A survey on variable neighborhood search methods for supply network inventory

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Abstract Over the past two decades, reverse logistics and closed-loop supply chain networks have gained substantial interest in business and academia. The dynamic lot sizing problem with product returns and recovery is one of the most extensively researched topics in inventory control literature. Several interesting generalizations of this optimization problem have lately emerged that include the multi-product case, the case with capacity constraints, and others. In this chapter, we present recent successful applications of Variable Neighborhood Search (VNS) for the efficient solution of such problems, review the state-of-the-art solution methods, and also discuss some future research directions.

1 Introduction

One of the most important issues that should be tackled by a company is the determination of the replenishment policy of the different goods (spare parts, raw material, components or finished goods) involved in the supply chain network. Manufacturers have started to integrate remanufacturing facilities into the regular production environment. Due to the increasing attention on the sustainability in industry, inventory control problems have been studied in the field of closed-loop systems. This problem becomes more complex when the demand varies with time through a finite planning horizon with T periods. Several interesting generalizations of this optimization problem have lately emerged.

Inventory theory constitutes a large part of supply chain optimization. Advances in inventory theory include, among others, advances in lot sizing methods [30], forecasting methods for product demands, mathematical models for optimal inventory

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levels, and others. Inventory optimization problems [6] arise in several cases as a part of modern supply chain networks. Inventory optimization problems can be classified into deterministic or stochastic (e.g., if the demand is assumed to be time varying and not constant). A deterministic optimization problem consists in minimizing a real-valued objective function $f(x)$, where x is a feasible solution belonging in the feasible set X which is part of the solution space S. Depending whether S is a finite but large set or equal to R^n, we define either a discrete or a continuous optimization problem, respectively. The optimality of a solution x^* belonging in X follows when $f(x^*)$ is no larger than $f(x)$ for all feasible solutions x.

The majority of real-world inventory optimization problems are $\mathcal{NP}$-hard problems and are computationally difficult to solve. This is the reason why, several researchers use heuristic and/or metaheuristic solution methods in order to compute optimal or suboptimal solutions in a short amount of time. Metaheuristics (based on local search procedures) try to continue the search by other means after finding the first local minimum.

This survey paper differentiates itself from prior research works in that, to date, the majority of VNS survey papers [4, 9, 10] have mainly focused on the methodological aspects. However, the interest of the researchers working with the VNS methodology has significantly grown [27] and the number of published works on modern supply chain network problems using VNS, has also significantly increased in the literature. Due to this reason, this work aims to present recent advances of VNS applications to various inventory optimization problems such as the Economic lot sizing problem with product returns and recovery (ELSR), the Multi-product dynamic lot-sizing problem with remanufacturing (MDLSRP), the Uncapacitated multilevel lot-sizing problems (MLLSs), the Capacitated lot-sizing problems (CLSPs), the Inventory-routing problems (IRPs), and the Location-Inventory-routing problems (LIRPs).

This work is structured as follows. Section 2 present a short overview of the VNS methodology. Section 3 present recent VNS contributions in supply network inventory. Finally, Sections 4 and 5 draw up concluding remarks and describe some future research directions, respectively.

2 Variable Neighborhood Search

Variable Neighborhood Search (VNS) is a metaheuristic based on systematic changes in the neighborhood structure. In spite of its simplicity, it has provided a huge number of successful applications in a wide range of areas. The success of VNS is based on three simple facts: i) A local minimum w.r.t. one neighborhood structure is not necessarily so for another, ii) a global minimum is a local minimum w.r.t. all possible neighborhood structure, and iii) for many problems, local minima w.r.t. one or several N_k are relatively close to each other. Let N_k, $(k = 1, \ldots, k_{max})$, a finite set of pre-selected neighborhood structures. $N_k(x)$ the set of solutions in the k^{th} neighborhood of x. Most local search heuristics use only one neighborhood structure, i.e., k_{max} = 1. An optimal solution x_{opt} (or global minimum) is a feasible solution where a minimum is reached. We call x' a local minimum with respect to N_k (w.r.t. N_k for short), if there is no feasible solution $x \in N_k(x')$ such that $f(x) < f(x_0)$.

