Calculating the Torque on the Shafts of the Antikythera Mechanism to determine the Location of the Driving Gear

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Abstract

The Antikythera Mechanism (AM) is considered by the scientific community to be the first computer ever built. Its operation is not entirely known, derived only by the existing fragments of the mechanism that have been found and studied. From the early comprehensive study of the mechanism by Derek De Solla Price to the Antikythera Research Project, which performed the CT scans of the AM, the input of the mechanism was assumed to be a to construct an exact copy of the mechanism found out that considerable torque should be applied to the presumed drive crown, in ordered to operate the mechanism. This paper tries to compute the transmission of torque from any of the mechanism's gears to the rest of the gear-train, to determine the best possible location of a driving gear or crown as the case may be. The computation takes into account an estimated rotation friction on each shaft that will result in further reducing the transmitted torque in the case of long gear trains.

Keywords: Antikythera mechanism, gear trains, torque computation, torque transfer, mechanism dynamics

1. Introduction

The Antikythera Mechanism (AM) is an extraordinary instrument from antiquity, containing more than 30 precision gears and more than7 dials. It was discovered off the shore of the island of Antikythera (hence the name) during the Easter of 1900, in a shipwreck lying at the bottom of the sea. It is dated at around the second half of the second century 150 BC [6]. The technology used to create the AM is so amazing that a mechanism of even half the gears of the AM has not been known form more that 1200 years afterwards. Derek de Solla Price was the first to extensively study the Antikythera mechanism for 25 years. He published his now famous papers of 1959 [3] and 1974 [4]. Although some researchers before Price presented various hypotheses about the operation of the mechanism, it was Price who was able to almost accurately count the teeth of each gear based on the radiographs of Dr. Karakalos, and give the first detailed description of the mechanism's operation. Although he misinterpreted the function of the gear-train that is now known to have emulated the Hipparchos lunar anomalies, his work is considered the most important in the explanation and appreciation of the mechanism.

De Solla Price specifies the input gear of the mechanism as Gear A, "a massive crown or concrete wheel" as he calls it, whose function is to transmit motion from a handle to the rest of the mechanism's gear [4].

Based on the CT scans performed by the Antikythera Research Project [6], we now know that some of De Solla Price's conjecturers, like the existence of a differential turntable, are not correct. Instead of a differential turntable, the AM contained an elaborate arrangement of 4 gears (named e5-k1-k2-e6) that are used to emulate the Hipparchos anomalies of the lunar orbit. These gears contain a pin and slot interlock, which produces a sinusoidal variation of a linear rotation. However, the ARP still considers the gear a1 to be the input gear, as shown in the schematic given in the paper [6].

Price had a physical model of the Antikythera mechanism built, according to the mechanical drawings he drafted, and in order to have a better conceptual idea of the operation of the mechanism. This model, with Price in the background, is shown in Figure 1 on the left. Neither his model, nor most of the physical models that were created subsequently operate flawlesly. This is attributed to the fact that the gears' bearings cannot be clearly distinguished in the CT scans and radiographs and, for the inner gears to be clearly visible, most reconstructions make the mounting plates less robust than they should

Figure 1: Physical Models of the mechanism, Price's (left) and Wright's (right)

be [5]. The first physical reconstruction that operated flawlessly was the one made by Michael Wright [11] but it deviates from Price's of AMP designs by adding some mounting plates. Wright's model is shown in Figure 1 on the right. As can be seen, both of these models have an input handle or knob on the side of the mechanism driving crown A1.

The most accurate models are built by K. Efstathiou and the Antikythera Investigation Group of Thessaloniki [5]. The first models built were not operational and it was then determined that trying to operate the mechanism from gear A1 presented an enormous stress on the shafts of the gears. M.M. Vicentini created a not very accurate physical model of the AM [10]. Although he used modern involute gear profiles (instead of the original triangular teeth) he still could not make the mechanism to operate smoothly when driven from the A1 crown. Instead, he added an additional three gear driving-train to drive the mechanism from gear d2.

The ingenuity of the gear placement and the economy of accuracy of gear teeth leads to the conclusion that the technician who created the AM must have put the driving gear at the most efficient position. The rest of the paper tries to identify this position as follows. Section 2 assumes that the driving

gear is any of the gears (including A1) that are part of the established model, in order to identify the shaft receiving the minimum torque in the complete gear-train.

As the length of the gear-train vary depending on the location of the driving gear, the next section assumes a constant reduction of the transmitted torque due to friction at each shaft, to determine the optimum torque transfer. Negative torque at any of the shafts would indicate that the mechanism cannot possibly be driven from the examined input gear. Section 4 adds a fixed-size driving gear at each of the model's gears, similar to the crown A1 to identify the location that gives the maximum minimum torque on any of the shafts. The last section discusses the results and proposes possible locations for the drive gear.

2. Torque Calculation

A sectional diagram of the AM gears is given in Figure 2, which is based on the findings of the ARP [6]. It should be noted that the lunar phase gearing differs from the corresponding gearing in [6] and conforms to the gearing used in the models constructed by Efstathiou [5]. Although these small gears have not been found on any of the AM fragments, the small crown on the lunar pointer faces away from the sun shaft (as found on the fragment) instead of towards the shaft as depicted in the ARP paper schematic. However, as the intermediate gears of the lunar phase have not been found, this part of the gear-train is not included in the subsequent analysis.

