**On Aggregate Composite Indicators**

Giannis Karagiannis

Professor, Department of Economics, University of Macedonia, 156 Egnatia Str. Thessaloniki, 54006 GREECE; [karagian@uom.gr](mailto:karagian@uom.gr); ++30-2310-891759

ABSTRACT: In this paper it is shown that the un-weighted arithmetic average is the theoretically consistent scheme to aggregate composite indicators, derived from the benefit-of-the-doubt (BoD) model, across decision-making units.

KEYWORDS: Composite Indicators; Benefit-of-the-doubt model; Aggregation across decision-making units

AKNOWLEDGMENT: This research has been co-financed by the European Union (European Social Fund ESF) and Greek national funds through the Operational Program “Education and Lifelong Learning” of the National Strategic Reference Framework (NSRF) – Research Funding Program: THALES-Investing in Knowledge Society through the European Social Fund.

December 2015

On Aggregate Composite Indicators

**1. Introduction**

The Benefit-of-the-Doubt (BoD) is one of the four statistical models recommended by the OECD (2008) for constructing composite indicators, with the other three being the data envelopment analysis (DEA), the principal component/factor analysis, and the unobserved components model.[[1]](#endnote-1) The BoD model has been used for the construction of several composite indicators, including the Human Development Index (e.g., Despotis, 2005), the Quality of Life Indicator (e.g., Morais and Camanho, 2011), the Internal Market Index (e.g., Cherchye *et al*., 2007), the Competitiveness Index (e.g., Bowen and Moesen, 2011), the Digital Access Indicator (e.g., Gaaloul and Khalfallah, 2014), the Technology Achievement Index (e.g., Cherchye *et al*., 2008), the Students’ Evaluation of Teaching indicator (e.g., de Witte and Rogge, 2011), the Health System Performance Index (e.g., Lauer *et al*., 2004), and the Environmental Performance Index (e.g., Zanella, Camanho and Dias, 2013). The main advantage of the BoD model is the endogenously determined and flexible (i.e., specific to each evaluated unit) weights that are used to value the relative importance of the considered performance sub-indicators.

There has been a long and still ongoing discussion in the literature regarding the extent of weights flexibility in the construction of composite indicators. The relevant choices range from the *ad hoc* imposition of equal weights to the flexible weights scheme opted by the BoD model, to common weights, and to restricted weights reflecting the preferences of experts, stakeholders, regulators, policy makers or even researchers. The latter can be accommodated in several ways in the BoD model using different approaches such as assurance regions, cross efficiency, value efficiency analysis, etc.; for more details see Cherchye *et al.* (2007).

The literature however remains silent regarding the aggregation of composite indicators across the evaluated units despite the fact that this turns to be a relevant and important issue as, in most of the cases, we are interesting on the performance of the group that these evaluated units belongs to. For instance, when assessing Students’ Evaluation of Teaching (STE) at the course level, a natural question that arises is how from these we may derive a teacher’s SET scores, especially if some of them lecture several courses and thus had more than one SET scores, i.e., one for each evaluated course. Similar are for example the problems of constructing a country Quality of Life indicator based on city-level results, or of estimating a Human Development Index for developed and less developed countries using country-level scores, or of estimating a continental Technology Achievement Index using country-level indices, or of aggregating Environmental Performance Indices across different cluster/groups of countries.

To answer these and similar questions for other composite indicators we need a theoretically consistent way to aggregate across all or some of the evaluated units. By theoretically consistent way we mean that the resulting aggregate composite indicator will have exactly the same intuitive interpretation as the individual ones. This necessitates the development of an aggregation scheme that is compatible with the BoD model and which involves the choice of aggregation weights as well as the type of average to be used. Morais and Camanho (2011), who are the only ones in the literature that have touched this aggregation question for the BoD model, noticed in passing that the use of the arithmetic average requires the weights for each sub-indicator to be equal across all the evaluated units. In what follows we examine both questions to derive a theoretically consistent aggregation scheme for the BoD model.

In particular, we verify that the un-weighted arithmetic average is indeed the theoretically consistent aggregation scheme for the BoD model without requiring however equal weights for all sub-indicators considered. To prove that we rely on Färe and Zelenyuk (2003) and Färe and Grosskopf (2004) results on aggregate Farrell efficiencies and the fact that the BoD model is essentially a radial, input-oriented DEA model with a single-constant input (see Cook and Kress, 1990; Caporaletti, Dula and Womer, 1999; Despotis, 2005). The latter implies that price independent weights can be used to aggregate the resulting input-oriented efficiency scores, which in the BoD model correspond to the values of the composite indicator, and since it is assumed that all the evaluated units have the same amount of the single input, namely one unit, the aggregation weights are equal to the inverse of the total number of the evaluated units.

