INTRA- AND INTER-GROUP COMPOSITE INDICATORS USING THE BoD MODEL

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ABSTRACT: We extent the radial DEA model with a single constant input, often referred to as the Benefit-of-the-Doubt (BoD) model, to account for environmental or contextual differences, defined to be the gap between the pooled and group frontiers and then we develop theoretically consistent aggregation rules to compute intra- and inter-group composite indicators. We verify that the simple arithmetic average remains the theoretically consistent aggregate for pooled and intra-group composite indicators and we develop a theoretically consistent aggregation scheme for the inter-group composite indicators. We applied the proposed methodology to analyze financial performance of public and private hospitals in Greece by means of financial ratios referring to their liquidity performance and to examine the extent of differences in the inter-group composite indicators between the two types of ownership both at the individual and the aggregate level.

January 2016

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1. Introduction

The Benefit-of-the-Doubt (BoD) is one of the optimization models suggested by OECD (2008) for constructing composite indicators. It is closely related to Data Envelopment Analysis (DEA) and in particular to a special form of it, namely a single constant input (or pure output) input-oriented model. Its main advantage is that it results in idiosyncratic weights to aggregate sub-indicators that vary both across sub-indicators as well as across the evaluated units. Thus each evaluated unit is allowed to choose that set of weights that maximizes its performance in terms of the resulting value of the composite indicator under the restriction that if the same set of weights is used by any other evaluated unit it will not result in a value of the composite indicator that is greater than one.

There are now a vast number of studies using the BoD model that span across different indicators, including the Human Development Index (e.g., Despotis, 2005), the Quality of Life Indicator (e.g., Morais and Camanho, 2011), the Internal Market Index (e.g., Cherchye *et al*., 2007a), the Competitiveness Index (e.g., Bowen and Moesen, 2011), the Digital Access Indicator (e.g., Gaaloul and Khalfallah, 2014), the Technology Achievement Index (e.g., Cherchye *et al*., 2008), Students' Evaluation of Teaching indicator (e.g., de Witte and Rogge, 2011), the Research Productivity Indicator (e.g., Karagiannis, 2016), the Health System Performance Index (e.g., Lauer *et al*., 2004), and the Environmental Performance Index (e.g., Zanella, Camanho and Dias, 2013).

To the best of our knowledge the BoD model has been used up to now to evaluate the performance of a set of evaluated units assuming implicitly a common set of environmental or contextual factors. These environmental factors are related to the conditions underlying or surrounding the performance of the evaluated units. They can refer to the level of development, income classes, organizational structure, ownership, geographical regions etc. Assessing performance by means of composite indicators across well-defined environmental or contextual factors suggest creating separate group for each factor and then analyze performance differences within and between groups.

Building on the DEA roots of the BoD model we provide such as an analytic framework elaborating on Charnes, Copper and Rhodes (1981) idea of programmatic efficiency. Programmatic efficiency accounts to intra-group efficiency differences and it measures the extent of inefficiency that remains after eliminating inter-group or managerial inefficiency. In the context of the BoD model, this will be referred to as intra-group differences in the value of the composite indicator as long as there are reasons to believe that the evaluated units should be included into different groups due to varying environmental factors.

We provide an illustrative example of the proposed framework by studying the performance of financial ratios of general hospitals in Greece. We hypothesize that the ownership status of the evaluated units more likely affects their financial performance and thus it may more accurate to evaluate public and private hospitals financial performance separately based on the notion of the inter-group composite indicator. Nevertheless, we are at the same time interesting (even more) on intra-group differences of composite financial ratios of the two groups of hospitals. For this purpose we have to use the notion of programmatic efficiency mentioned above. In the context of the BoD model, this for each evaluated unit will be given by the ratio pooled and inter-group composite indicators.

Besides developing the idea of intra- and inter-group composite indicators we also explore the aggregate measures of these indicators by providing theoretically consistent aggregates of intra- and inter- group composite indicators. In particular, following Karagiannis (2016), one can verify that the simple arithmetic average is the theoretically consistent aggregate for the intra-group composite indicator while the aggregation procedure for the inter-group composite indicator is a little bit more complex. We derive however a quite simple alternative, which is given by the ratio of aggregate pooled composite indicator to aggregate intra-group composite indicator, each of which is measured by the simple arithmetic average of the relevant group.

2. Theoretical Framework and Main Results

The BoD is essentially a tool for aggregating performance sub-indicators without explicit reference to the inputs used to achieve such performance (Cherchye *et al.*, 2007b). For each decision-making unit (DMU), it results in an estimated value of the composite indicator that is equal to the maximum weighted arithmetic average of the considered sub-indicators, with the weights being determined endogenously and thus are allowed to vary across the evaluated units, sub-indicators and time. In addition, the weights are constrained to be non-negative to reflect that the composite indicator is a non-decreasing function of sub-indicators (i.e., non-negatively constraint) and if are used by any other evaluated unit they would not result in a value of the composite indicator that is not greater than one (i.e., normalization constraint). In mathematical terms, the BoD model is given as:

$$
\begin{aligned}
I^k = \max_{s_i^k} & \sum_{i=1}^{N} s_i^k I_i^k \\
st & \sum_{i=1}^{N} s_i^k I_i^j \leq 1^j \quad \forall\, j = 1, \dots, K \\
& s_i^k \geq 0 \qquad\qquad \forall\, i = 1, \dots, N
\end{aligned}
\tag{1}
$$

where I^k is the composite indicator of the k$^{\text{th}}$ DMU, I_i^k is the i$^{\text{th}}$ sub-indicator of the k$^{\text{th}}$ DMU, s_i^k are the DMU-specific weights to be estimated, $j = 1, \dots, k, \dots, K$ is used to index DMUs and $i = 1, \dots, N$ to index sub-indicators. By construction, I^k takes values in the [0,1] interval, with the upper value indicating best practice.

