Financial Markets during highly anxious time: Multifractal fluctuations in asset returns

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Abstract

Building on the notion, that systems and in particular complex systems such as stock exchange markets reveal their structure better when they're under stress we analyze the multifractal character and non linear properties of four major stock market indices during financial meltdowns by means of the multifractal detrended fluctuation analysis (MF-DFA). The three distinct financial crises under investigation are the Black Monday, the Dot-Com and the Great Recession. Scaling and Hurst exponents are derived as well as the singularity spectra. The results show that all indices exhibit strong multifractal properties. The complexity of the markets is higher under the Black Monday event revealed by the width of the singularity spectrum and the higher α_0 parameter.

Key Words: Financial Crisis**,** Stock Market, Multifractality

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1.0 Introduction

For the last thirty years, the world has witnessed a plethora of financial crises causing major economic turbulence and distress across the financial markets, while disrupting world trade of goods and services. Amongst those crises, three are very memorable, initiated in the heart of the U.S. economy: The 1987 crash, became known as Black Monday crisis, the 1999 Dot-Com bubble and lastly the 2008 Great Recession triggered in the real estate market and spread to world financial markets, and to Europe in the form of a banking and sovereign crisis. These extreme events prompted researchers to focus especially on the origins of the crises and also in providing helpful insights into underlying complexity of the system. By identifying the statistical properties of the financial market events under stress, researchers hope to enhance our understanding of the mechanisms determining the dynamics and to develop diagnostic models in predicting financial meltdowns.

A large body of work has dealt with the non-linear properties of a complex dynamical stock market systems and with the stylized facts of long-memory, fat tails, power law correlations and multifractal signature [1-7]. Building on the notion, that systems and in particular complex systems such as stock exchange markets reveal their structure better when they're under stress [8] our aim is to study the stock market structure and their statistical properties, during specific periods in crisis. Four major stock exchange general indices are chosen, namely the S&P 500, the Nasdaq Composite index, the FTSE 100 of London Market Exchange and the Nikkei 225. Two indices are selected from the U.S. financial markets, one from the U.K. and one from Japan.

The effects of extreme events on financial markets have received modest attention thus far. Drozdz et al. [9] analyzed 30 companies of DAX index, within a period of 11 years, and observed that draw downs are always accompanied by a sizeable separation of one strong eigenstate of the correlation matrix which, at the same time, reduces the variance of the noise state. The draw ups on the other hand turn out to be more competitive. In this case the dynamics spread more uniformly over the eigenstates of the correlation matrix resulting in an increase of the total information entropy. Therefore, increases are more competitive, and less collective, and thus more nonlinear correlated than decreases. Oswiecimka et al. [10] analyzed periods of upward and downward trends of the DAX index and show that multifractal spectra are broader during bull market than during its bear phase where bear market is more persistent than the bull market irrespective of the sign of fluctuations[1]. Siokis [12] analyzed extreme economic events of troubled European Economies needed financial assistance. With the use of the of the national stock market index as a forward looking indicator they try to reveal if this financial assistance had helped the economies to come back in track. More recently Cao and Zhang [12] focused on the comparative analysis of extreme values in the Chinese and American stock markets and report that the range of extreme value of Dow Jones Industrial Average is smaller than that of Shanghai composite index, and the extreme value of Dow Jones Industrial Average is more time clustering. Also based on the multifractal detrended cross-correlation analysis algorithm they find that extreme events have nothing to do with the cross-correlation between the Chinese and American stock markets. On the foreign exchange market, Schmitt et al. [12] considered the scaling and multifractal properties of the Chinese currency against the US dollar and the euro. They have shown that both foreign exchange rates possess multifractal properties. By dividing the time series in to several samples to reveal any statistical differences, they report a change in the power spectra during the pegged system, and also, after the end of the pegged system, when the value of the exchange rate was steadily decreasing, fluctuations was still scaling with multifractal exponents very far from the previous ones. Wang et al. [13] investigated the yuan exchange rate index after China's exchange rate system reform on the 21st July 2005. By dividing the time series into two parts according to the change in the yuan exchange rate regime in July 2008, they compare the statistical properties of the yuan exchange rate index during these two periods. They report that the change in China's exchange rate regime gave rise to the different multifractal properties of the yuan exchange rate index in these two periods, and thus has an effect on the effective exchange rate of the yuan after the exchange rate reform on the 21st July 2005. Lastly, Oh et al. [14] analyzed the multifractal spectra of various currencies with respect to the U.S. dollar, during the turbulent period from 1991 until 2005. They discover that the return time series show multifractal spectrum features for all four cases and especially after the Asian crisis, some currencies experienced a significant increase in multifractality.