The rest of this paper proceeds as follows: our main theoretical results are presented in the next section. In the third section we illustrate in a case study how our theoretical results can be used to obtain a composite research productivity indicator at the department level based on the individual achievements of its faculty members. Concluding remarks follow in the last section.

**2. The Main Results**

The BoD is a model facilitating the (linear) aggregation of a number of quantitative sub-indicators into a single composite indicator when exact knowledge of the weights is not available. In estimating these weights, the BoD model assumes that each decision-making unit (DMU) attaches less (more) importance to those aspects or dimensions of performance, reflected in the different sub-indicators, on which it is demonstrably a weak (strong) performer relative to the other evaluated units. This in turn allows the estimated weights to vary across DMUs, sub-indicators and time. Moreover, assessment is based on best practice (i.e., the observed DMUs’ data) rather than on external references and each evaluated unit’s benchmark is endogenously determined, implying that in general will differ across DMUs.

In the BoD model, the estimated value of each DMU’s composite indicator is equal to the ratio of two metrics, reflecting respectively actual and benchmark overall performance, both of which are given by a (albeit different) weighted sum of the considered sub-indicators. Since the composite indicator is designed to take values in the [0,1] interval, benchmark overall performance attains by construction the maximum value of one (Cherchye *et al.*, 2007). Thus the resulting composite indicator reflects actual overall performance and is a non-decreasing function of sub-indicators implying non-negative weights (non-negativity constraint). In determining actual overall performance, the weights are selected in such a way as to maximize the value of the composite indicator of the evaluated unit. This in turn guarantees that any other set of weights would worsen the ranking position of the evaluated unit. In addition, if the derived set of weights is used by any other evaluated unit it would not result in a value of the composite indicator that is greater than one (normalization constraint).

Taking all these into consideration the BoD model can be written in the form of a linear programming problem as:

where is the composite indicator of the kth DMU, is the ith sub-indicator of the kth DMU, are the (unit-specific) weights to be estimated, is used to index DMUs and to index sub-indicators. Thus the BoD model may be viewed as a tool for aggregating performance sub-indicators without explicit reference to the input(s) used to achieve such performance (Cherchye *et al.*, 2007).[[2]](#endnote-2) Indeed it can be shown (see Cook and Kress, 1990; Caporaletti, Dula and Womer, 1999; Despotis, 2005) that the BoD model is a special case of the Charnes, Cooper and Rhodes (1978) input-oriented constant-returns-to-scale (CRS) DEA model when there is a single constant input that takes the value of one for all DMUs.[[3]](#endnote-3) To verify this notice that the multiplier form of the input-oriented CRS DEA model is given as:

where *x* refers to input quantities, *v* to their relative weights (input multipliers) and is used to index inputs. However, in the case of the BoD model, there is only one input (r=1) and in addition, for all DMUs (). This implies that for all and for every DMU. Under these circumstances (2) is reduced to (1). On the other hand, the envelopment form of the Charnes, Cooper and Rhodes (1978) input-oriented CRS DEA model is given as:[[4]](#endnote-4)

where refers to intensity variables. For the BoD model where r=1 and for all DMUs, (3) simplifies to:

This implies that the value of the composite indicator is in fact equal to the sum of the estimated intensity variables.[[5]](#endnote-5)

To derive a theoretically consistent aggregate composite indicator, using the BoD model, we rely on Färe and Zelenyuk (2003) and Färe and Grosskopf (2004, pp. 115-19) results for aggregating radial efficiency indices, which for the case of a single input are similar to those of Bjurek, Hjalmarsson and Førsund (1990). Their point of departure is Koopmans’ (1957) theorem and its cost and revenue corollaries (see Färe and Grosskopf, 2004, pp. 96-100).[[6]](#endnote-6) For our purposes we use the cost corollary of Koopmans’ (1957) theorem since as we have shown the BoD model is essentially a special case of the input oriented model, which is compatible with cost minimization. Koopmans’ (1957) cost corollary states that aggregate or industry minimum cost is equal to the sum of firms’ minimal costs as long as all the evaluated units face the same input prices. Given this we may define aggregate cost efficiency as:

where *w* refers to input prices and are vectors rather than observations of data consisting of for . In the case where there is only a single input that takes the value of one for all DMUs, (5) simplifies to:

which indicates that aggregate cost efficiency is given by the arithmetic average of individual cost efficiencies.[[7]](#endnote-7) In addition, the presence of a single input implies that there is no input allocation inefficiency and thus the above result applies to technical efficiency, which in terms of model (4) yields:

That is, the aggregate composite indicator equals the simple un-weighted arithmetic average of the estimated individual composite indicators.[[8]](#endnote-8) This rather simple aggregation rule renders the use of the average technical efficiency from a radial input-oriented DEA model with a single constant input an accurate measure of aggregate performance.