The operation proceeds as follows: The presumed input, crown A_1, drives gear B_1 which is the Sun gear. The sun pointer is attached to this shaft and shows the date in the upper (front) dial. On the same shaft is gear B_2 which drives the rest of the mechanism. First, rotation is transferred to shaft L (L_1-L_2) which will drive all the back dial pointers. From L_2 the rotation is transferred to shaft M (M_1-M_2-M_3) which drives both N_1 and subsequent gears-trains, and $E_3$and subsequent gear trains. From shaft N (N_1-N_2-N_3) rotation is transferred to both shaft P (P_1-P_2) and O (O_1). Shaft N itself carries the pointer of the Metonic dial on the back side of the AM. Shaft P drives Q_1 through P_2. Q (Q_1) carries the pointer for the Callippic dial on the back side of the AM. Similarly, Shaft N drives O_1 through N3. O (O_1) carries the pointer for the Staphanitic Games dial on the back side of the AM.

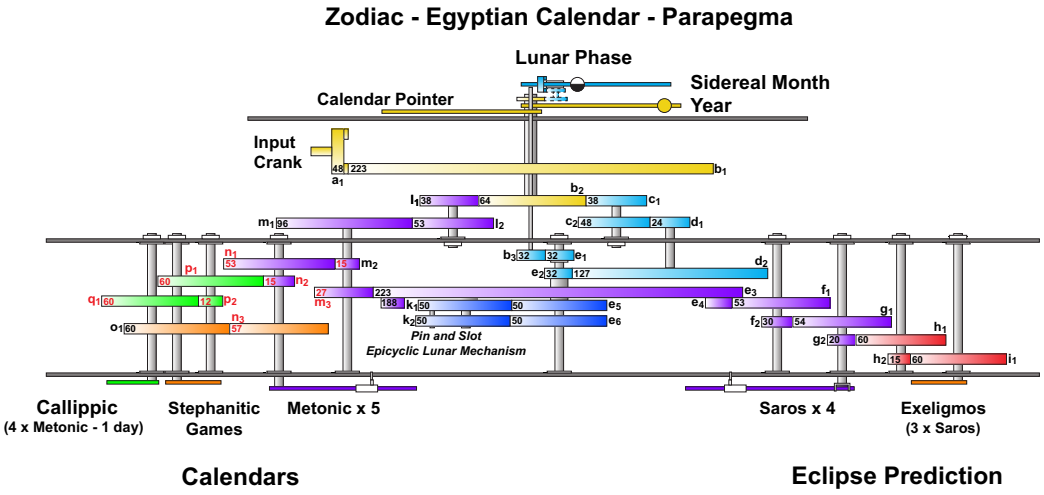

Figure 2: diagram of the AM

As can be seen in the diagram, gear E_4 is directly attached on gear E_3 and transfers rotation to the lower part of the back side through shaft F (F_1-F_2). Shaft F drives shaft G, which carries the Saros dial via interlock F_2/G_1. Shaft G (G_1-G_2) drives shaft H (H_1-H_2) through the interlock G_2/H_1. Finally, gear H_2 drives gear I_1 which carries the Exeligmos dial on the back side of the AM.

Going back to gear B_2, it drives shaft C (C_1-C_2), which drives shaft D (D_1-D_2), which drives shaft E_2 (E_2-E_5). This is the outer shaft as shown in the diagram, through which shaft E_1 (E_1-E_6) passes. Rotation is transferred transfers rotation to shaft B_3, which carries the lunar pointer on the same dial as the Sun, pointing to the location of the Moon on the celestial sphere and indicating the lunar phase through the rotating lunar sphere.

Of particular importance for the computation of the torque transfer is the Pin&Slot arrangement connecting gears K_1 and K_2 and transferring the torque from one to the other. The gears K_1 and K_2 have almost the same diameter and the exact same number of teeth (50). They are both mounted on freely rotating axles on gear E_3 as shown in Figure 3(a) but their axes are offset by a distance a. Gear K_1 has a pin at distance r from its axis, which engages a slot in gear K_2. As gear K_1 rotates, the pin forces gear K_2 to rotate by transferring the torque through the slot as shown in Figure 3(b).

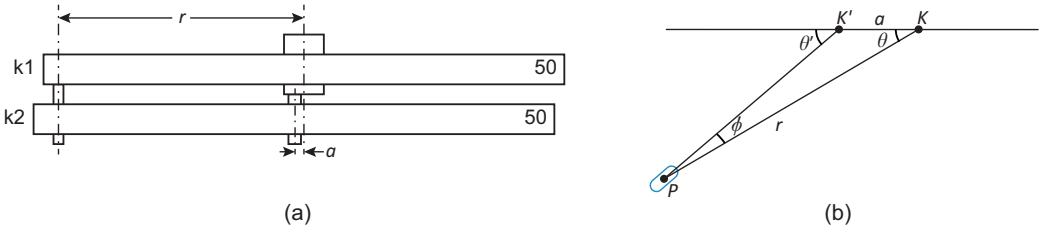

Figure 3: The Pin and Slot arrangement of a gears K_1-K_2: (a) Sectional diagram and (b) Pin&Slot parameters.

The minimum torque transfer occurs when the pin is on the line passing through the two axes KK'. As the distance between the axes a is 1.1 mm and the distance of the pin is 9.6 mm [6], the minimum torque transfer from K_1 to K_2 is 9.6/(9.6-1.1)=1.1294, while the minimum torque transfer from K_2 to K_1 is (9.6+1.1)/9.6=1.1146.