[1] An interesting presentation of complex systems as well as summarizing the findings of these papers and the most up to date review of related issues can be found in J. Kwapien and S.Drozdz [11]

2.0 Data and Methodology

We examine the non linear features of the S&P 500, Nasdaq Composite, FTSE 100 and Nikkei 225 indices during three major crises: the Black Monday, Dot-Com and the Great Recession. Since a financial crisis has its own time length and magnitude, which makes it hard to mark the beginning and ending of the crisis we utilize the TED index in an attempt to identify the peak of the crisis. The highest point reached in the TED index is taken as the peak of the crisis and consequently as the middle point of each sample. The TED index is the difference between three-month LIBOR and three-month Treasury bills and used by the investors as an indicator of perceived risk in the financial markets and in the economy in general[2]. As figure 1 depicts, the peak for the Black Monday event took place in Oct 20th 1987 where TED spread reached 302 basis points (bps). For the Dot-com crisis the highest level achieved on Oct 8th,1999 with spread reaching 150 bps, and, for the Great Recession crisis the Oct. 10th, 2008 with 458 bps respectively. Strangely enough all crises occurred during the month of October. The sample for each event consists of 1320 trading days ≈ almost 5 years divided equally before and after the peak of the TED index. The data are taken from Bloomberg consisting of daily returns of the stock market indices calculated as

$$r_t = \ln p_t - \ln p_{t-1} \qquad (1)$$

where $p(t)$ is the price of the index on day t and r is the rate of return. The multifractal concept is used as a feature of the financial complex systems and we're investigating the multifractal properties of the indices based on the periods of high financial stress as depicted by the TED spread indicator. Although numerous procedures were produced in calculating multifractality, we utilize the Multifractal-Detrended Fluctuation Analysis (MF-DFA) developed by Kantelhardt et al. (2002) [17], which reduces noise effects, removes local trends and avoids spurious detection of correlations that are artifacts of nonstationarities in the time series.

[2] LIBOR is the interbank lending rate and as it rises relative to risk-free Treasury bills, the perceived credit risk is increasing and liquidity events take place. As the value of the index increases so the risk involved in the financial markets and investors allocate their capital to safer markets.

The multifractal generalization of the MF-DFA procedure can be briefly sketched as follows. The MF-DFA operates on the time series $x(k)$, where $k = 1, 2, ..., N$ and N is the length of the series. We assume that $x(k)$ are increments of a random walk process around the average $\langle x \rangle$ and the profile is given by the integration of the signal

$$Y(i) = \sum_{k=1}^{N} [x(k) - \langle x \rangle], \quad i = 1,..., N. \tag{2}$$

Next, the time series $Y(i)$ is divided into $N_S \equiv \mathrm{int}(N/s)$ non-overlapping segments of equal length s, starting from both beginning and the end of the time series. Each segment v has it own local trend that can be approximated by least-squares fitting of the series. Then we determine the variance

$$F^2(v,s) \equiv \frac{1}{s}\sum_{i=1}^{s} \{Y[(v-1)s+1] - y_v(i)\}^2 \tag{3}$$

for each segment v, $v = 1, ..., N_S$ and

$$F^2(v,s) \equiv \frac{1}{s}\sum_{i=1}^{s} \{Y[N-(v-N_s)s+1] - y_v(i)\}^2 \tag{4}$$

for $v = N_S + 1, ..., 2N_S$. Here, $y_v(i)$ is the fitting line in segment v. Then, detrend the series and average over all segments to obtain the qth order fluctuation function

$$F_q(s) \equiv \{\frac{1}{2N_s}\sum_{v=1}^{2N_s}[F^2(v,s)]^{q/2}\}^{1/q} \tag{5}$$

The property of $F_q(s)$ is that for a signal with fractal properties, it reveals power-law scaling within a significant range of s

$$F_q(s) \propto s^{h(q)} \tag{6}$$

and the variable q can take any real value other than zero.

In general the exponent $h(q)$ will depend on q. For stationary time series, $h(2)$ is the well-defined Hurst exponent H and thus, $h(2)$ is the generalized Hurst exponent. Multifractal (MF) scaling exponent $\tau(q)$ is related to $h(q)$ through

$$\tau(q) = qh(q) - D_f \qquad (7)$$

where Df is the fractal dimension of a geometric support of the multifractal measure and $Df = 1$. The exponent $\tau(q)$ represents the temporal structure of the time series as a function of the various moments q, or τ reflects the scale-dependence of smaller fluctuations for negative values of q, and larger fluctuations for positive values of q. If $\tau(q)$ increases nonlinear with q, then the series is multifractal.