The BoD model is a specially tailored version of DEA, and in particular of the Charnes, Cooper and Rhodes (1978) input-oriented model when there is a single constant input that takes the value of one for all DMUs (see Cook and Kress, 1990; Caporaletti, Dula and Womer, 1999; Despotis, 2005).[1] One can verify this by using the multiplier form of this DEA model, i.e.,

$$
\begin{aligned}
I^k = \max_{s_i^k} & \sum_{i=1}^{N} s_i^k I_i^k \\
st & \sum_{r=1}^{M} v_r^k x_r^j - \sum_{i=1}^{N} s_i^k I_i^j \geq 0 \quad \forall\, j = 1, \dots, K
\end{aligned}
\tag{2}
$$

$$\sum_{r=1}^{M} v_r^k\, x_r^k = 1$$

$$s_i^k \geq 0 \qquad\qquad \forall\, i = 1, \dots., N$$

$$v_r^k \geq 0 \qquad\qquad \forall\, r = 1, \dots, M$$

where *x* refers to input quantities, *v* to their component weights (input multipliers) and $r = 1, \dots, M$ is used to index inputs.[2] If there is only a single input with $x_1^j = 1$, for all DMUs $(j = 1, \dots, K)$ then $v_1^j = 1$ for all $j = 1, \dots, K$ and $\sum v_r^k x_r^k = v_r^k = 1$ for every DMU. In this case, (2) is reduced to (1). Then, using the envelopment form of (2), namely

$$\begin{aligned} &\min_{\lambda_j^k} \quad I^k \\ &s.t. -\sum_{j=1}^{K} \lambda_j^k x_r^j + I^k x_r^k \geq 0 \ \ \forall\ \ r = 1, \dots, M \\ &\sum_{j=1}^{K} \lambda_j^k I_i^j \geq I_i^k \qquad \forall\ \ i = 1, \dots, N \\ &\lambda_j^k \geq 0 \qquad\qquad \forall\ \ j = 1, \dots, K \end{aligned} \tag{3}$$

for r=1 and $x_1^j = 1$ for all DMUs, we can derive the envelopment form of the BoD model as:

$$\begin{aligned} I^k = \min_{\lambda_j^k} &\sum_{j=1}^{K} \lambda_j^k 1^j \\ st &\sum_{j=1}^{K} \lambda_j^k I_i^j \geq I_i^k \qquad \forall\, i = 1, \dots, N \\ &\lambda_j^k \geq 0 \qquad\qquad \forall\, j = 1, \dots, K \end{aligned} \tag{4}$$

where λ refers to intensity variables. From (4) we can see that the estimated value of the composite indicator equals the sum of the intensity variables. If the k[th] DMU is "efficient" in the sense of best practicing then $\lambda_k^k = 1$ with all other $\lambda_j^k = 0$ for $j \neq k$ and thus $I^k = \lambda_k^k = 1$ while if it is "inefficient", $\lambda_k^k = 0$ and $I^k = \sum_{j \neq k} \lambda_j^k < 1$.

For our purposes we estimate (1) and/or (4) using data from the entire sample, i.e., $j = 1, \dots, K$, to construct what we refer to as the *pooled composite indicator*, I^k, of the kth DMU, i.e., the composite indicator without considering any contextual differences (e.g., production practices, organizational structure, ownership status, etc.) that may affect the performance of the evaluated units. When such differences are taken into account the evaluated DMUs can be portioned into $g = 1, \dots, G$ groups as follows: $[1, \dots, k_1], [k_1 + 1, \dots, k_2], \dots, [k_k + 1, \dots, K]$, where each group does not necessarily contain the same number of DMUs. Then we estimate (1) and/or (4) separately for each group to construct what we refer to as the *intra-group composite indicator*, ${}^{g}I_m^k$, of the kth DMU that belongs to gth group.

We provide two alternative procedures for constructing what we refer to as the *inter-group composite indicators*. Both of them treat inter-group composite indicators in a manner analogous to programmatic efficiency in a DEA framework where each DMU posses a single unitary input that is used to steer all sub-indicators towards their maximum levels. Inspired from the procedure proposed by Charnes, Cooper and Rhodes (1981), the first alternative involves three steps: *first*, estimate intra-group composite indicators separately for each group using (1) or (4). *Second*, use the intra-group composite indicators to adjust the "unitary input" to eliminate any intra-group inefficiency. To do this notice that due the radial input-oriented form of the BoD model ${}^{g}I_m^j = \tilde{1}^j / 1^j = \tilde{1}^j \leq 1$ for $j = 1, \dots, K$, where a tilde over a variable refers to its potential (i.e., inefficiency adjusted) value. That is, the adjusted "unitary input" for each DMU equals its intra-group composite indicator. *Third*, estimate (2) or (3) using the entire sample of DMUs and the adjusted "unitary input" to construct the inter-group composite indicators.