VNS has shown to be very successful for solving hard either combinatorial or global optimization problems. Moreover, a large number of VNS variants have been already proposed in the literature, e.g., Reduced VNS (RVNS), Basic VNS (BVNS), Variable Neighborhood Descent (VND), General VNS (GVNS), Skewed VNS (SVNS), Nested VNS (NVNS), Variable Neighborhood Decomposition Search (VNDS). All these VNS variants exhibit differences either in the intensification and/or in the diversification phase. Generally speaking, a VNS method alternately executes the improvement phase (intensification phase) and the shaking procedure (diversification phase) together with the neighborhood change step until some stopping criterion is fulfilled.

3 VNS contributions to supply chain optimization problems

3.1 VNS for the ELSR

The economic lot sizing problem with product returns and recovery has the following problem parameters: a number of periods (T), setup/holding costs (k_M, k_R, h_M, h_R), the demand for each time period ($D(t)$) and also, the number of returned items per period and product ($R(t)$) that can be completely remanufactured and sold as new. The ELSR aims to compute the number of new $x_M(t)$ and/or remanufactured products $x_R(t)$ and also, the inventory level of serviceable items $y_M(t)$ and/or items that can be remanufactured $y_R(t)$, per period. The objective of the problem is to minimize the total cost due to manufacturing and/or remanufacturing setup cost, and also, holding cost for the serviceable items and/or recoverable items per unit time.

In [26] several new neighborhoods for this combinatorial optimization problem were presented and an efficient local search method for exploring them was described. The computational results obtained on an established set of benchmark problems with 6,480 instances shown that the VNS metaheuristic algorithm outperformed previous state-of-the-art heuristic methods from the literature.

3.2 VNS for the MDLSRP

An interesting generalization occurs by considering multiple products and thus now there is a number of periods (T), a number of different products (K), setup/holding costs ($k_M(k)$, $k_R(k)$, $h_M(k)$, $h_R(k)$), the demand for each time period and product ($D(k,t)$) and also the number of returned items per period and product ($R(k,t)$) that can be completely remanufactured and sold as new. The MDLSRP aims to compute the number of new $x_M(k,t)$ and/or remanufactured products $x_R(k,t)$ and also, the inventory level of serviceable items $y_M(k,t)$ and/or items that can be remanufactured

$y_R(k,t)$, per period. The objective of the problem is again to minimize the total cost due to manufacturing and/or remanufacturing setup cost, and also, holding cost for the serviceable items and/or recoverable items per unit time.

In [24, 25], a subset of the most efficient neighborhoods for the ELSR were appropriately generalized for the MDLSRP, since the local search part now requires more time. The authors proposed and solved a new, larger, benchmark set with 300 products, 52 periods (publicly available from: `http://users.uom.gr/~sifalera/benchmarks.html`). Furthermore, the VNS approach outperformed the state-of-the-art Gurobi optimizer. Also, the authors have implemented a simple constructive Heuristic for the MDLSRP, where the total demand is fulfilled by a single lot (without remanufacturing units) for each product in the first period, in order to find an initial solution to the MDLSRP. This approach did not depend on any other, commercial or not, solver for either computing a starting solution or an intermediate computation. Thus, it was a self-contained solver without any link to other callable library API.

3.3 VNS for MLLSs

Xiao et al. [32] in 2011 presented a reduced variable neighborhood search algorithm for solving uncapacitated MultiLevel Lot-sizing (MLLS) problems. Such uncapacitated MLLS problems constitute a basic form of several other extended versions of the MLLS problem under various constraints. The authors used the modified Randomized Cumulative Wagner–Whitin (RCWW) method for computing initial solutions and also applied six other techniques in order to enhance the efficiency of the searching process. The same authors [33] in 2012, investigated the use of three indexes, i.e., distance, changing range, and changing level, for characterizing the neighborhood of the incumbent solution in three aspects, and improving the efficacy of a VNS algorithm.

Seeanner et al. [23] in 2013 combined the variable neighborhood decomposition search method and the MIP-based Fix & Optimize heuristic for solving problems that typically arise in the consumer goods industry. Such problems require to simultaneously determine lot-sizes and sequences for parallel lines and multiple production stages. The authors defined different neighborhood structures based on product, (micro)period, production line decompositions, and bill-of-materials.