3.0 Multifractal Results

We calculate the Hurst exponent $h(q)$ determined by fits in the regime $10 \leq s \leq N/4$. In figure 2, (panels a.I, b.I, c.I) when $q \in [-5,5]$ increases from −5 to 5, the Hurst exponent , $h(q)$ decreases characterizing typical multifractality form in time series[3] [4]. In addition the richness in multifractality is associated with high variability of $h(q)$ and the degree can be quantified as

$$\Delta h = h(q_{\min}) - h(q_{\max}) \qquad (8)$$

The region of small $|q|$ produces a linear behavior with narrow slope while at higher/lower values of q, the generalized Hurst exponent $h(q)$ increases/decreases with a much higher slope. The difference between $h(q)$ at q = 0 and $q = \pm 5$ varies between 9% and 37% with most of the cases to be greater than 15%, while the difference w.r.t $q = \pm 2$ is between 3% and 18%. Based on the differences associated with the variability of $h(q)$, one could strongly argue that the indices for all events exhibit strong multfractal behavior. Furthermore, (**table 1)** the mean values and the standard deviations of the $h(q)$ curve, along with the calculated values of the maximum $h(q)$ achieved with respect to the mean value, point not to monofractal but

[3] The determination of the scaling range is important since the scaling behavior could be hidden by nonlinearity. Therefore, a narrow range of q values prevents a potential distortion of the results by the so called "freezing" phenomenon which manifests itself in the linearization of $h(q)$ at large q as observed for a log-normal cascade[18-20].

[4] Due to limited space we do not present results of the type of multifractality of the series at hand. But in all cases the width of the singularity spectrum decreases after shuffling the series indicating that the source of multifratality comes from broad probability density function and long term correlations.

rather to multifractal behavior. For the U.S. indices, the strongest variability, according to the standard variation and the *Δmax* values, is recorded during the Black Monday period, while for the FTSE and Nikkei Indices is recorded during the Great Recession event.

Next, we examine the multifractal properties by converting q and $\tau(q)$ to α and $f(\alpha)$ by a Legendre transform as

$$f(\alpha) \equiv aq - \tau(q), \qquad a \equiv \frac{d\tau(q)}{dq} \tag{9}$$

where $f(\alpha)$ is the fractal dimension of the time series. The width of the $f(\alpha)$ spectra, defined as the difference between the α_{max} and the α_{min} conventionally quantifies the degree of multifractality, while $f(\alpha)$ tells how frequently events with α scaling exponent occur. **Figure 2**, (panels a.II, b.II and c.II) depicts the singularity spectra of the indices for the three events. All indices during the Black Monday event exhibit wider degree of multifractality, meaning higher heterogeneity in the scaling indices, and strong asymmetry. The asymmetry in all spectra $f(\alpha)$, with the left width to be larger than the right side, i.e. $\alpha_0 - \alpha_{min} > \alpha_0 - \alpha_{max}$, indicates the dominance of large fluctuations in the distribution. On the other hand, as table 2 depicts, the singularity spectra of the Nasdaq and Nikkei indices for the Dot-Com crisis exhibit larger right width (values>1) signifying that small fluctuations have greater impact on the distribution. Based on the information provided by **table 2**, the Black Monday event had much greater impact on the indices as the higher value of α_0 ensures larger complexity and richer process in structure. Therefore, a signal with a high value of α_0, a wide range of fractal exponent and a right-skewed shape is more complex than one with less or the opposite characteristics (21).

Next, in an attempt to measure the impact of large negative/ positive percentage changes on the multifractal singularity spectra we replace the percentage changes greater than $\lambda\sigma$, where σ is the standard deviation of the time series and λ=2,5,6 , with percentage changes equal to their relative standard deviation, i.e. $|r_t| > \lambda\sigma$ to $|r_t| = \lambda\sigma$. Other researchers (16, 22) have replaced the large magnitudes with random numbers drawn from a normal distribution or with the returns re-sampled randomly from the return series, with returns lower than the standard deviation. By means of this truncated method comparability of different singularity spectra seems possible.

Figure 3 depicts the spectra of the indices for the three crises. For all indices, the singularity spectrum increases as the threshold λ increases from 2σ to 6σ. It is apparent that for the crises of Black Monday and Great Recession the increase of the spectrum comes from the left hand side which means that as the threshold level decreases higher price changes are diminishing. Only in the case of the Dot-com crisis the changes come from the right hand side. Therefore, with the above exercise we show that the multifractality is stronger when high values of returns in the series are present.

3.1 Time Depended Analysis

We now turn into a time dependent analysis trying to capture the dynamic aspect of the stock market indices and calculate the temporal evolution of multifractality. A sliding window of specific events methodology is employed, keeping the time interval of each window constant to 1320 days~5 years worth of trading days, hence ensuring results comparability, and by including the temporal dynamics of the variability pattern. The shift of the window is set to 25 trading days ~1month. We calculate the generalized Hurst exponent as well as the maximum α_0 and the Width parameters by fitting the local multifractal spectrum by a quadratic function.