The same formula can also be used to aggregate composite indicators within sub-groups of DMUs. For example, aggregation may be conducted in terms of any relevant contextual variable, being size, location, ownership type, organization form, etc. In addition, (7) is not limited to linear programming models but it can be used even if the BoD model is estimated by means of a stochastic frontier formulation. This is so since (7) is a result stemming from Koopmans’ (1957) theorem and it is not related to or induced by the linear programming structure of (4). Last but not least, (7) is readily applicable to other single constant input DEA models apart from BoD. These include *first*, models that use ratio variables (e.g., financial ratios) to evaluate performance all of which are treated as “outputs” (see Yang *et al*., 2014) and *second,* multiple criteria decision making (MCDM) types of DEA models such as those for inventory classification (i.e., Ramanathan, 2006; Zhou and Fan, 2007; Hadi-Vencheh, 2010; Chen, 2011), supplier selection (i.e., Seydel, 2006; Sevkli *et al*., 2007), and service quality evaluation (i.e., Lee and Kim, 2014; Charles and Kumar, 2014).

**3. A Case Study**

In this section we rely on the above results to assess performance at the department level based on the individual achievements of its faculty members by using a rather simple research productivity indicator (RPI) consisting of two sub-indicators, which correspond to two different types of publications. The data refer to the department of Economics at the University of Macedonia, Greece during the period 2000-2006. Each of its 25 faculty members is viewed as a “helmsman” endowed with one unit of the single input that is used to produce two outputs, namely, journal articles and other publications. As “journal articles” we consider all publications in outlets referenced in the Journal of Economic Literature and as “other publications”, papers published in journals not referenced in the Journal of Economic Literature, chapters in books, and edited book volumes.[[9]](#endnote-9) All the relevant data are taken from the University of Macedonia’s *Guide of Published Research Work* (2009).

In Figures 1 and 2 we portray respectively the average annual number of publications and the average number of publications per faculty member over the period 2000-2006. On average, each faculty member published almost one journal article per year during the period 2000-2003 while the average publication record improved a little during the period 2004-2006 and exceeded one (see Figure 1).[[10]](#endnote-10) The corresponding figures for other publications are well below one for the whole period, with a tendency to decline significantly in the last two years. However, both types of publications are unevenly distributed among faculty members (see Figure 2). There are few faculty members with satisfactory achievements in terms of journal article publications (more than two and a half on average per year) and one faculty member with similar performance in terms of other publications but most of the faculty members are more or less around the departmental averages.

Before estimating the model we have to make two adjustments to our data set. As we can see from Figure 2 there were two faculty members that had no journal articles published during the whole period under consideration and only one other publication each. These are expected to cause problems in estimating (1) or (4) as for all but one year they violate the minimum data requirement, namely that of having annually at least one positive output per faculty member. For this reason we have excluded them from the empirical analysis. Nevertheless, the number of zero entries was still present due to the irregular frequency of publications between adjacent years. For this reason we decided to use three-years moving averages running from 2000-02 to 2004-06. In addition, we disregarded from the assessment the three new faculty members that joined the department in 2003 and for whom there were no data for the whole period under consideration. Thus we end up with a balanced panel data of 20 DMUs (i.e., faculty members) over five three-years rolling windows from 2000-02 to 2004-06.

The empirical results concerning the estimates of the RPI are presented in Figures 3 and 4. In Figure 3 we report the average RPI for each faculty member over the period under consideration. From there we can see that there are two peers with an average over time RPI score of almost one and in addition, two other faculty members who are well above average. However, the great majority (16 faculty members) had an average RPI score of less than 0.4, very close to the departmental average. There are also three faculty members with comparatively low achievements and RPI scores below 0.2. Nevertheless, it seems that on the average there is a tendency for performance improvement over time. On the other hand, the average annual RPI scores, which according to our theoretical results are equal to aggregate (departmental) RPI scores, are rather low and in the range of 0.36 to 0.43 (see Figure 4) indicating the relative heterogeneity across faculty members. Thus the department has achieved less than half of its potential output with its current composition of faculty members. That is, the current faculty members could have “produced” on average more than twice as much research output as they did during the period 2000-2006.