The DEA model involved in the third step of the above procedure may be viewed as modified version of the BoD model suitable for constructing inter-group composite indicator. To verify this first substitute $x^k = \tilde{1}^k = {}^{g}I_m^k$ into (2) and (3). Then the equality constraint in (2) implies $v^k = 1/{}^{g}I_m^k$ for r=1; that is, the input multiplier of the evaluated DMU is equal to the inverse of its intra-group composite indicator. Based on this, the first inequality constraint in (2) is written as ${}^{g}I_m^j / {}^{g}I_m^k - \sum_{i=1}^{N} s_i^k I_i^j \geq 0$ for all $j = 1, \dots, K$ and $g = 1, \dots, G$. With these changes included in (2) and (3) the multiplied form of modified BoD model is given as:

$$I_p^k = \max_{s_i^k} \sum_{i=1}^{N} s_i^k I_i^k$$

$$st \sum_{i=1}^{N} s_i^k I_i^j \leq ({}^g I_m^j / {}^g I_m^k) \quad \forall\, j = 1, \dots, K, g = 1, \dots, G \qquad (5)$$

$$s_i^k \geq 0 \qquad \forall\, i = 1, \dots, N$$

and its envelopment form as:

$$I_p^k = \min_{\lambda_j^k} \sum_{j=1}^{K} \lambda_j^k\, ({}^g I_m^j / {}^g I_m^k)$$

$$st \sum_{j=1}^{K} \lambda_j^k I_i^j \geq I_i^k \quad \forall\, i = 1, \dots, N \qquad (6)$$

$$\lambda_j^k \geq 0 \qquad \forall\, j = 1, \dots, K$$

Thus the modified BoD model for constructing inter-group composite indicators I_p^k in (5) and (6) may be viewed as the conventional BoD model in (1) and (4) with 1^j being replaced by the relative intra-group composite indicators ${}^g I_m^j / {}^g I_m^k$.

Due to the DEA structure of (5) and (6), the estimated intra-group composite indicator will take values in the range [0, 1]. From (6) we can see that I_p^k is given by the weighted sum of the intensity variable in the modified BoD model, where the weights are the corresponding relative intra-group composite indicators.[3] If the k[th] DMU is "efficient" in the sense of best practicing then $\lambda_k^k = 1$ with all other $\lambda_j^k = 0$ for $j \neq k$ in (6) and thus $I_p^k = \lambda_k^k({}^g I_m^k / {}^g I_m^k) = 1$ while if it is "inefficient", $\lambda_k^k = 0$ and $I_p^k = \sum_{j \neq k} \lambda_j^k\, ({}^g I_m^j / {}^g I_m^k) < 1$.

The second alternative procedure we propose for constructing inter-group composite indicators is inspired from the work of Cook, Kazakov and Roll (1994) and also involves three steps: *first*, estimate intra-group composite indicators separately for each group using (1) or (4). *Second*, estimate the pooled composite indicator using (1) or (4) and the entire sample of DMUs. *Third*, for each DMU the inter-group composite indicator is calculated residually as:

$$I_p^k = \frac{I^k}{{}^{g}I_m^k} \tag{7}$$

namely, by the ratio of its pooled and intra-group composite indicators. The inter-group composite indicator reaches the maximum value of one only if both the pooled and the intra-group composite indicators are at their maximum value.

Even though there is no clear preference for one of the two alternatives, it seems that the second one is easier to implement, but it provides no insights about the identification of peers and/or the estimated component weights as the values of the inter-group composite indicator are calculated residually instead of being estimated as in the first alternative. In addition, it does not provide any insights for the aggregation of inter-group composite indicators, in which we will turn now.

The consistent aggregation across DMUs of the pooled, intra-group and inter-group composite indicators relies on the fact that the BoD model is, as we have shown above, an input-oriented model with a single constant input that takes the value of one for all DMUs, and on the denominator rule (see Färe and Karagiannis, 2013), which may be viewed as the empirical counterpart of Koopmans' (1957) theorem.[4] The denominator rule states that consistency in aggregation of ratio-type performance measures, including efficiency indices, is ensured as long as the aggregation weights are defined in terms of the denominator variable of the relevant index. For the input-oriented model, these will be the actual cost or input shares, depending on whether multiple or a single input are used (Färe and Zelenyuk, 2003; Färe and Karagiannis, 2014). Furthermore, for the single constant input case, as Karagiannis (2016) has shown, these are common to all DMUs and equal to the inverse of their total number in the sample. Then, the arithmetic average is the theoretically consistent aggregation rule for the single constant input DEA model and thus for the BoD model. This in turn implies that the aggregate pooled composite indicator is equal to the simple (un-weighted) arithmetic average of the estimated individual pooled composite indicators; that is,

$$\mathrm{I} = \frac{1}{\mathrm{K}} \sum_{\mathrm{k}=1}^{\mathrm{K}} I^k \tag{8}$$

Similarly, the aggregate intra-group composite indicator is given by the arithmetic average of individual intra-group composite indicators in the particular group; that is,

$$ {}^{g}I_m = \frac{1}{k_g}\sum_{k=\alpha}^{k_g} {}^{g}I_m^k \tag{9} $$

for each $g = 1, \dots, G$, where $\alpha = k_{g-1} + 1$. This simple aggregation rule cannot however be used for the inter-group composite indicators as the modified BoD model in (5) and (6) differs from the conventional BoD model in (1) and (4).