Figure 4 shows the variation in time of two parameters, α_0, the value where the spectrum has its maximum, $f(\alpha)=1$ and the W, the width of the spectrum[5]. Both parameters exhibit strong variability, a sign of a strong variability of the multifractal character of the times series, while one of the most interesting feature is in all indices there is a significant time pattern change before and after each crisis. In other words, the parameter α_0 is significantly higher during the crisis, indicating that there is a higher local fractal dimension, than the other time periods. After the crisis the parameter decreases in value. A very striking feature, especially for the Black Monday plot, is the significant drop of the α-value as soon as the impact of the crash vanishes away. This performance is also evident by the width (W) of the spectrum, measuring the multifractality degree. Especially for the S&P index, the width parameter is at the highest level during the Black Monday event and decreases sharply after the crisis. It could be explained by the sharp increase of the α_{min} and the simultaneous decrease of the α_{max} as shown in figure 5. For better understanding the

[5] We also depict a 5-month moving average line.

changes in width of the spectra, figure 5 shows the maximum and the minimum α-values (α_{max} and α_{min}) of all four indices. The difference between α_{max} and α_{min} is much broader during the crises, especially for the Black Monday event, indicating a change from homogeneous to heterogeneous dynamics. The minimum α-value increases immediately as we're getting out of the crisis, which is more pronounced for the S&P and to a lesser degree for the Nasdaq index. For the Dot-Com crisis the impact is significantly stronger for U.S. and FTSE Indices. Worth noting is that FTSE index seems not to be greatly impacted-given the degree of multifractality-from the sterling forced withdrawal from the European Exchange Rate Mechanism (ERM) after the exchange rate crisis of September 1992.

4.0 Conclusion

We investigate scaling and multifractal properties of stock market indices under three recent "extreme" financial events: the Black Monday, the Dot-com and the Great Recession events. The multifractal detrended fluctuation analysis (MF-DFA) shows that stock market indices exhibit consistent increments of multifractality, for all events. All indices (except the Nikkei index) under the Black Monday event are characterized by a wider multifractality spectrum and asymmetry compared to the other two events. It seems that the Black Monday event had much greater impact on the indices pertaining larger complexity and richer process in structure.

Next in an attempt to measure the impact of large negative/ positive percentage changes on the multifractality singularity spectra we find that for the Black Monday and Great Recession crises, the singularity spectra increase from the left hand side, meaning that as the threshold level decreases higher price changes are diminishing.

Lastly with the assistance of time varying analysis of the evolution of multifractality characterized by the $\alpha 0$, width, symmetry and the two set of values of a-max and a-min, we conclude that financial crises such as the three under investigation are characterized by a dynamical change from heterogeneous to homogeneous as soon as the crisis fades away. This is translated into a loss of multifractality.

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Figure 1: The TED spread index. Three events are showing in circle. The Black Monday crisis, the Dot-com and the Great Recession crises.

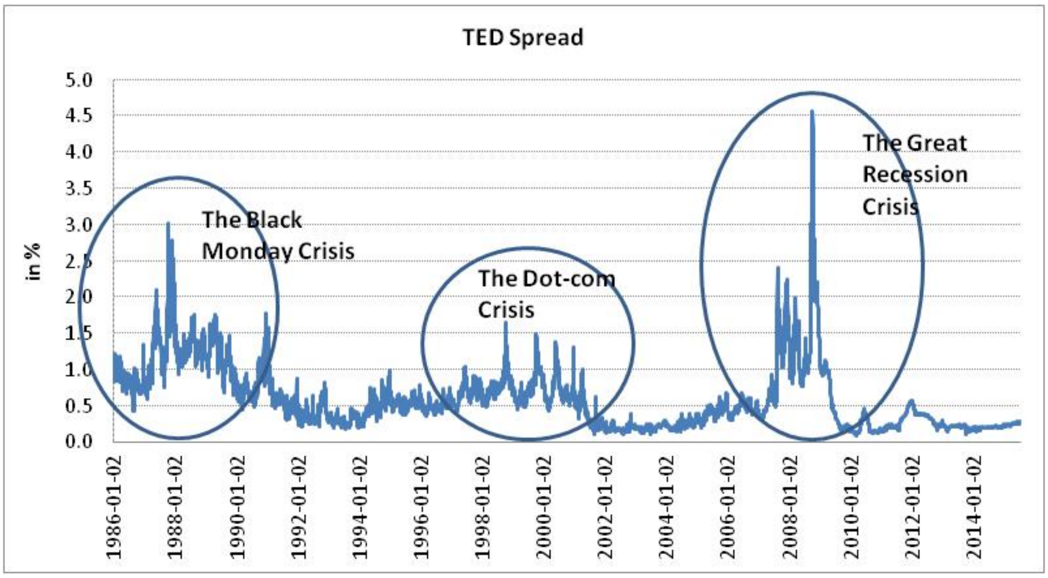

Figure 2. The Generalized Hurst Exponent *h(q)* and Fractal dimensions. Panels (a), (b) and (c) , (I) show the Generalized Hurst exponent, h(q) as a function of q, and(II) the fractal dimensions *f(α)* for Black Monday, Dot-Com and Great Recession crises, respectively.