Following the operational sequence, four gear-trains can be identified in this diagram. Starting from the, until today regarded as input, crown A_1 we identify the trains a) A-$B_{1,2}$-L-M-N-P-Q, b) A-$B_{1,2}$-L-M-N-O, 3) A-$B_{1,2}$-L-M-E_3-F-G-H-I, and 4) A-B1-C-D-$E_{2,5}$-K_1-Pin&Slot-K_2-$E_{1,6}$-B_3. The gear-trains are shown in Figure 4.

The teeth of the gears of AM do not have an involute profile. Instead, they are triangular as shown in Figure 5. Since triangular teeth slide on each other as they are engaged, the transferred torque is not constant. Minimum torque is transferred when the tip of the driving tooth first engages the opposite tooth at the clearance point and the maximum torque is transferred when the driving tooth reaches the disengaging point (the addendum circle in modern gears), right before the next tooth engages. This difference can be quite large if the number of teeth is small (15 is the smallest tooth count in the AM), but cannot be estimated with any degree of accuracy, as the particular parameters, like clearance and tooth depth, cannot be measured accurately from the available CT scans. However, as shown in Figure 4, the gear-trains are quite long, and the maximum torque transfer at one engagement may coincide with a minimum torque transfer at a subsequent one. Thus, as the objective of this research is to determine the optimum driving gear, and not the exact torque at each shaft, taking into account that the driving force is not even known, we consider the average torque transfer for each gear interlock which, occurs at the pitch radius of each gear.

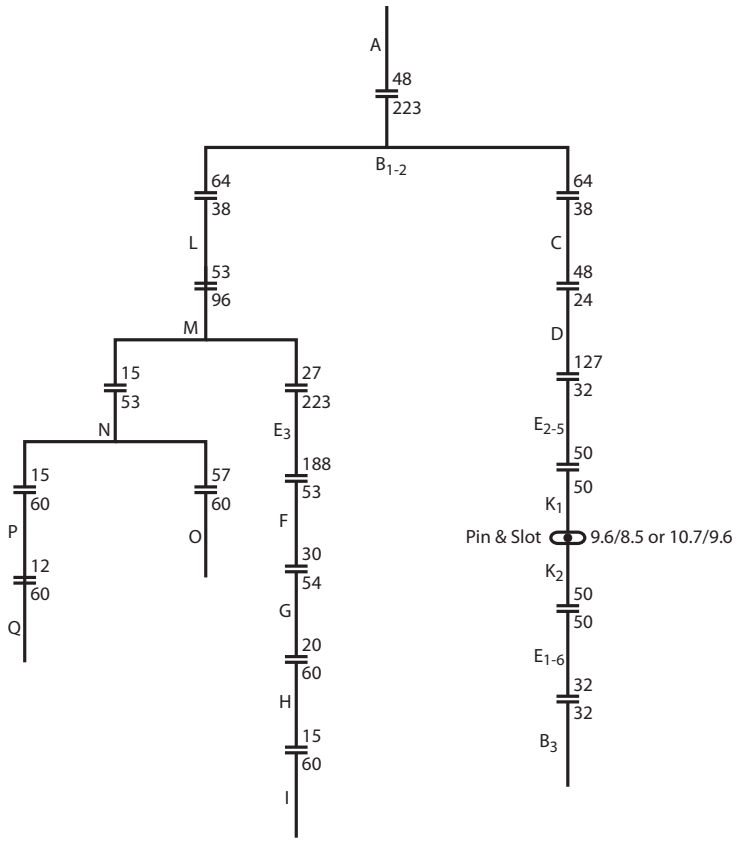

Figure 4: Schematic diagram of the gear-trains with transmission ratios.

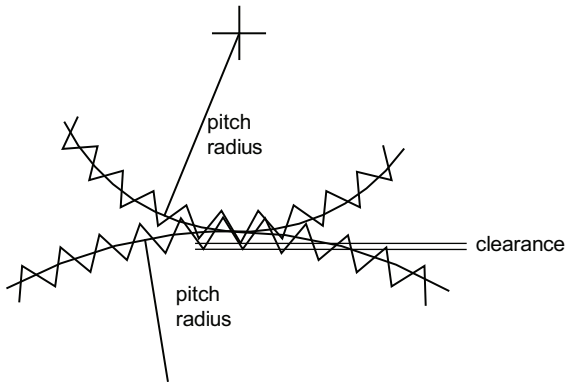

Figure 5: Gear parameters.

It follows that for the toque ratio transferred from one to the next is the same as the ratio of the pitch radii of the interlocking gears, which is the same as the teeth ratio of the interlocking gears. This teeth ratio is shown in shown Figure 4 for the torque transfer from one shaft to the next. For example, from shaft M to N the torque is transferred from gear M_2 to N_1 and the ratio is given as 15/53, while from shaft M to E_3 the torque is transferred from gear M_3 to E_3 and given as 27/223.

Depending on the driving gear, torque can be transferred from K_1 to K_2, in which case the ratio is 9.6/8.5, or from K_2 to K_1, in which case the ratio is 10.7/9.6. It should be noted that gears K_1 and K_2 cannot possibly be the driving gears, as their axles are attached to gear E_3 and rotate along with it. However, they are included in the calculation because their different than 1 torque ratio should be taking into account and for completeness.

To calculate the torque transferred from any driving gear to all the others, we need to take into account the torque split at the shafts that drive more than one gear-trains. For example, if the driving gear is A_1 or B_1 or B_2, torque is transferred to both gears C_1 and L_1. The amount of torque transferred to each direction depends only on the gear ratio between B_2-C_1 and B_2-L_1 if there is no friction or resistance at any of the shafts. However, as we introduce friction in subsequent sections, it is better to compute the torque transfer formally by using the superposition method. Thus, for any driving gear, we start at the end of the gear-trains leading to the driving gear and compute the torque transferred to the driving gear from any of the terminal gears. These torques are superimposed at the driving gear, and by reversing the torque calculation we can compute the torque transferred from the driving gear to any of the others.