Furthermore, Xiao et al. [34] in 2014, developed the Ancestors Depth-first Traversal Search (ADTS), which is an effective local search method and can be embedded in the VNS framework for significant improvements of the solution quality. Additionally, the authors proposed a new formulation for the MLLS problem which allowed them to devise a Recursive Accumulation of the Cost Reduction (RACR) method for fast updating of the objective function after a change of an incumbent solution.

3.4 VNS for CLSPs

Almada-Loboa and James [1] in 2010, addressed the multi-item capacitated lot-sizing and scheduling problem with sequence-dependent setup times and costs, which is an extension of the Wagner–Whitin model taking also into account capacity constraints. The authors applied a construction heuristic that contains five, forward and backward, steps that are able to efficiently compute feasible solutions and and then employed a VNS method for the improvement of this solution for this $\mathcal{NP}$-hard problem. Three different types of moves (neighborhoods) were used in the improvement phase of the VNS: i) an insertion move which takes a job and inserts it before another job in the sequence, ii) a fractional insertion move which furthermore allows the option of splitting a job into two jobs, and iii) a swap move which takes two jobs in the sequence and swaps their location in the sequence. According to the authors, if sufficient time was available, they found that the proposed VNS method outperformed the tabu search method for the same problem.

Chen [5] in 2015, proposed a new fix-and-optimize (FO) approach for two dynamic multi-level capacitated lot sizing problems (MLCLSP). Based on their FO method, the authors also develop a VNS approach for the MLCLSP without setup carryover, which can further improve the solution obtained by the FO, by diversifying the search space. Also, numerical experiments on benchmark instances showed that, their VNS approach outperforms other previous FO approaches.

Recently, Liuxi et al. [17] in 2017, motivated from a real-world steel enterprise problem, proposed a piecewise linear approximation method for solving two nonlinear mathematical models for stochastic multi-item capacitated lot-sizing problems with and without setup carryovers. Moreover, the authors presented an integrative approach combining a FO method and VNS (FO-VNS), for the efficient solution of these approximation models.

3.5 VNS for IRPs

Lejeune [16] in 2006, considered a three-stage supply chain which combines an inventory–production–distribution plan over a multi-period horizon. The authors proposed a VNDS method where, the decomposition scheme is related to the relaxation of the integrality constraints on the integer decision variables. The authors validated their VNDS method using a real-industrial life problem faced by a large North American chemical company and outperformed other commercial MIP solvers.

Liu & Lee [19] in 2011, applied a hybrid method that integrates VNS and Tabu Search, in order to address the inventory routing problem with time windows. Later in 2012, Liu & Chen [18] proposed a VNS for simultaneously solving an integrated model for the inventory routing and scheduling problem (IRSP) rather than sequentially and separately taking the inventory, routing, and scheduling decisions, based on the minimal cost criterion. Also in 2012, Popović et al. [22] applied a Randomized Variable Neighborhood Descent (RVND) method for minimizing the total cost of a multi-product multi-period Inventory Routing Problem (IRP) in fuel delivery with multi-compartment homogeneous vehicles, with a deterministic consumption that varies with each petrol station and each fuel type. Thus, this IRP resulted from the combination of a Vehicle Routing Problem (VRP) in fuel delivery, including petrol stations inventory management. Their proposed stochastic RVND method was based on random changes of the neighborhoods rather than changing of neighborhoods in a deterministic way as for example in the VND method.

Mjirda et al. [20] in 2014 presented a two-phase VNS metaheuristic method for solving a multi-product inventory routing problem. The authors applied a VNS method in the first phase, for solving a capacitated VRP at each period and finding an initial solution without taking into account the inventory. Afterward, an iterative improvement of the initial solution while minimizing both the transportation and inventory costs is taking place in the second phase. The proposed VNS used four neighborhood structures while the shaking part alternated between only two neighborhood structures.

Hasni et al. [11] in 2017, proposed a GVNS method for minimizing both the transportation and inventory holding costs of a multi-product deterministic version of the IRP, with a feet of heterogeneous vehicles and known demands. Their GVNS approach utilized six different neighborhoods, i.e., insertion moves in the same route or between two routes from the same period, or between two routes in different periods, and exchange moves within the same route, or between two routes from the same period, or between two routes from different periods. The authors demonstrated the efficiency of their approach against a branch & cut algorithm, using benchmark instances ranging from 10 to 100 customers, and with a maximum of seven periods, five vehicles, and five products.