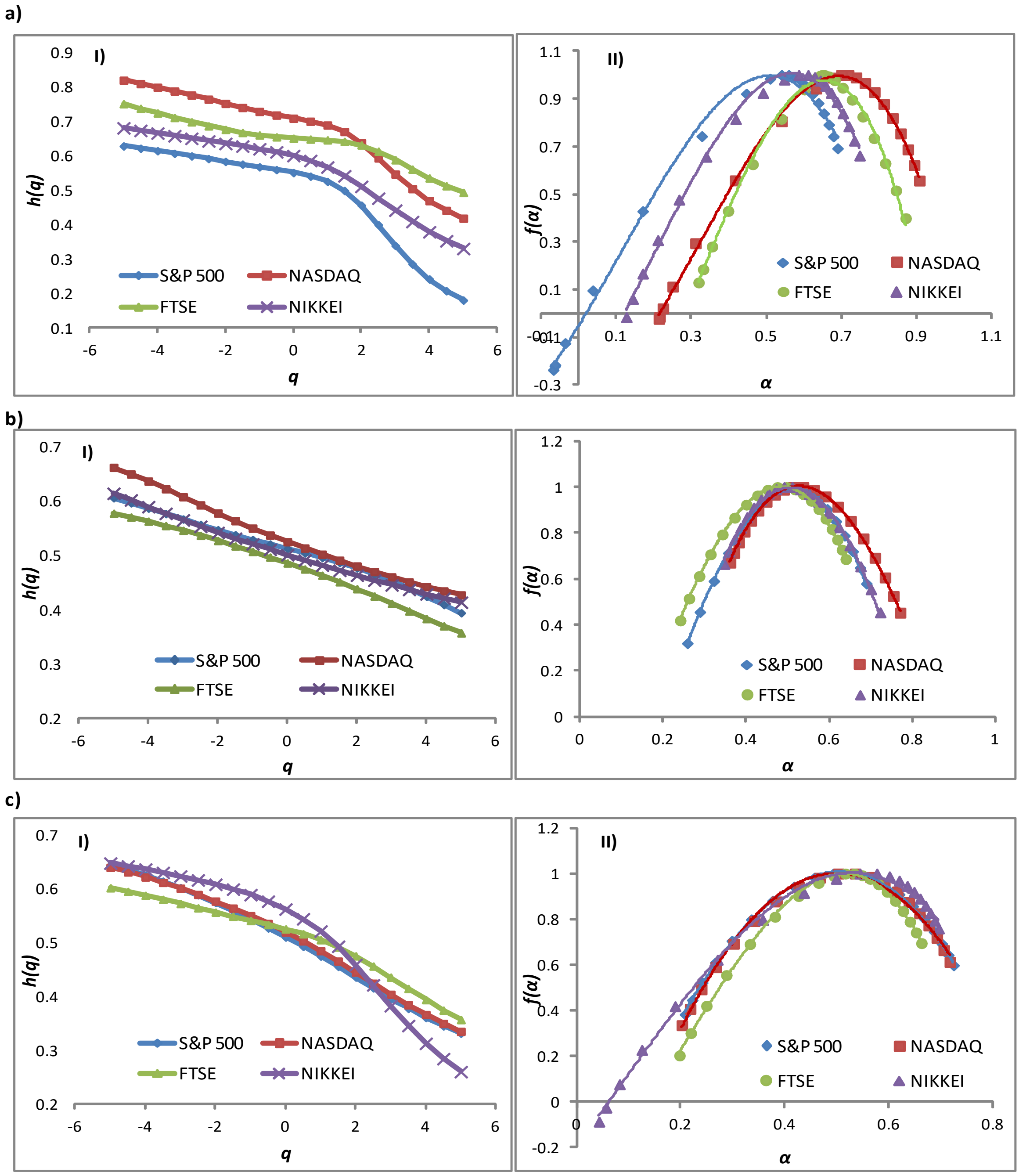

Figure 3: Multifractal spectra of the stock market indices a) S&P, b) Nasdaq, c) FTSE, d) Nikkei, for Black Monday (I), Dot-Com(II) and Great Recession(III) events based on threshold level, where $|r_t|$ **>λσ,** λ=2,5,6, are replaced by the respective λσ value.