In order to derive a theoretically consistent aggregation rule for the inter-group composite indicators we rely again on the denominator rule and the input-oriented form of the modified BoD model. As a result, the aggregation weights of the inter-group composite indicator should be defined in terms of the adjusted "unitary input", which is the corresponding denominator variable. But as we have shown above this for each DMU is equal to its intra-group composite indicator. Thus, the aggregate inter-group composite indicator for the g^th^ group is given as:

$$ {}^{g}I_p = \sum_{k=\alpha}^{k_g}\left(\frac{{}^{g}I_m^k}{\sum_{k=\alpha}^{k_g} {}^{g}I_m^k}\right) {}^{g}I_p^k = \frac{1}{k_g}\sum_{k=a}^{k_g}\left(\frac{{}^{g}I_m^k}{{}^{g}I_m}\right) {}^{g}I_p^k \tag{10} $$

where the second equality is obtained by using (9). That is, the aggregate inter-group composite indicator for the g^{th} group is given by the weighted arithmetic average of the individual inter-group composite indicators, with the weights being the ratio of the individual intra-group composite indicator to their sum in this particular group.

In addition, one may start from (8) to show that at the group level there is a relationship similar to that at the individual level given in (7). Specifically,

$$ \begin{aligned} {}^{g}I &= \frac{1}{k_g}\sum_{k=\alpha}^{k_g} {}^{g}I^k = \frac{1}{k_g}\left(\sum_{k=\alpha}^{k_g} {}^{g}I_m^k\right)\frac{\sum_{k=\alpha}^{k_g} {}^{g}I_m^k\, {}^{g}I_p^k}{\sum_{k=\alpha}^{k_g} {}^{g}I_m^k} \\ &= \left(\frac{1}{k_g}\sum_{k=\alpha}^{k_g} {}^{g}I_m^k\right)\sum_{k=\alpha}^{k_g}\left(\frac{{}^{g}I_m^k}{\sum_{k=\alpha}^{k_g} {}^{g}I_m^k}\right) {}^{g}I_p^k = {}^{g}I_m\, {}^{g}I_p \end{aligned} \tag{11} $$

where the last equality follows from (9) and (10). Then, the aggregate inter-group composite indicator for the g[th] group may be measured as:

$$ {}^{g}I_p = \frac{{}^{g}I}{{}^{g}I_m} = \frac{(\frac{1}{k_g}) \sum_{k=\alpha}^{k_g} {}^{g}I^k}{(\frac{1}{k_g}) \sum_{k=\alpha}^{k_g} {}^{g}I_m^k} \tag{12} $$

That is, the aggregate inter-group composite indicator is given by the ratio of the average pooled composite indicator to the average intra-group composite indicator. Notice however that the aggregate inter-group composite indicator is not an average of the individual inter-group composite indicators; that is, ${}^{g}I_p \neq (1/k_g) \sum {}^{g}I_p^k$. Instead, using the Olley and Pakes (1996) decomposition, one can show that:

$$ {}^{g}I_p = \left(\frac{1}{k_g}\right) \sum_{k=\alpha}^{k_g} {}^{g}I_p^k + Cov({}^{g}I_p^k, {}^{g}I_m^k / \sum_{=\alpha}^{k_g} {}^{g}I_m^k) \tag{13} $$

where the covariance term reflects the relationship between individual inter-group composite indicators and their relative intra-group composite indicator. As a result, the aggregate inter-group composite indicator is greater (less) than the average inter-group composite indicator as long as inter- and intra-group composite indicators are positively (negatively) correlated. That is, if the DMUs with above average intra-group scores also have above average inter-group scores and the DMUs with below average intra-group scores also have below average inter-group scores, then the aggregate inter-group composite indicator is greater than the average inter-group composite indicator. On the other hand, if DMUs with above average intra-group scores have below average inter-group scores and the DMUs with below average intra-group scores have above average inter-group scores, then the aggregate inter-group composite indicator is less than the average inter-group composite indicator. They are equal if intra- and inter-group composite indicators are not correlated.