More recently Gruler et al. [8] in 2018, presented a simulation–optimization (simheuristic) approach integrating Monte Carlo Simulation (MCS) within a VNS framework to solve the multiperiod IRP with stochastic customer demands. The authors aimed at computing the optimal refill policies for each customer–period combination. Thus, a constructive procedure using MCS techniques was applied for generating an initial solution, and afterwards it was iteratively improved by the VNS framework.

3.6 VNS for LIRPs

Turan et al. [29] in 2017 tackled a two-echelon inventory routing problem with uncertain demand, where perishable products are distributed from one central warehouse to multiple retail stores. The authors proposed a VNS procedure, which was extended with a dynamic programming sub-routine to handle inventory allocation. The intensification (local search) phase consisted of randomly selecting one of the following operators (neighborhoods): Intra-route Two-Opt, Intra-route Swap, and Intra-route Relocate. The diversification (shaking) phase, was similarly based on a random selection of one of the following operators: Interroute Two-opt, Inter-route Swap, and Inter-route Relocate.

Recently, Karakostas et al. [14] in 2018 proposed two BVNS metaheuristic approaches for the solution of a more realistic LIRP version which integrates economic and environmental decisions. This new variant is denoted as the Pollution LIRP (PLIRP). The same authors [15] proposed in 2019 a GVNS metaheuristic method for the solution of the Location Inventory Routing with Distribution Outsourcing (LIRPDO). This new problem variant represents a more realistic situation, in which a company is required to outsource its distribution operation, (e.g., in cases where it cannot afford vehicles acquisition). Therefore, more decisions, such as the most efficient allocation of the company's opened depots to the selected providers and the selection of the proper vehicles providers, needs to be made.

3.7 VNS for supplier selection and performance evaluation in inventory control

Since traditional methods often fail to identify the factors affecting supplier selection and evaluation, Vahdani et al. [3] in 2017 proposed a novel hybrid approach based on continuous GVNS and Least Square-Support Vector Machine (LS-SVM). The authors applied GVNS to improve the generalization performance in searching the support vector machine model, which is used to build the non-linear relationship between selection, evaluation criteria and performance rating of suppliers. Additionally, a real-world case study of a supplier selection and evaluation problem from the cosmetics industry is shown for demonstrating the performance of the proposed integrated model.

3.8 VNS for sustainable order allocation (inventory) and sustainable supply chain

Govindan et al. [7] in 2015 applied a novel multi-objective hybrid optimization approach in order to simultaneously minimize the total costs and environmental effects, occurred by strategically designing a sustainable forward supply chain network (SCN) with stochastic demand. The authors combined an Adaptive Multi-Objective Variable Neighborhood Search (AMOVNS) method as a local search together with an adapted multi-objective electromagnetism mechanism algorithm. Their proposed supply chain network was composed of five echelons including suppliers classified in different classes, plants, distribution centers, and cross-docks. Furthermore, a real-world case study from an automobile industry was used to demonstrate the applicability of this approach.

3.9 VNS for EOQ-based inventory problems

Recently, Đorđević et al. [21] in 2017 presented a BVNS method for tackling the static time-continuous multiproduct Economic Order Quantity (EOQ) based inventory management problem with the storage space constraints. Also, the authors examined the efficiency of their approach using a preliminary computational analysis with randomly generated instances with the same number of products.

3.10 VNS for joint replenishment problems

Wang et al. [31] in 2018, approximately obtained lower and upper bounds of Joint Replenishment and Delivery problem (JRD)problem using a bounding procedure. Afterward, they applied a VNS method utilizing several new neighborhood structures, with the bounding method to solve the JRD problem. This hybrid VNS was shown to perform better than the best known heuristic and metaheuristic approaches for JRD in terms of accuracy, using randomly generated examples.

3.11 VNS for vendor managed inventory problems

Hemmelmayr et al. [12] in 2010, applied a VNS approach for optimal planning delivery routes for the supply of blood products to hospitals by a blood bank. Their method allowed for low cost and robust plans that hedge against the natural uncertainty associated with blood product usage at hospitals. The authors used 15 neighborhoods that were defined from combinations of three operators, i.e., move, cross-exchange, and change-combination.