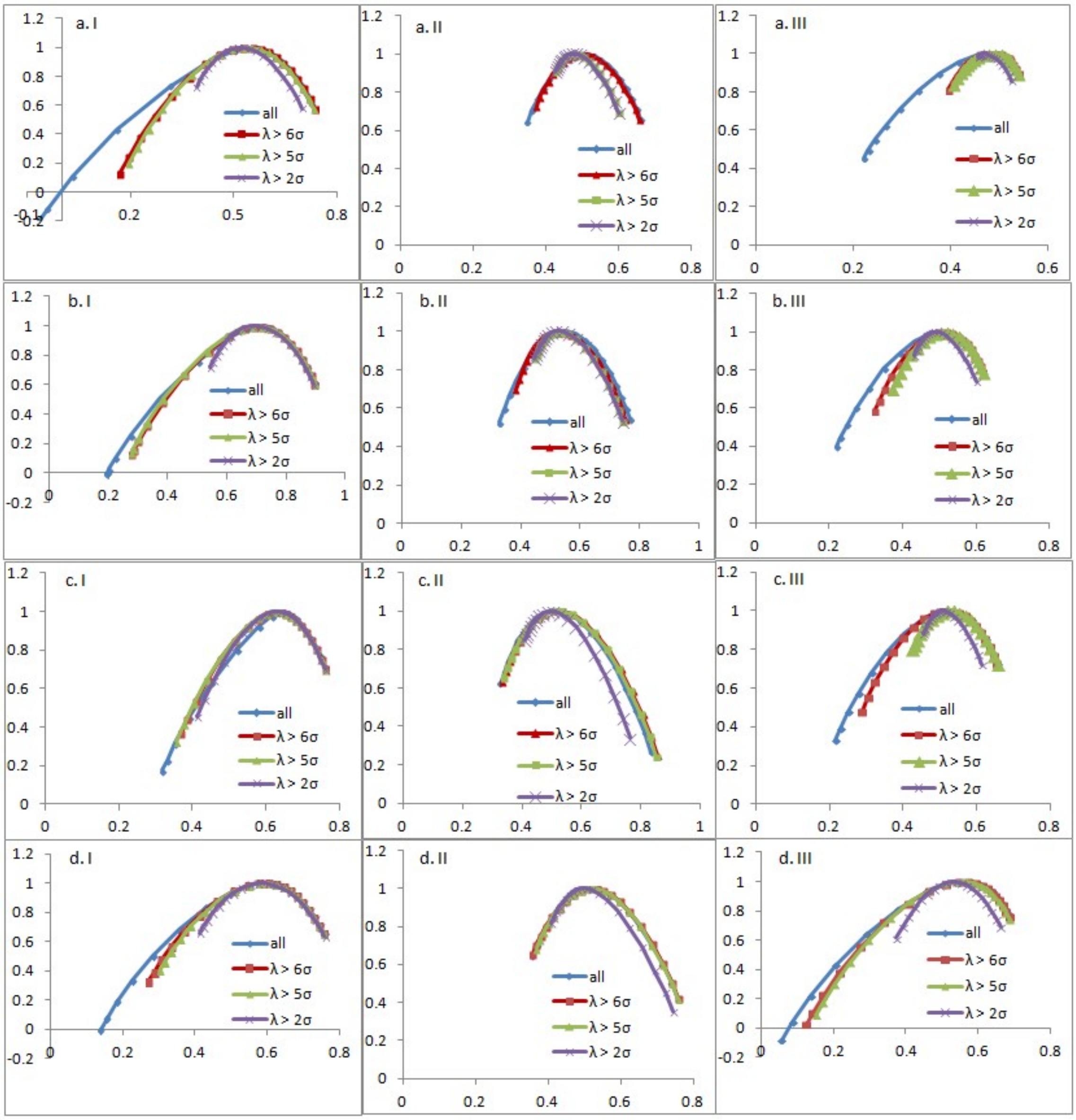

Figure 4: The parameters α_0 and W, quantifying the multifractality of a) S&P 500, b) Nasdaq, c) FTSE 100 and d) Nikkei indices as a function of time, with a 5-month moving average shown by the solid line.