For the torque calculation we used a unit torque of 10 $N \cdot mm$ (1 $N \cdot mm$ would give very small numbers) at the driving shaft and computed the torque transferred to each of the other gears. This was performed for all possible shafts of the AM. The reason we used shafts instead of gears is that the torque on the driving shaft is the same as the torque on all the gears attached to that shaft. Using these parameters, the torque on each of the other gear shafts is given in Table 1.

In Table 1, the minimum torque transfer for each driving shaft is shown in bold. As can be seen, the smallest torques are transferred to shafts E_3, K_1, K_2, E_1, and B_3. These results are also shown as a log-linear chart in Figure 6. As can be seen, the torque transferred to some of the shafts is quite large, which means that these shafts cannot possibly be the driving shafts.

Table 1: Torque transfer for a 10 N.mm driving torque

	A	B1	L	M	N	O	P	Q	E3	F	G	H	I	C	D	E2	K1	K2	E1	B3
A	10	46.46	27.58	49.96	176.5	185.8	706.1	3530	412.6	116.3	209.4	628.2	2512	27.58	13.79	3.48	3.48	**3.08**	**3.08**	**3.08**
B1	2.11	10	5.94	10.75	38.00	40.00	152	760	88.83	25.04	45.07	135.2	540.8	5.94	2.97	0.75	0.75	**0.66**	**0.66**	**0.66**
L	3.55	16.84	10	18.11	64.00	67.37	256	1280	149.6	42.17	75.91	227.7	911	10	5	1.26	1.26	**1.12**	**1.12**	**1.12**
M	1.96	9.30	5.52	10	35.33	37.19	141.3	706.6	82.59	23.28	41.91	125.7	502.9	5.52	2.76	0.70	0.70	**0.62**	**0.62**	**0.62**
N	0.55	2.63	1.56	2.83	10	10.53	40.00	200	23.38	6.59	11.86	35.59	142.3	1.56	0.78	0.20	0.20	70.17	**0.17**	**0.17**
O	0.53	2.50	1.48	2.69	9.50	10	38	190	22.21	6.26	11.27	33.81	135.2	1.48	0.74	0.19	0.19	0.17	**0.17**	**0.17**
P	0.14	0.66	0.39	0.71	2.50	2.63	10	50	5.84	1.65	2.97	8.90	35.59	0.39	0.20	0.05	0.05	**0.04**	**0.04**	**0.04**
Q	0.03	0.13	0.08	0.14	0.50	0.53	2	10	1.17	0.33	0.59	1.78	7.12	0.08	0.04	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**
E3	0.24	1.13	0.67	1.21	4.28	4.50	17.11	85.56	10	2.82	5.07	15.22	60.89	0.67	0.33	0.08	0.08	**0.07**	**0.07**	**0.07**
F	0.84	3.99	2.37	4.29	15.17	15.97	60.70	303.5	35.47	10	18.00	54.00	216	2.37	1.19	0.30	0.30	**0.26**	**0.26**	**0.26**
G	0.47	2.22	1.32	2.39	8.43	8.87	33.72	168.6	19.71	5.56	10	30	120	1.32	0.66	0.17	0.17	**0.15**	**0.15**	**0.15**
H	0.16	0.74	0.44	0.80	2.81	2.96	11.24	56.20	6.57	1.85	3.33	10	40	0.44	0.22	0.06	0.06	**0.05**	**0.05**	**0.05**
I	0.04	0.18	0.11	0.20	0.70	0.74	2.81	14.05	1.64	0.46	70.83	2.50	10	0.11	0.05	0.01	**0.01**	**0.01**	**0.01**	**0.01**
C	3.55	16.84	10	18.11	64	67.37	256	1280	149.6	42.17	75.91	227.7	911	10	5	1.26	1.26	**1.12**	**1.12**	**1.12**
D	7.10	33.68	20	36.23	128	134.7	512	2560	299.2	84.35	151.8	455.4	1821	20	10	2.52	2.52	**2.23**	**2.23**	**2.23**
E2	28.18	133.6	79.3	143.7	508	534.7	2032	10160	1187	334.7	602.5	1807	7230	79.38	39.69	10	10	**8.85**	**8.85**	**8.85**
K1	28.18	133.6	79.38	143.7	508	534.7	2032	10160	1187	7334	602.5	1807	7230	79.38	39.69	10	10	**8.85**	**8.85**	**8.85**
K2	25.28	119.9	71.21	129	455.7	479.7	1823	9115	1065	300.3	540.6	1621	6487	71.21	35.61	**8.97**	**8.97**	10	10	10
E1	25.28	119.9	71.21	128.9	455.7	479.7	1823	9115	1065	7300	7540	1621	6487	71.21	35.61	**8.97**	**8.97**	10	10	10
B3	25.28	119.9	71.21	129	455.7	479.7	1823	79115	1065	7300	7540	1621	6487	71.21	35.61	**8.97**	**8.97**	10	10	10

