest \( p \)-value for which \( p(k) \leq ka/m \) holds.

Since the PMIME does not bear significance testing, we empirically compensate for multiple testing by making more strict the termination criterion in the mixed embedding vector formation, i.e. setting \( \alpha = 0.01 \) rather than the standard choice of \( \alpha = 0.05 \). In this way, the mixed embedding vector tends to have less components and thus it is more likely to obtain \( \text{PMIME} = 0 \).

The two global network quantities considered in this study, the average degree and the average shortest path length, are calculated on networks from both the CGCI and the PMIME. The degree of a node is the number of the corresponding connections associated with it. The in-degree \( k_{i\text{in}} \) of node \( i \) is the sum of inward connections, while its out-degree \( k_{i\text{out}} \) is the sum of outward ones. Since our adjacency matrix is not symmetric, the in-degree and the out-degree differ. The node degree \( k_i \) is then defined as the sum of the in-degree and the out-degree of a node, i.e. \( k_i = k_{i\text{in}} + k_{i\text{out}} \). The average degree (AD) is the mean of all the node degrees, i.e. \( \text{AD} = 1/N \sum_{i=1}^{N} k_i \).

The average shortest path length (ASPL) or characteristic path length is the mean of all shortest path lengths between any two nodes, where the shortest path length between two nodes is the smallest number of connections in order to get from one node to another one. A distance matrix which contains lengths of shortest paths between all pairs of nodes is formed based on the adjacency matrix and ASPL is calculated as the global mean of the distance matrix, excluding lengths between disconnected nodes (and distances on the main diagonal), i.e. \( \text{ASPL} = \frac{1}{N} \sum_{i,j} L_{i,j} / N(N-1) \), where \( L_{i,j} \) is the shortest path length between node \( i \) and \( j \). (Therefore ASPL= 1 does not indicate a fully connected network). It is a measure of the informational efficiency on a network. In principle, the shorter the average path length, the faster the information diffusion within a network, but exceptions may occur depending on the nature of dynamics that affect the network in times of
important disturbances.

Both network measures are computed from the binary adjacency matrices on the sliding windows.

IV. Results

For the implementation of the CGCI, the order $P$ of the VAR model is set to 1, according to the Bayesian Information Criterion \cite{37}, applied to randomly chosen data windows. For the free parameter $L_{max}$ in PMIME, we chose the value 3 which is sufficiently large in view of the selected VAR order. The number of nearest neighbors is $k = 5$ (standard choice in the estimation of PMIME).

The considered time series lengths $n$ range from 250 to 3000. The setting $n = 250$ may be insufficient for estimating nonlinear causal effects using PMIME. On the other hand, for as large $n$ as 1500 and 3000, it is rather unlikely that a stationary market mechanism is not altered over such long periods (almost 6 and 12 years, respectively). Therefore, we focus merely on results for $n = 500$ and 750.

The mean out-degree for each country, averaged over all segments, shows that US strongly influences the other countries, and this is more pronounced with the CGCI (Fig. 1). Outstanding driving variables, but to a lesser extent compared to the US are Mexico and France based on the CGCI, and Argentina and Denmark based on the PMIME. The nonlinear measure may be capturing broad information composing of both global aggregate shocks and market-specific large events.

The AD seems to be more informative than the ASPL for this data set. With regard to the CGCI, an increase of the AD is observed during the years of the financial crisis of 2008-2009 considering all segment lengths (except for $n = 250$). The AD presents a slight upward trend after 2011, but compared to the AD values during the crisis, seems to stay at the same level as before the crisis (Fig. 2a). A raise of ASPL during the financial crisis is also detected and it is best observed for $n = 500$ and 750 (Fig. 2b). A smaller rise of ASPL during 2002 can be seen. In order to account for the heterogeneity of the time series, we also validated the CGCI for $n = 500$ using a randomization (surrogate) significant test instead of the parametric Fisher test, by randomly time-shifting the driving time series (for details see e.g. \cite{30, 29}). To compensate for not using FDR (since FDR requires a very large number of surrogates), we use 100 surrogates and set the significance level of the test to $\alpha = 0.01$ (for the one-sided test, rejection requires that the CGCI value for the original time series is larger than all 100 CGCI values for the surrogates). Similar results are obtained for

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{The mean out-degree for each country from networks computed over overlapping segments based on the CGCI in (a) and on the PMIME in (b).}
\end{figure}
Figure 2: (a) AD and (b) ASPL of the networks formed at each overlapping segment based on the CGCI. The points denote the center of each respective segment with respect to the real time shown in the x-axis.