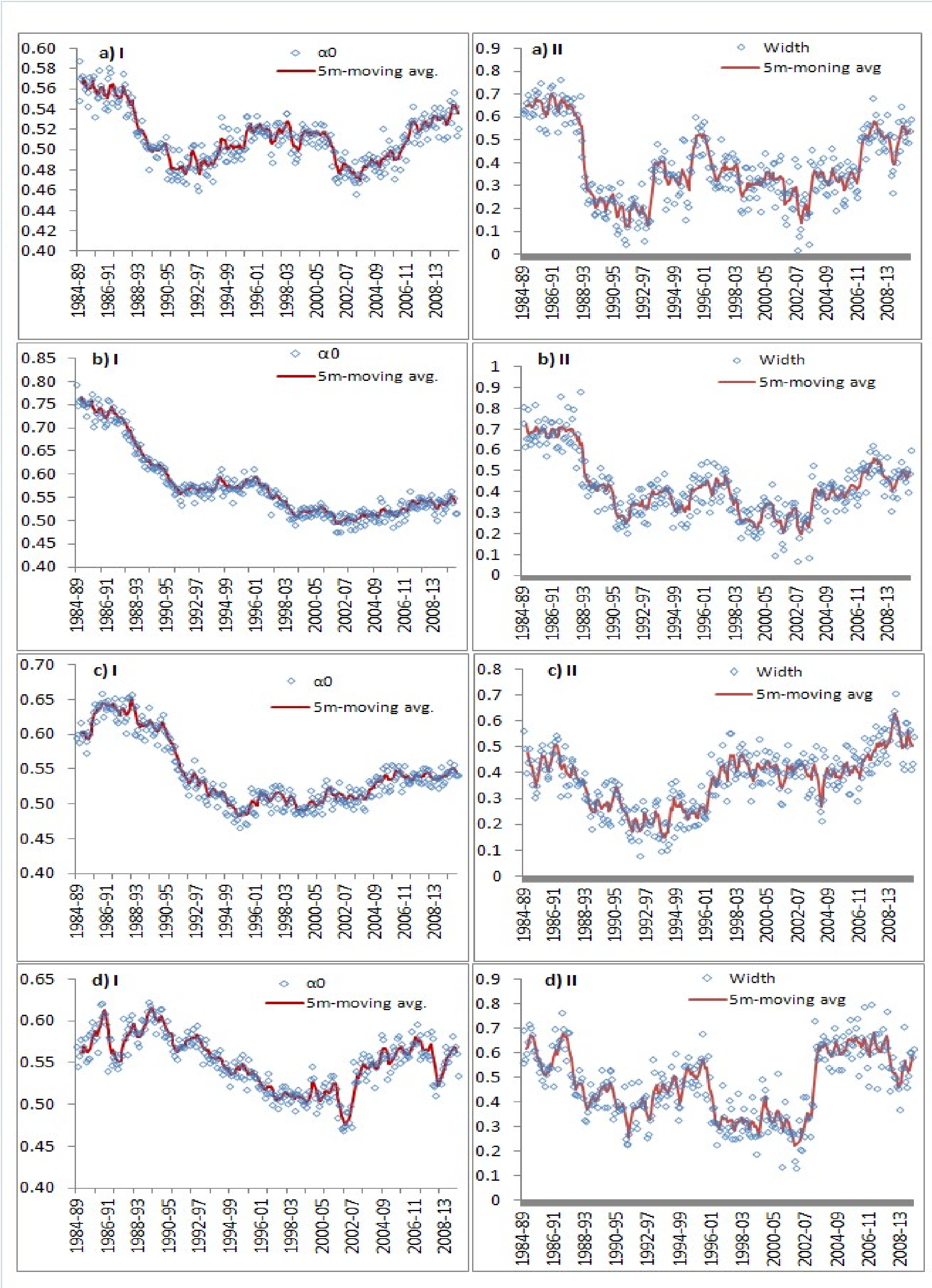

Figure 5: The maximum and minimum values of the parameter α, for a) S&P 500, b) Nasdaq Composite, c) FTSE 100 and D) Nikkei 225 indices. The solid line depicts a 5-month moving average.