3.12 VNS for multi-criteria inventory classification

Kaabi et al. [13] in 2015 proposed an Automatic Learning Method (ALM) that combined the benefits of Artificial Intelligence (AI) and MultiCriteria Decision Making (MCDM) techniques. The authors applied a continuous VNS to infer the criteria weights of the ABC analysis used in inventory management, in order to produce a classification of items that minimizes the inventory cost. Furthermore, the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method was used to compute the items score, using an aggregation function combining the item evaluations on the different criteria and their weights. Also, the performance of the proposed ALM with respect to some others ABC inventory classification models, was evaluated based on a benchmark data set of 47 items from an Hospital Respiratory Therapy Unit.

4 Conclusions

This chapter, presented recent successful applications of VNS for the efficient solution of inventory optimization problems that often arise in modern closed-loop supply chain networks. A thorough literature review on several state-of-the-art VNS solution methods is intended as a reference for both scientists and practitioners in these disciplines. Table 1 summarizes several research works and provides a taxonomy based on different problem variants and their characteristics.

Table 1 Key literature VNS contributions on supply network inventory problems

Reference	Problem type	Demand type	Commodity type	Level type	Capacitated version	Closed-loop supply chain	VNS variant
[26]	Inventory	Deterministic	Single	Single	-	✓	GVNS
[24]	Inventory	Deterministic	Multiple	Single	-	✓	VND
[25]	Inventory	Deterministic	Multiple	Single	-	✓	GVNS
[32]	Inventory	Deterministic	Multiple	Multilevel	-	-	RVNS
[23]	Inventory	Deterministic	Multiple	Multilevel	Product lines	-	VNDS
[33]	Inventory	Deterministic	Multiple	Multilevel	-	-	Iterated VNS
[34]	Inventory	Deterministic	Multiple	Multilevel	-	-	VNS modification
[1]	Inventory	Deterministic	Multiple	Single	Product lines	-	GVNS
[5]	Inventory	Deterministic	Multiple	Multilevel	Product lines	-	Hybrid VNS with fix-and-optimize heuristic
[17]	Inventory	Stochastic	Multiple	Single	Product lines	-	Hybrid VNS with fix-and-optimize heuristic
[18]	Inventory / Routing / Scheduling	Deterministic	Single	Single	Vehicles	-	VNS modification
[19]	Inventory Routing	Deterministic	Single	Single	Vehicles	-	Hybrid VNS with Tabu Search
[16]	Inventory Routing	Deterministic	Single	Single	Vehicles	-	VNDS
[22]	Inventory / Routing	Deterministic	Multiple	Single	Reservoir for fuel	-	RVND
[20]	Inventory Routing	Deterministic	Multiple	Single	Vehicles	-	VND, VNS
[11]	Inventory Routing	Deterministic	Multiple	Single	Customers, Vehicles	-	GVNS
[8]	Inventory Routing	Stochastic	Multiple	Single	Retail centers, Vehicles	-	Simheuristic VNS with Monte Carlo
[29]	Location / Inventory / Routing	Stochastic	Multiple	Single	-	-	Hybrid VNS with dynamic programming
[14]	Location / Inventory / Routing	Deterministic	Single	Single	Depots, Vehicles	-	BVNS
[15]	Location / Inventory / Routing	Deterministic	Single	Single	Depots, Vehicles	-	GVNS

5 Future research guidelines

Skouri et al. [28] in 2018 recently presented a survey of open problems in green supply chain modeling and optimization with carbon emission targets. Nowadays, research on pollutant emissions management is very important and a large number of, computationally difficult, inventory lot sizing optimization problems arise, that could improve the effectiveness of carbon management in the supply chain.

In order to tackle such hard optimization problems, a future research direction consists in exploiting parallelism in multi-core CPU+GPU systems. Antoniadis & Sifaleras [2] in 2017 proposed a hybrid parallel VNS method using a CPU-GPU system via OpenMP and OpenACC on a set of recent benchmark problem instances for the multi-product dynamic lot sizing problem with product returns and recovery, which appears in reverse logistics and is known to be $\mathcal{NP}$-hard.

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