3. Case Study: Public versus Private Hospitals in Greece

In this section we apply the measures of intra- and inter-group composite indicators, developed in the previous section, to assess the financial performance of 203 general hospitals in Greece, of which 120 are public and 83 are private, during the 2012 fiscal year. The empirical analysis is based on financial ratios related to hospitals' liquidity possibilities, which reflect how well each hospital is positioned to meet its short-term obligations. The ability of each one to meet its current obligations demands adequate liquidity, which is achieved by the sequential conversion of inventories and supplies into healthcare services, of healthcare services into patient and/or social security organization accounts receivables, and of net accounts receivables into cash in hand. If there are not enough assets, hospitals could not meet their current obligations resulting in increased deficits, limited creditor's trust and potential bankruptcy of private units. Therefore, performance in terms of financial liquidity offers hospital's managers the opportunity to evaluate the efficient use and the adequacy of working capital relative to the provided healthcare services.

Calculation of liquidity ratios involves items belonging to current assets and liabilities as the conversion of current assets into cash is expected to provide all or part of the funds that will be needed to pay off the current liabilities outstanding at the end of the fiscal year. Current assets are equal to the sum of net patient accounts receivable, inventories and supplies, cash, and cash equivalents. *Net patient accounts receivable* represents money owed to hospitals for services that they have already provided by patients and/or social security organizations.[5] *Inventories and supplies* refer mainly to hospital's purchases of medical supplies. As it is not in hospital's best interest to hold excessive inventories, there is a certain level of supplies necessary to meet medical needs and to maintain a safety stock to guard against unexpected surges in use. Anything above this level create unnecessary cost and for this reason most hospitals hold relatively small levels of inventories. *Cash* represents actual cash in hand plus money held in commercial checking accounts (demand deposits). *Cash equivalents* are short-term security investments that are readily convertible into cash.[6] On the other hand, *liabilities* represent claims (i.e. unpaid wages and salaries, unpaid taxes, unpaid supplies, loans etc.) that must be paid within one accounting period.

Three key ratios fall into the category of liquidity performance (Zelman *et al*., 2009; Gapenski, 2012): the current ratio, the quick ratio and cash or acid test ratio. The *current ratio* indicates whether a hospital has enough resources to pay its debt over the next 12 months. It is an indicator of hospital's market liquidity and ability to meet creditors' demands, and it is calculated by dividing current assets by current liabilities:

$$Current\ Ratio = \frac{Current\ Assets}{Current\ Liabilities} \tag{14}$$

It indicates the cash that would be provided by the liquidation of a hospital's current assets at book value per unit of current liabilities. In general, the higher the current ratio is, the greater hospital's liquidity and therefore, the better able to meet current obligations using liquid assets. If the current ratio is less than unity, then a hospital may have problems meeting its short-term obligations.[7]

The *quick ratio* measures the ability of a hospital to use its near cash or quick assets to extinguish its current liabilities immediately. As quick assets are considered those current assets that presumably can be converted almost instantaneously into cash at close to their book value. These current assets are cash, cash equivalents and net patient accounts receivable. The quick ratio is calculated by dividing the sum of quick assets by current liabilities (Zelman *et al*., 2009):

$$Quick\ Ratio = \frac{Cash + Cash\ Equivalents + Net\ Account\ Receivable}{Current\ Liabilities} \tag{15}$$

A value close to unity could be satisfactory only if the net patient accounts receivable do not include doubtful accounts and its receivable/payment period is less than a year. A hospital with a quick ratio less than unity cannot fully pay back its current liabilities and it needs extra funds (i.e. loans or state subsidies).[8]

The *acid test ratio* provides the most stringent test of liquidity. It measures how much cash is on hand or readily available from short-term securities to pay off all current liabilities. The acid test ratio is calculated by dividing the sum of cash and cash equivalents by current liabilities (Zelman *et al.*, 2009):

$$Acid\ Test\ Ratio = \frac{Cash\ and\ Cash\ Equivalents}{Current\ Liabilities} \tag{16}$$

It indicates how many times the cash on hand would cover current liabilities. If its value is close to unity, it implies that hospitals have a high level of cash liquidity. This ratio is particularly usefulness when current liabilities contain a high percentage of accounts that must be paid off soon (e.g., salaries or medicine supplies) and/or collection of net patient accounts receivables is slow.

Descriptive statistics for the three financial ratios are given in Table 1. From there we can see that on average the figures for the current ratio and the quick ratio are quite satisfactory while there seem to be problems with the acid test ratio as its average value is less than one and thus cash and cash equivalents cannot cover current liabilities. However, for all three financial ratios, there are hospitals with values less than one as indicated by their minimum values reported in Table 1. In addition, as the mean for each of the three financial ratios is greater than its median, their distributions are skewed to the right, with the majority of hospitals having a lower than average value of the corresponding financial ratio.

On the other hand, public hospitals seem to perform on average better than private ones by means of all three financial rations. As a result it is quite likely that public hospitals will outperform the private hospitals overall and this is supposed to be depicted by the estimated values of the inter-group composite indicator. However, there are no significant differences in within group heterogeneity. For the figures in Table 1 we can see that performance dispersion in terms of the current ratio and the quick ratio is a little greater for private than public hospitals while the opposite is true in terms of the acid test ratio.