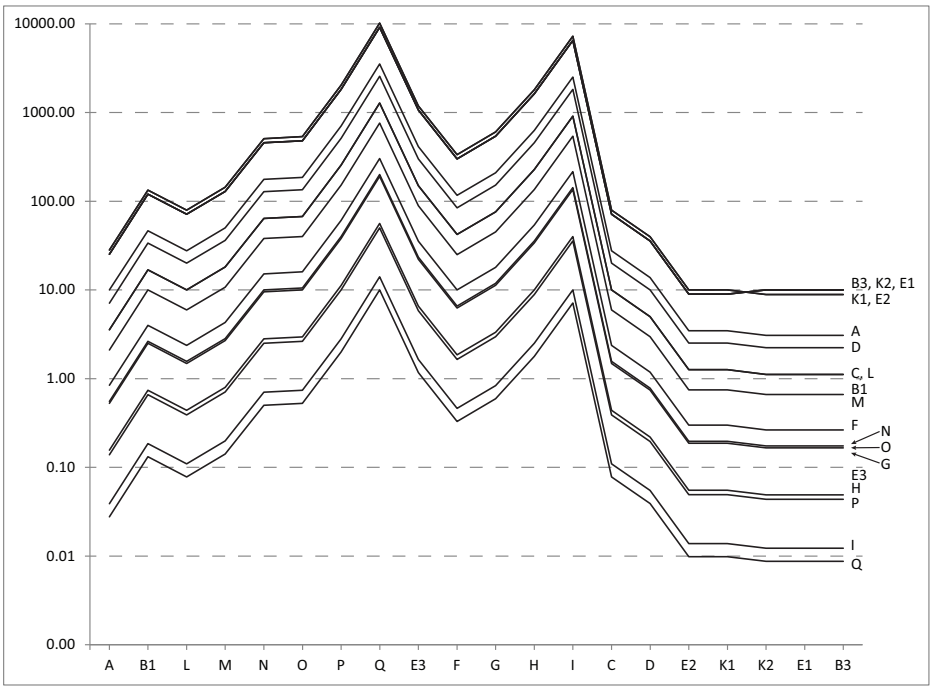

Figure 6: Log-Linear torque at the shafts for any driving gear.

3. Friction Calculation

The calculation of the previous section computed the maximum torque transferred from each input shaft, if there is no resistance or friction. However, if there is friction at each of the shafts of the gear-trains, long gear-train are more susceptible to seizure due to friction than short ones, if the non friction torque transferred to the last shaft is the same. Thus, it would me more accurate to computer the torque transfer in the presence of resistance, in the form of friction at each shaft.

It is known that the material of the AM gears, shafts, axles, and supporting plates is bronze rather than brass [4]. As given in [2, 9] the coefficient of friction of bronze against bronze is quite small, in the range of 0.04 to 0.06. Maybe this is one of the reasons of selecting this material for the construction of the AM, in addition, of course, to its hardness. As the exact bronze composition of the AM is not known, and after being in the bottom of the sea for more than 2000 years, it is impossible to tell wether a lubricant was used at the bearings of the shafts or the gears ar their axles. Thus, the average coefficient of friction of $\mu_s = 0.05$ was used to calculate an average friction of $0.2\ N \cdot mm$ at each shaft bearing based on the examples of clock frictions given in [7, 8]. It should be noted that this is an estimation of the average friction, as the exact parameters of the bearings cannot be accurately estimated from the CTs of the AM.

The average friction at each shaft and axle was used to calculate the torque transferred to each of the other gear shafts for a driving torque of 10 $N \cdot mm$ from any shaft. The torques are given in Table 2.

As can be seen in the table, some of the shafts receive a negative torque, which, of course, is not possible. This simply means that the combination of a certain driving shaft with a torque of 10 $N \cdot mm$ and an average friction of $0.2\ N \cdot mm$ at each shaft leads to seizure of thew mechanism. The negative torques are indicated by bold letters in the Table, along with the corresponding driving shafts. It follows that the shafts in bold cannot possibly be driving shafts.

The torque diagram for each driving gear is shown in Figure 7, only for shafts E_3, K_1, K_2, E_1, and B_3 which have been found to receive the smallest torque.