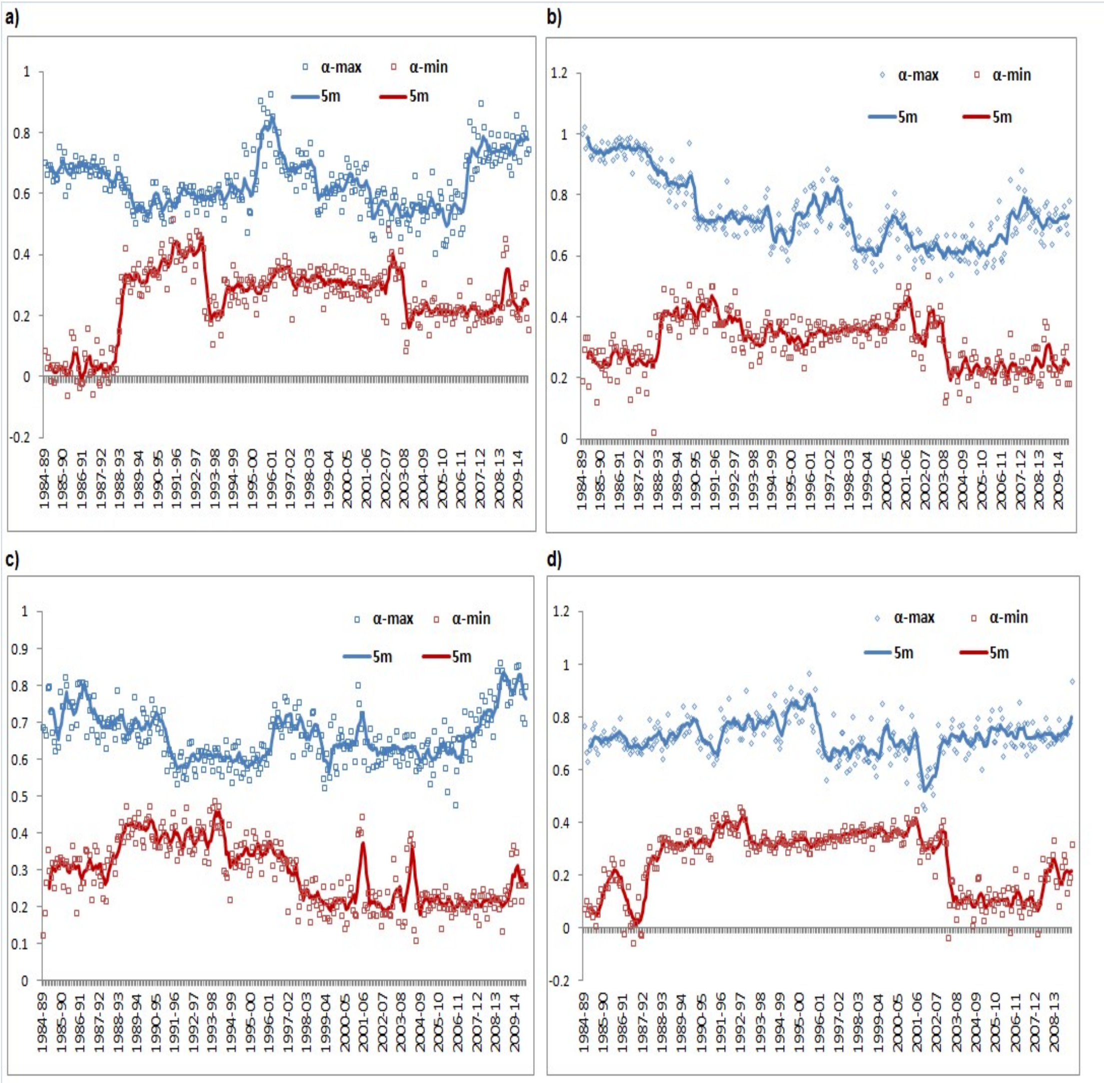

Table 1. The mean, standard deviation for h(q) in each event $\Delta\max \equiv |\max\{h(q)\} - h\langle(q)\rangle| / h\langle(q)\rangle$, and, the H2 values.

Event	*<h(q)>*	*σ(h(q))*	Δ_{max}	H2
Black Monday				
S&P 500	0.484	0.147	0.628	0.456
NASDAQ	0.670	0.126	0.376	0.638
FTSE 100	0.642	0.071	0.232	0.631
NIKKEI	0.555	0.113	0.405	0.511
Dot-Com				
S&P 500	0.509	0.062	0.225	0.476
NASDAQ	0.533	0.075	0.241	0.480
FTSE 100	0.479	0.069	0.241	0.439
NIKKEI	0.505	0.062	0.254	0.463
Great Recession				
S&P 500	0.500	0.102	0.337	0.436
NASDAQ	0.505	0.099	0.336	0.445
FTSE 100	0.506	0.075	0.295	0.475
NIKKEI	0.512	0.128	0.492	0.459

Table 2. Parameters, α_0, W and S derived from the respective multifractal spectrum.

Event	α0	W	S
Black Monday			
S&P 500	0.559	0.753	0.210
NASDAQ	0.718	0.694	0.376
FTSE 100	0.654	0.552	0.645
NIKKEI	0.611	0.621	0.284
Dot-Com			
S&P 500	0.521	0.431	0.644
NASDAQ	0.537	0.409	1.330
FTSE 100	0.497	0.398	0.561
NIKKEI	0.511	0.376	1.294
Great Recession			
S&P 500	0.527	0.516	0.619
NASDAQ	0.535	0.515	0.546
FTSE 100	0.533	0.465	0.385
NIKKEI	0.577	0.652	0.223