**4. Concluding Remarks**

In this paper we show that the simple arithmetic average is the theoretically consistent aggregation rule for the BoD model and thus, for the input-oriented single constant input DEA model. This is so as the value of the composite indicator is equal to the degree of technical efficiency in a radial, input-oriented DEA model with a single constant input and several sub-indicators as outputs. As a result, the value of the aggregate composite indicator is equal to arithmetic average of DMUs’ composite indicators derived from the BoD model as aggregate efficiency is equal to average efficiency in a radial, input-oriented DEA model with a single constant input. This rather simple aggregation rule can be applied without requiring the relative weights of each sub-indicator to be the same across DMUs, as Morais and Camanho (2011) suggested. Moreover, the proposed aggregation rule holds regardless of whether weight restrictions are imposed or not when estimating the composite indicator for each DMU. The only thing that is required is to have a single unitary input.

Figure 1: Average Number of Publications per Year, 2000-2006



2000 2001 2002 2003 2004 2005 2006

Figure 2: Average Number of Publications per Faculty Member, 2000-2006

Figure 3: Average RPI Score per Faculty Member, 2000-2006

Figure 4: Aggregate (Departmental) RPI Scores, 2000-2006

**References**

Bjurek, H., Hjalmarsson, L. and F.R. Førsund. Deterministic Parametric and Non-parametric Estimation of Efficiency in Service Production: A Comparison, *Journal of Econometrics*, 1990, 46(**1-2**), 213-27.

Bowen, H.P. and W. Moesen. Composite Competitiveness Indicators with Endogenous versus Predetermined Weights: An Application to the World Economic Forum’s Global Competiveness Index, *Competitiveness Review: An International Business Journal*, 2011, 21(**2**), 129-51.

Caporaletti, L.E., Dula, J.H. and N.K. Womer. Performance Evaluation Based on Multiple Attributes with Nonparametric Frontiers, *Omega*, 1999, 27(**6**), 637-45.

Charles, V. and M. Kumar. Satisficing Data Envelopment Analysis: An Application to SERVQUAL Efficiency, *Measurement*, 2014, 51, 71-80.

Charnes, A., Cooper, W.W. and E. Rhodes. Measuring the Efficiency of Decision Making Units, *European Journal of Operational Research*, 1978, 2(**6**), 429-44.

Chen, J.X. Peer-estimation for Multiple Criteria ABC Inventory Classification, *Computers and Operations Research*, 2011, 38(**12**), 1784-91.

Cherchye, L., Lovell, C.A.K., Moesen, W. and T. van Puyenbroeck. One Market, One Number? A Composite Indicator Assessment of EU Internal Market Dynamics, *European Economic Review*, 2007, 51(**3**), 749-79.

Cherchye, L., Moesen, W., Rogge, N. and T. van Puyenbroeck. An Introduction to “Benefit of the Doubt” Composite Indicators, *Social Indicators Research*, 2007, 82(**1**), 111-45.

Cherchye, L., Moesen, W., Rogge, N., van Puyenbroeck, T., Saisana, M., Saltelli, A., Liska, R. and S. Tarantola. Creating Composite Indicators with DEA and Robustness Analysis: The Case of the Technology Achievement Index, *Journal of Operational Research Society*, 2008, 59(**2**), 239-53.

Cook, W.D. and M. Kress. A Data Envelopment Model for Aggregating Preference Rankings, *Management Science*, 1990, 36(**11**), 1302-10.

Despotis, D.K. A Reassessment of the Human Development Index via Data Envelopment Analysis, *Journal of Operational Research Society*, 2005, 56(**8**), 969-80.

de Witte, K. and N. Rogge. Accounting for Exogenous Influences in Performance Evaluations of Teachers, *Economics of Education Review*, 2011, 30(**4**), 641-53.

Färe, R. and S. Grosskopf. *New Directions: Efficiency and Productivity*, Kluwer Academic Publishers, 2004, Massachusetts.

Färe, R. and V. Zelenyuk. On Aggregate Farrell Efficiencies, *European Journal of Operational Research*, 2003, 146(**3**), 615-20.

Gaaloul, H. and S. Khalfallah. Application of the “Benefit-of-the-Doubt” Approach for the Construciton of a Digital Access Indicator: A Revaluation of the “Digital Access Index”, *Social Indicators Research*, 2014, 118(**1**), 45-56.

Hadi-Venchech, A. An Improvement to Multiple Criteria ABC Inventory Classification, *European Journal of Operational Research*, 2010, 201(**3**), 962-65.