The results from both causality measures are robust with respect to the significance level. Although there might be slightly different number of interconnections when changing the significance level, these persist across all segments, so that the changes of the respective financial networks over
the time are not altered.\footnote{Results for different significance levels can be obtained from the authors upon request.}

In Figs. 4 and 5 indicative plots of the financial networks for $n = 500$ are presented for the CGCI and the PMIME, respectively, regarding the following periods: (a) 9/2005-8/2007 (before the financial crisis), (b) 7/2008-6/2010 (including the financial crisis), (c) 6/2010-5/2012 (after the financial crisis), (d) 5/2012-4/2014 (well after the financial crisis). For both causality measures, prior (period a) and posterior (period c) to the period covering the 2008-2009 stock market collapse (period b), the resulting networks (period c) present less dense connectivity. Most importantly, PMIME suggests that several years after the crisis (period d) connectivity among stock markets does not revert to the before-crisis state (period a). In contrast, the linear causality measure CGCI suggests that interconnection returns to its original state shortly after the turbulent period of 2007-2009. Additionally, the strong driving effect of the US on the other stock markets is apparent when visualizing the networks. This is particularly notable for CGCI for which far fewer connections are present than for the PMIME overall.

To stress the similarities and differences between the CGCI and PMIME results, we present in Fig. 6 the network measure profiles for both CGCI and PMIME together and for $n = 500$, a window length found to give reliable information\footnote{Results for different significance levels can be obtained from the authors upon request.}. Left and right axes display the AD for the CGCI and the PMIME in Fig. 6a, respectively, and the same for ASPL in Fig. 6b. The increase of the connectivity during the financial crisis is apparently larger when using the CGCI. This is
particularly notable for CGCI for which far fewer connections appear than for the PMIME overall, so that when more connections occur during the 2008-2009, the change of the AD level is stronger.

Figure 5: As Fig. [a] but for PMIME.

Figure 6: Network measures as a function of time for $n = 500$: (a) AD, (b) ASPL. Left and right axes display the AD in (a) and the ASPL in (b) for the CGCI and the PMIME, respectively.
V. Conclusion

The standard linear Granger causality test has been utilized effectively in various applications. However, the observed anomalies and stylized facts in financial time series explain why, nonlinear indicators of connectivity are able to quantify adequately the dynamics of real networks. Further, each causality index and network measure obey different degrees of sensitivity, and therefore a joint investigation would be the optimal approach in order to formulate reliable interpretations in case of real time series, where we do not know a priori the direction of the causal effects and the formation of the networks.

Comparisons of different causality measures, except PMIME, showed no clear superiority on real time series in neuroscience \[38\] and finance \[49\]. On the other hand, the PMIME turns out to perform best on simulated nonlinear systems, outperforming well-known causality measures such as the standard linear CGCI and the partial transfer entropy, e.g. see \[29\] \[41\]. Additionally, it has been effectively utilized mainly in neurophysiology \[22\] \[19\]. Our empirical findings provide evidence that the PMIME performs well also on financial time series, when applying sliding windows to address non-stationarity.

We note here that future work will focus on the investigation of the performance of both causality measures when time series are very stochastic, containing volatility clusters and outliers. In case they are significantly influenced by the shape of the data, the non-stationarity in the variance and the presence of high peaks, they may produce spurious results.

Regarding the different network measures, as expected, both reveal the qualitative characteristics of the data \[31\]. Most importantly, when combining the PMIME with a network measure, a high level of discrimination of the coupling structures is reached \[18\] \[19\]. Since the goal of the application is not the comparison of various network measures, we estimate two of the most effective network measures: the average degree (AD) and the average shortest path length (ASPL).

In short, through the application of both CGCI and PMIME it comes out that the latter detects richer linkages among the 21 stock indexes. Additionally, both methods indicate a strong influence of the US market on the other markets, while the high values of network measures during the 2007-2009 crisis regime are in accordance with the observed interdependence among the international stock markets. Finally, the nonlinear method indicates that several years after the financial crisis connectivity among stock markets does not revert to the before-crisis states and therefore there is a dense structure between the international stock markets.

The existence of distinct phases in the topology of our network is in line with the empirical findings of works in \[14\] and \[26\], which show that a module structure of the S&P500 network in times of prosperity gives way to a centralized node organization during the 2007-2009 financial crisis. This similarity puts forward the major role of the US market as signal transmitter. In this spirit, the authors in \[16\] argue that the US stock market has predictive power for the performance of other global equity markets. Further, the work in \[43\] suggests that the leading contributor to the recent global instability was the financial sector, while the work in \[6\] underlines the role of the S&P500 as a precursor of changes in the business cycle. Finally, as pointed out by the authors in \[17\], financial markets are highly interconnected, and as such, a shock can activate significant cascading failures in the entire economic system.

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