Table 2: Torque transfer for a 10 N.mm driving torque and friction at each shaft

	A	B1	L	M	N	O	P	Q	E3	F	G	H	I	C	D	E2	K1	K2	E1	B3
A	10	46.26	27.27	49.19	173.6	182.5	694.1	3470	406	114.2	205.4	616.2	2464	27.27	13.43	3.18	2.98	2.64	2.44	2.24
B1	2.11	10	5.74	10.19	35.81	37.50	143	715	83.98	23.48	42.06	125.9	503.6	5.74	2.67	0.47	0.27	0.24	0.04	**-0.16**
L	3.51	16.64	10	17.91	63.09	66.21	252.1	1260	147.7	41.45	74.42	223	892	9.68	4.64	0.97	0.77	0.68	0.48	0.28
M	1.85	8.76	5.32	10	35.13	36.78	140.3	701.4	82.39	23.03	41.25	123.5	494	5	2.30	0.38	0.18	0.16	**-0.04**	**-0.24**
N	0.40	1.91	1.25	2.63	10	10.53	40	200	23.38	6.59	11.86	35.59	142.3	0.93	0.27	-0.13	-0.33	**-0.29**	**-0.49**	**-0.69**
O	0.36	1.72	1.14	2.43	9.30	10	37	184.8	19.89	5.41	9.53	28.40	113.3	0.82	0.21	-0.15	-0.35	**-0.31**	**-0.51**	**-0.71**
P	**-0.02**	**-0.12**	0.05	0.45	2.30	2.22	10	49.80	3.52	0.79	1.23	3.49	13.74	**-0.27**	**-0.33**	**-0.28**	**-0.48**	**-0.43**	**-0.63**	**-0.83**
Q	**-0.14**	**-0.66**	**-0.27**	**-0.13**	0.25	0.06	1.80	10	**-1.27**	**-0.56**	**-1.20**	**-3.81**	**-15.44**	**-0.59**	**-0.50**	**-0.32**	**-0.52**	**-0.46**	**-0.66**	**-0.86**
E3	0.08	0.40	0.36	1.01	3.37	3.35	13.29	66.23	10	2.62	4.51	13.34	53.17	0.04	**-0.18**	**-0.25**	**-0.45**	**-0.39**	**-0.59**	**-0.79**
F	0.68	3.25	2.05	4.07	14.18	14.73	56.53	282.4	35.27	10	17.80	53.20	212.6	1.73	0.66	**-0.03**	**-0.23**	**-0.21**	**-0.41**	**-0.61**
G	0.29	1.39	0.95	2.08	7.13	7.31	28.34	141.5	18.8	5.36	10	29.8	119	0.63	0.11	**-0.17**	**-0.37**	**-0.33**	**-0.53**	**-0.73**
H	**-0.16**	**-0.76**	**-0.33**	0.44	1.35	1.22	5.18	25.72	4.36	1.34	3.13	10	39.80	**-0.65**	**-0.52**	**-0.33**	**-0.53**	**-0.47**	**-0.67**	**-0.87**
I	**-0.28**	**-1.32**	**-0.67**	**-0.17**	**-0.82**	**-1.06**	**-3.47**	**-17.56**	**-0.70**	**-0.09**	0.57	2.30	10	**-0.99**	**-0.69**	**-0.37**	**-0.57**	**-0.51**	**-0.71**	**-0.91**
C	3.51	16.64	9.68	17.34	61.05	64.07	244	1219	142.9	40.11	72	215.7	862.9	10	4.80	1.01	0.81	0.72	0.52	0.32
D	6.99	33.15	19.48	35.09	123.7	130	494.8	2474	289.5	81.44	146.3	438.9	1755	19.80	10	2.32	2.12	1.88	1.68	1.48
E2	27.92	132.4	78.46	141.9	501.2	527.3	2004	10023	1171	330.1	594.1	1782	7128	78.78	39.49	10	9.80	8.68	8.48	8.28
K1	27.92	132.4	78.46	141.9	501.2	527.3	2004	10023	1171	330.1	594.1	1782	7128	78.78	39.49	10	10	8.85	8.65	8.45
K2	24.46	116	68.71	124.2	438.8	461.7	1755	8775	1026	289	520.1	1560	6240	69.03	34.61	8.77	8.97	10	9.80	9.60
E1	23.95	113.6	67.28	121.6	429.7	452.1	1718	8593	1004	283	509.2	1527	6110	67.60	33.90	8.59	8.79	9.80	10	9.80
B3	23.45	111.2	65.86	119	420.6	442.5	1682	8410	983.4	277	498.4	1495	5980	66.18	33.19	8.41	8.61	9.60	9.80	10

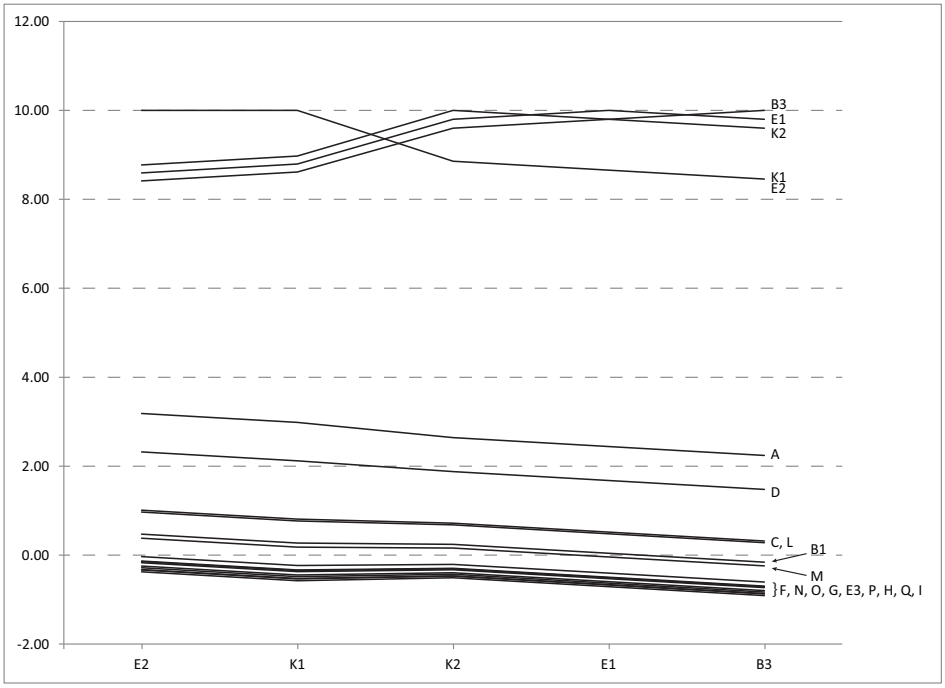

Figure 7: Torque at shafts E_2 to B_3 for any driving gear and friction.