Estimates of pooled and intra-group composite liquidity indicators are given in Table 2. From the average value of the pooled composite indicator (0.329), which also reflects the aggregate composite indicator for the whole sample, we can see that the evaluated units as a group could have done almost three times better compared to the performance of its own best hospitals. There are only two hospitals, both of them from the public ones, that have achieved the maximum value of the one while more than 50% of hospitals are in the range of 0.2 to 0.4 and only 3.5% of them with scores greater than 0.7. Thus the large majority of hospitals consist of low performers and there are few exceptional ones that constitute the high performers group (see also Figure 1).

Based on intra-group composite indicators, public hospitals tend on aggregate to be relatively more efficient than private hospitals and also more homogeneous in terms of performance dispersion. Although the differences in terms of their means are not so pronounced, based on the Mann-Whitney rank-sum test (see e.g., Brockett and Golany, 1996) we can reject the hypothesis that public and private hospitals come from a common distribution, as the calculated test statistics is -7.04.[9] This is also evidence from the frequency distributions reported in the low panel of Table 2. From there we can see that the distribution of the intra-group composite indicators for the private hospitals is more skewed to the left than the corresponding distribution for the public hospitals. Specifically, around 50% of public hospitals are in the range of 0.2 to 0.6 while around 50% of the private hospitals are in the range of 0.1 to 0.3 (see also Figure 1). In addition, standard deviation figures reported in Table 2 indicate that the distribution of the intra-group composite indicators for the private hospitals has longer tails, especially in the upper side.

In terms of group best practice peers, the results reported in Table 2 show that there are two in the public group, which by the way are the same with those that achieved a value of one in the pooled composite indicator case, and four in the private group. Notice that none of these efficient units is a self-evaluator and each one contributes with different weights in determining best practice for the inefficiency units. Based on Johnson and Zhu (2003) benchmarking share of the k^{th} efficient unit, defined as:[10]

$$\delta^k = \frac{\sum_{j=1}^{K} \lambda_j^k}{\#\ of\ inefficient\ DMUs} \tag{17}$$

we can infer that hospital # 25 for the public group and hospital # 188 for the private group are by difference the leading units (see Table 3), as larger values δ^k indicate that there is a higher concentration of performance levels around this particular peer's figures. Moreover, these two hospitals appear most times (118 and 70 respectively) as peers in their group.

The empirical results regarding the inter-group composite indicators are given in Table 4. For these first notice that the inter-group composite indicators for all public hospitals are found to be equal to one and thus, the public group and the pooled frontiers coincide. In addition, there is no a single private hospital with a value of the inter-group composite indicator equal to one and thus, the entire group frontier of private hospitals lie below that of public hospitals' group frontier. Consequently, the environmental or contextual conditions related to the operation/organization of public hospitals are such that do not impose any constraint in achieving the best possible score in terms of the pooled composite indicator. Any deviation from best practice for public hospitals is due to within group difference in managerial ability to deal with liquidity financial aspects. On the other hand, the environmental or contextual conditions related to the operation/organization of private hospitals impose rather severe constraints in achieving the best possible performance. On average they have deteriorated performance by 26.5%. However, the vast majority of private hospitals fall in the range of 0.6 to 0.9 in terms of the inter-group composite indicator and definitely above 0.5 (see also Figure 2).

From Table 4 we can also see that the average inter-group composite indicator is greater than the aggregate inter-group composite indicator for the private hospitals group and thus the covariance term from (13) is negative and in particular, equal to -0.027.[11] Its magnitude is however rather small as it only accounts for 3.8% of the magnitude of the aggregate inter-group composite indicator. The negative sign of the covariance term implies that private hospitals with relatively high values of intra-group composite indicators achieved relatively low values of inter-group composite indicators and/or private hospitals with relatively low values of intra-group composite indicators achieved relatively high values of inter-group composite indicators. This does not mean that all hospitals in the private group followed these patterns but only that did the majority of its hospitals.

4. Concluding Remarks

In this paper we extend the BoD model to account for environmental or contextual differences and we propose the estimation of three types of indicators: the pooled composite indicator, using the entire sample of DMUs, the intra-group composite indicator, used to account for within group differences, and the inter-group composite indicator, which accounts for between groups differences. We then also examine how these composite indicators are aggregated. In that respect we verify that the simple arithmetic average remains the theoretically consistent aggregate for pooled and intra-group composite indicators and we develop a theoretically consistent aggregation scheme for the inter-group composite indicators.

We provide an illustrative example of the proposed framework by studying the liquidity performance of general hospitals in Greece. Our main empirical findings may be summarized as follows: *first*, public and private hospitals seem on average to be equally well managed in terms of liquidity financial ratios but the former seem to have an advantage in achieving higher financial liquidity standards, as the relevant inter-group composite indicator for private hospitals is less than one. This may be due to the fact that public hospitals receive regular funding from the government to cover part of their operational expenses while private hospitals cover their liabilities mainly by payments from patients and/or social security organizations, which though may delay their reimbursement for more than a year. *Second*, the overall performance of private hospitals is, on average, 25% below that of public hospitals while as a group private hospitals are 30% below public hospitals. *Third*, the between differences are more pronounced than the within group differences and *fourth*, the private hospitals with relatively high values of intra-group composite indicators tend to have relatively lower values of inter-group composite indicators, and *vice versa*.