Koopmans, T.C. *Three Essays on the State of Economic Science*. McGraw-Hill, New York, 1957.

Lauer, J. A., C. A. K. Lovell, C. J. L. Murray and D. B. Evans. World Health System Performance Revisited: The Impact of Varying the Relative Importance of Health System Goals, *BMC Health Services Research,* 2004, 4(**1**). <http://www.biomedcentral.com/1472-6963-4-19>

Lee, H. and C. Kim. Benchmarking of Service Quality with Data Envelopment Analysis, *Expert Systems with Applications*, 2014, 41(**8**), 3761-68.

Liu, W.B., Zhang, D.Q., Meng, W., Li, X.X. and F. Xu. A Study of DEA Models without Explicit Inputs, *Omega*, 2011, 39, 472-80.

Lovell, C.A.K. and J.T. Pastor. Radial DEA Models without Inputs or without Outputs, *European Journal of Operational Research*, 1999, 118(**1**), 46-51.

Morais, P. and A.S. Camanho. Evaluation of Performance of European Cities with the Aim to Promote Quality of Life Improvements, *Omega*, 2011, 39(**4**), 398-409.

OECD. *Handbook on Constructing Composite Indicators: Methodology and User Guide*, 2008, Paris.

Ramanathan, R. ABC Inventory Classification with Multiple-criteria Using Weighted Linear Optimization, *Computers and Operations Research*, 2006, 33(**3**), 695-700.

Sevkli, M., Koh, S.C.L., Zaim, S., Dermibag, M. and E. Tatoglu. An Application of Data Envelopment Analytic Hierarchy Process for Supplier Selection: A Case Study of BEKO in Turkey, *International Journal of Production Research*, 2007, 45(**9**), 1973-2003.

Seydel, J. Data Envelopment Analysis for Decision Support, *Industrial Management and Data Systems*, 2006, 106, 81-95.

University of Macedonia. *Guide of Published Research Work*, 2009, Thessaloniki.

Yang, G., W. Shen, D. Zhang and W. Liu. Extended Utility and DEA Models Without Explicit Input, *Journal of the Operational Research Society*, 2014, 65(**8**), 1212-20.

Zanella, A., Camanho, A.S. and T.G. Dias. Benchmarking Countries’ Environmental Performance, *Journal of Operational Research Society*, 2013, 64(**3**), 426-38.

Zhou, P. and L. Fan. A Note on Multi-criteria ABC Inventory Classification Using Weighted Linear Optimization, *European Journal of Operational Research*, 2007, 182(**3**), 1488-91.

**Footnotes**

1. The OECD (2008) Handbook also mentioned another set of methods that can be used to estimate composite indicators. These are referred to as participatory methods and are the budget allocation process, the analytic hierarchy process and the conjoint analysis. [↑](#endnote-ref-1)
2. This has been interpreted as a “helmsman”, namely a collective decision-making apparatus underlying every unit being assessed, that attempts to steer all of the sub-indicators towards their maximum levels (Lovell and Pastor, 1999). [↑](#endnote-ref-2)
3. More on the radial DEA models with a single constant input can be found in Lovell and Pastor (1999), Caporaletti, Dula and Womer (1999), and Liu *et al*. (2011). Notice also that the unitary input DEA models are equivalent to DEA models without explicit inputs. [↑](#endnote-ref-3)
4. Since the radial, single constant input DEA model exhibits constant returns to scale (Lovell and Pastor, 1999) the composite indicator can also be estimated by using the output-oriented formulation of the model. [↑](#endnote-ref-4)
5. If the kth DMU is efficient then with all other for . In turn (4) implies that . [↑](#endnote-ref-5)
6. Koopmans’ (1957) theorem states that industry maximum profit is equal to the sum of firms’ maximal profits as long as all firms face the same input and output prices. [↑](#endnote-ref-6)
7. In general however industry cost efficiency is equal to the share-weighting sum of DMU’s cost efficiencies, with the aggregation weights being their share to industry total (observed) cost (see e.g., Färe and Grosskopf, 2004, pp. 118-19). [↑](#endnote-ref-7)
8. Alternatively one can use the general cost efficiency aggregation rule as stated in footnote 7 and then notice that the input shares (i.e., in this case simplify to 1/K as all DMUs have the same amount (namely, one unit) of the single input and face the same input price. [↑](#endnote-ref-8)
9. We disregard books (textbooks or others) because in their majority have not gone through a formal referee process. [↑](#endnote-ref-9)
10. This increase is related to three new faculty members that jointed the department in 2003 and their journal article publication record was above the departmental average. [↑](#endnote-ref-10)