4. Most Probable Driving Gear

Having computed the torque transferred at each gear in the presence of friction, it is now time to identify the possible location of the driving gear of the AM. All the calculation so far started from the torque at a driving shaft. However, a handle cannot be attached directly on any of the shafts to provide the driving torque as most of tt the shafts have pointers on them (shafts B_1, B_3, O, Q, G and I), while an extension of the other shafts (M, P, F, and H) to add a handle will interfere with the pointers on the first set. A third set of shafts (E_2, C, D, and L) are internal to the mechanism, having bearings on internal plates, and therefore, cannot be fitted with a handle.

The only shaft that has bearing on the the back plate and may not interfere with the pointers is shaft E_1 - E_6. Anastasiou [1] has shown that the pointers of the Metonic and Saros dials carry a needle pointing downwards, which follows the grooves of the spirals and for this reason the pointers are quite long. However, a fragment of the AM containing the part of the back plate where the two spiral dials touch has not been found, and the exact distance of the outer perimeters of the spirals cannot be computed. In addition, axis E_1 - E_6 is not on the same plane as the axes M and G that carry the Saros and Metonic pointers. This leaves a possibility that there may be space for a handle attached to shaft E_1-E_6, close to the middle of the back plate. If this is the case, then a handle on shaft E_1 - E_6 is a good candidate, because it results in large minimum torques, as can be seen in Figure 7.

If there is no space to put a handle on shaft E_1-E_6. or the designer of the AM did not want to deface the back plate with the handle, we can only assume that the mechanism was operated from the side. A handle attached to a driving crown, similar to the one believed until today to be the driving crown (or even the same one somehow misplaced) as driving one of the shafts. Therefore, the next step is to start from s driving torque on a crown that drives any of the shafts and compute the torque transferred to the other shafts or gears. For the size of the crown it is only logical to assume that it was similar to the one already fount on the AM, that is, a crown with 48 teeth.

The driving crown can be interlocked with any of the gears (except, of course K_1 and K_1 which have moving axles). However, since our purpose is to drive the AM with ease, i.e. with the smallest possible driving torque, for the shafts with more that one gears, we consider only the case that the crown is interlocked to the largest one.

Table 3: Torque transfer for a 10 N.mm driving torque

	A	B1	L	M	N	O	P	Q	E3	F	G	H	I	C	D	E2	K1	K2	E1	B3
A	10	46.26	27.27	49.19	173.6	182.5	694.1	3470	406	114.2	205.4	616.2	2464	27.27	13.43	3.18	2.98	2.64	2.44	2.24
B1	9.79	46.46	27.38	49.40	174.3	183.3	697.2	3485	407.8	114.7	206.3	618.9	2475	27.38	13.49	3.20	3.00	2.66	2.46	2.26
L	3.88	18.40	11.04	19.80	69.76	73.23	278.8	1394	163.3	45.85	82.32	246.7	986.8	10.72	5.16	1.10	0.90	0.80	0.60	0.40
M	3.81	18.06	10.84	20	70.47	73.98	281.6	1408	164.9	46.31	83.16	249.2	996.9	10.52	5.06	1.08	0.88	0.78	0.58	0.38
N	0.46	2.18	1.41	2.93	11.04	11.62	44.17	220.8	25.81	7.28	13.10	39.29	157.1	1.10	0.35	-0.11	-0.31	-0.28	-0.48	-0.68
O	0.50	2.35	1.51	3.10	11.68	12.50	46.50	232.3	25.44	6.97	12.35	36.85	147.1	1.20	0.40	-0.10	-0.30	-0.27	-0.47	-0.67
P	0.01	0.05	0.15	0.63	2.93	2.88	12.50	62.30	4.99	1.21	1.97	5.71	22.64	-0.17	-0.29	-0.27	-0.47	-0.42	-0.62	-0.82
Q	-0.13	-0.62	-0.25	-0.09	0.38	0.19	2.30	12.50	-0.98	-0.47	-1.05	-3.36	-13.66	-0.57	-0.49	-0.32	-0.52	-0.46	-0.66	-0.86
E3	0.95	4.51	2.80	5.43	18.97	19.77	75.67	378.1	46.46	12.90	23.02	68.85	275.1	2.48	1.04	0.06	-0.14	-0.12	-0.32	-0.52
F	0.77	3.66	2.29	4.52	15.76	16.39	62.85	314	38.97	11.04	19.68	58.83	235.1	1.98	0.79	0.00	-0.20	-0.18	-0.38	-0.58
G	0.35	1.67	1.11	2.37	8.19	8.42	32.55	162.5	21.26	6.05	11.25	33.55	134	0.79	0.20	-0.15	-0.35	-0.31	-0.51	-0.71
H	-0.12	-0.57	-0.22	0.64	2.05	1.96	7.99	39.77	6.00	1.80	3.97	12.50	49.80	-0.54	-0.47	-0.32	-0.52	-0.46	-0.66	-0.86
I	-0.27	-1.28	-0.64	-0.13	-0.64	-0.88	-2.77	-14.05	-0.29	0.03	0.78	2.93	12.50	-0.96	-0.68	-0.37	-0.57	-0.51	-0.71	-0.91
C	3.51	16.64	9.68	17.34	61.05	64.07	244	1219	142.9	40.11	72	215.7	862.9	10	4.80	1.01	0.81	0.72	0.52	0.32
D	18.67	88.59	52.40	94.71	334.4	351.8	1337	6687	782	220.2	396.2	1188	4754	52.72	26.46	6.47	6.27	5.55	5.35	5.15
E2	29.09	138	81.76	147.9	522.3	549.6	2089	10446	1221	344.1	619.2	1857	7429	82.08	41.14	10.42	10.22	9.05	8.85	8.65
K1	29.09	138	81.76	147.9	522.3	549.6	2089	10446	1221	344.1	619.2	1857	7429	82.08	41.14	10.42	10.42	9.22	9.02	8.82
K2	25.51	121	71.68	129.6	457.8	481.7	1831	9155	1070	301.5	542.6	1627	6510	71.99	36.10	9.15	9.35	10.42	10.22	10.02
E1	25.01	118.6	70.25	127	448.7	472.1	1794	8972	1049	295.5	531.8	1595	6380	70.57	35.39	8.97	9.17	10.22	10.42	10.22
B3	15.02	71.28	42.12	76.10	268.6	282.6	1074	5372	628.3	176.9	318.2	954.6	3818	42.44	21.32	5.42	5.62	6.27	6.47	6.67