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Table 1 Descriptive statistics of Liquidity Ratios

	Current Ratio	Quick Ratio	Acid Test Ratio
	Pooled		
Average	1.859	1.783	0.157
Median	1.771	1.665	0.096
Min	0.303	0.291	0.001
Max	6.021	5.930	1.647
St. deviation	0.977	0.953	0.211
	Public		
Average	2.154	2.045	0.182
Median	2.019	1.922	0.123
Min	0.368	0.291	0.016
Max	6.021	5.930	1.647
St. deviation	0.879	0.860	0.227
	Private		
Average	1.434	1.405	0.121
Median	1.155	1.118	0.051
Min	0.303	0.291	0.001
Max	5.012	4.960	1.002
St. deviation	0.960	0.959	0.180

Table 2 Estimates Pooled and Intra-group Composite Indicators

	Pooled	Public	Private
	Descriptive Statistics		
No. of hospitals	203	120	83
Average	0.329	0.376	0.368
Median	0.310	0.340	0.280
Min	0.050	0.070	0.060
Max	1.000	1.000	1.000
St. deviation	0.175	0.162	0.255
	Frequency distribution		
0-0.10	12	2	4
0.10-0.20	33	7	19
0.20-0.30	46	25	24
0.30-0.40	57	47	7
0.40-0.50	26	20	8
0.50-0.60	13	10	7
0.60-0.70	9	4	2
0.70-0.80	2	1	3
0.80-0.90	1	0	4
0.90-1.00	2	2	1
1.00	2	2	4

Table 3 Benchmarking Share for Peers in Public and Private Group

	Pooled	Public	Private
hospital #25	0.253	0.289	
hospital #118	0.068	0.076	
hospital #173			0.065
hospital #187			0.082
hospital #188			0.177
hospital #190			0.012

Table 4 Inter-group Composite Indicators

Descriptive statistics	
No. of hospitals	83
Average	0.735
Median	0.757
Min	0.510
Max	0.882
St. deviation	0.098
Aggregate	0.708
Frequency distribution	
<0.50	0
0.50-0.60	8
0.60-0.70	20
0.70-0.80	24
0.80-0.90	31
0.90-1.00	0
1.00	0

Figure 1 Frequency distribution of Pooled and Intra-group Composite Indicators

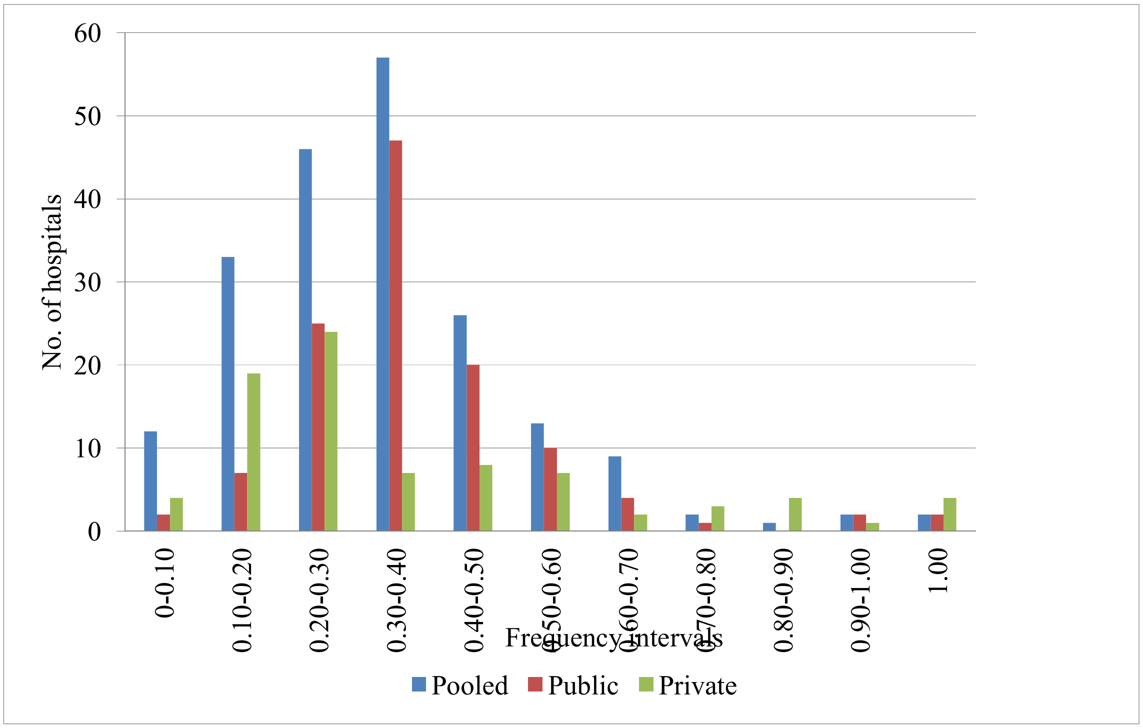
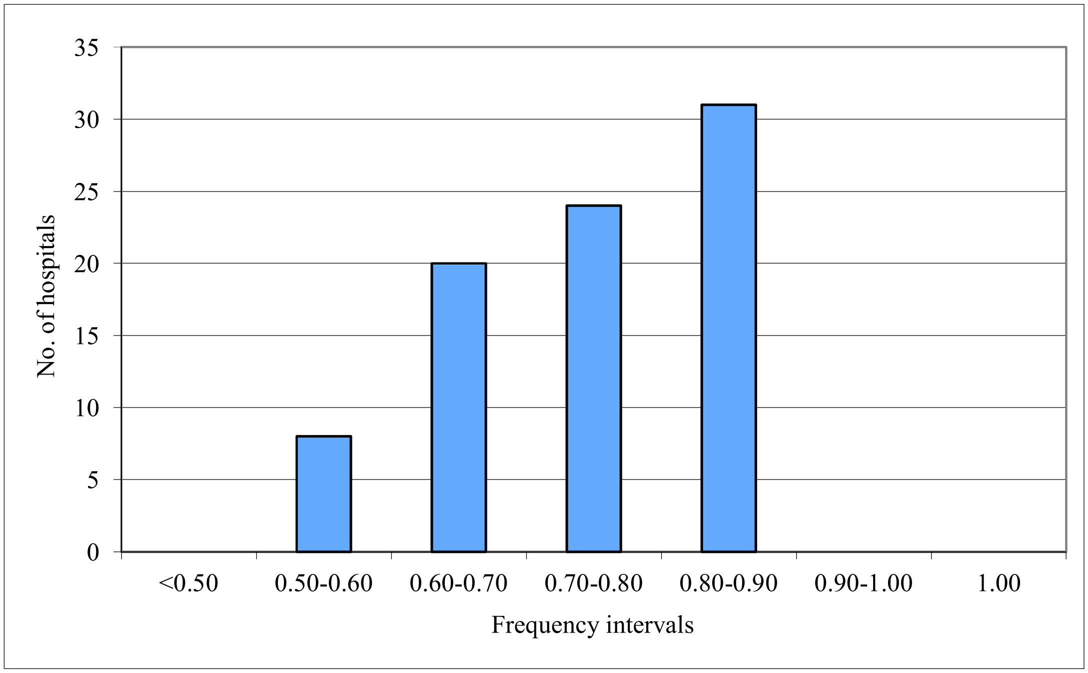