In summary, taking into account that:

1. The driving crown has 48 teeth
2. The driving torque is 10 $N \cdot mm$
3. There is friction at each shaft or axle of 0.2 $N \cdot mm$
4. The crown is attached tot the largest gear of each shaft

the torque transferred to each shaft or gear is given Table 3.

Again, some of the shafts receive a negative torque, which, of course, is not possible, and simply means that the combination of a certain driving shaft with a torque of 10 $N \cdot mm$ and an average friction of 0.2 $N \cdot mm$ at each shaft leads to seizure of thew mechanism. The driving shafts which lead to seizure are indicated by bold letters in the Table. It follows that the shafts in bold cannot possibly be driving shafts.

The torque diagram for each driving gear is shown in Figure 8, again only for shafts E_3, K_1, K_2, E_1, and B_3 which have been found to receive the smallest torque.

We can now proceed to identify the most probable gear for attaching the driving crown. The shaft that give the maximum minimum torques are shafts E_1-E_6 and E_2-E_5. For shaft E_2-E_5 that crown must be attached to the largest gear that is gear E_5. This does no seem to be possible since gear E_5 is very close to gear E_6 from one side and gear E_3 from the other, leaving no space for a crown gear (see Figure 2). This leaves gear E_6 as the best candidate, with space for the crown gear between E_6 and the back plate.

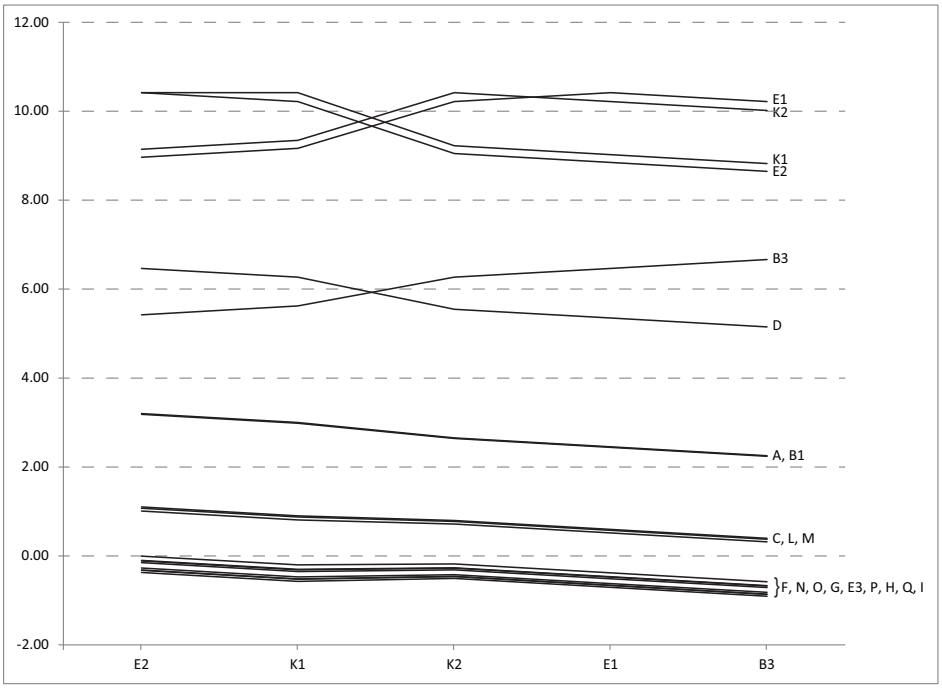

Figure 8: Torque at shafts E_2 to B_3 for a driving gear with 48 teeth at any of the gears and friction.

Gears B_3 and shaft D_1-D_2 are the next good candidates. But, as can be seen in Figure 2 gear B_3 is a very small gear with only 32 teeth located in a gear D_2 is one of the best candidates as it is a large gear and it is relatively easily reached with a driving crown.

The next candidate is gear B_1, the one already assumed to be the driving gear. However, this results in a minimum torque about 5 times smaller than the one provided by gear E_6 and about one third of the torque provided by gear D_2. Thus, if the friction at each shaft or axle is larger than the estimated $0.2\ N \cdot mm$, driving the mechanism by gear B_1 may be difficult.

5. Conclusions

The Antikythera Mechanism is an exceptional example of craftsmanship, coming from more that 2000 years ago. Its construction becomes even more amazing by the fact that the early physical models of the mechanism cannot

be operated due to difficulty in turning the handle at the assumed location. This paper attempted to determine the driving gear by calculating the torques transferred from any input gear to the rest of the gear-trains. It was found that although the gear assumed to be the driving one is a good candidate it is not the best possible one. There are two other candidate gears that give from three to five times largest torque at the mort difficult to turn shafts of the mechanism. It is expected that these input gears will be considered in the next physical models to be constructed.

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