Figure 2 Frequency distribution of programmatic efficiency for private hospitals



Endnotes

[1] For radial DEA model without explicit inputs or with single unitary input see Lovell and Pastor (1999), Caporaletti, Dula and Womer (1999) and Liu et al. (2011).

[2] Notice that in this setup the sub-indicators play the role of outputs in the original formulation.

[3] Notice that in the cases of the pooled data and the intra-group composite indicators that was equal to the sum of the corresponding intensity variables.

[4] Koopmans' (1957) theorem states that industry maximum profit is equal to the sum of firms' maximal profits as long as all firms face the same input and output prices.

[5] In Greece, social security organizations, as third-party payers, make most payments for healthcare services, and these payments often take weeks, months or years to be billed, processed, and ultimately paid.

[6] Many private hospitals use short-term debt to finance seasonal or cyclical working capital needs. The temporary increase in current assets typically is financed by a bank loan of some type. Public and private hospitals also fund accounts payable that represent amounts due to vendors for supplies purchases. Suppliers offer their customers credit, which allow payment sometime after the purchase is made.

[7] Nevertheless, if the current ratio is far greater than one, then a hospital may not be efficiently using its current assets (i.e. low inventories and supplies turnover, doubtful patient accounts receivables, increased obligations with decreased revenues at the same time) or its short-term financing facilities. Such a case is not however reported in our sample.

[8] Also notice that a great variation between the current and quick ratio implies the existence of exceed inventories or supplies.

[9] The rank-sum test is a nonparametric statistical test based on the ranking of data. This test serves to examine the hypothesis that two groups belong to the same distribution or they differ significantly. Let the k_1 public hospitals be index by $j = 1, \dots, k_1$ and the k_2 private hospitals by $j = k_1 + 1, \dots, K$. Then the rank-sum test is given as $T = \left[S - \left(\frac{k_1(k_1+k_2+1)}{2}\right)\right] / \sqrt{(k_1 k_2 (k_1 + k_2 + 1)/12}$, where S is the sum of rankings in the one group (i.e., public hospitals) that follows approximately a normal distribution with mean $k_1(k_1 + k_2 + 1)/2$ and variance $k_1 k_2 (k_1 + k_2 + 1)/12$ for $k_1, k_2 \geq 10$. We reject the hypothesis that the two groups have the same intra-group

composite indicator if $T \leq -T_{\alpha/2}$ or $T \geq T_{\alpha/2}$, where $T_{\alpha/2}$ corresponds to the upper $\alpha/2$ percentile of the standard normal distribution for any level of significance α.

[10] In words, the benchmarking share is equal to the sum the values of the intensity variables corresponding to this unit over all inefficient DMUs for which appears as a peer and divide it by the number of inefficient units.

[11] For the public hospitals group, the covariance term is equal to zero as both the average and the aggregate intra-group composite indicators are equal to one.