EOQ with independent endogenous supply disruptions

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Abstract

We consider an inventory installation, controlled by the periodic review base stock (S, T) policy and facing a fixed-rate deterministic demand which, if unsatisfied, is backordered. The supply process is unreliable, so supply deliveries may fail according to an independent Bernoulli process; we refer to such failures reflecting the supply service quality and being internal to the supply chain, as endogenous disruptions. We seek to jointly determine the two policy variables, so to minimize long-run average cost. While an approximate model for this problem was recently analyzed, we present an exact analysis, valid for two common accounting schemes for inventory cost evaluation: continuous and end-of-cycle costing. After developing a unified (and exact) average cost model for both costing schemes, the cost for each scheme is analyzed. In both cases, the optimal policy variables and cost prevail in closed-form, having an identical structure to those of EOQ (with backorders). In fact, under continuous costing, the optimal solution reduces to EOQ for perfect supply. Analytical properties, demonstrating the impact of deteriorating supply quality on the optimal policy, are established. Moreover, computations reveal the cost impact of deploying heuristics that either ignore supply disruptions or rely on inaccurate costing information.

Keywords: Single-echelon; EOQD; Stochastic; Uncertainty; Newsvendor.

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1. Introduction

Supply uncertainty directly affects the performance of production-inventory networks, increasing operating costs and reducing customer service. It is not surprising, therefore, that the development of models for encapsulating and effectively managing all types of supply uncertainly has evolved into an active research objective with a rich body of results. In general, the associated literature can be divided into two broad areas; namely, supply under *random yield* and supply under *disruptions*. Since the problem we study here relates with both these areas, we start with a brief review of findings in both areas.

Yield uncertainty occurs when the quantity supplied differs from that ordered by a random amount. Most studies in the field restrict attention to stochastically proportional yield models, where the yield (a random variable) is a fraction of the quantity ordered. Silver (1976) proposed one of the earliest modifications of the basis EOQ model to accommodate proportional yield. Assuming that defectives are replaced on receipt, the optimal lot-size is obtained in closed-form. An extension of this model to incorporate complete and partial backorders was proposed in Karlo and Gohil (1982). Also assuming a finite inspection time, Salameh and Jaber (2000) provided a further extension, which has effectively formed the basis for an ongoing surge of supplementary findings (e.g. Voros, 2013, Hauck and Voros, 2015, Alamri et al., 2016). A review of EOQ-type proportional yield models is presented by Khan et al. (2011).

Proportional yield uncertainty has also been studied in association with stochastic demand. Gerchak et al. (1988) first studied an inventory model where both the demand and yield rate are uniformly distributed, while Henig and Gerchak (1990) extended this work for general form of yield uncertainty. Because of the practical difficulties imposed by stochastic demand, several heuristic approaches have also appeared; good examples include Bollapragada and Morton (1999), Li et al. (2008) and Huh and Nagarajan (2010). For comprehensive (but somewhat dated) review of stochastic demand uncertain yield models, we refer to Grosfeld-Nir and Gerchak (2004).

Turning to *supply disruptions*, these correspond to sudden interruptions of the supply process and the associated research was instigated by Parlar and Berkin (1991). They studied an EOQ-type model under "wet" and "dry" supply periods of random length. Supplies are available during “wet” periods, while they unavailable throughout “dry” periods. A correction of this model was later published by Berk and Arreola-Risa (1994), while Parlar and Perry (1996) allowed for non-zero reorder points and backorders (only) during “dry” periods. For this model, Heimann and Waage (2007) proposed an approximate solution, while Snyder

(2014) proposed a simple but effective approximation to the optimal order quantity for exponentially distributed "wet" and "dry" periods. Turning to stochastic demand models, Gupta (1996) extended Parlar and Perry (1996) for Poisson demand and a continuous review (r, Q) control policy, while Mohebbi (2004) allowed for compound Poisson demand and for the "wet" and "dry" periods to follow a general and a hyperexponential distribution respectively. Under the periodic review (S, T) policy, Schmitt et al. (2010) considered disruptions that follow an infinite-state discrete-time Markov chain, each state resulting from a fixed number of consecutive disrupted periods. They showed that the optimal order-up-to level prevails by solving a classic Newsboy formulation.

In the models above, supply disruptions are effectively exogenous to the supply chain, not necessarily related to the supplier efficiency (e.g. natural disasters, terrorist attacks, accidents etc). There are also models, however, where the disruptions are endogenous, so reflect the inherent quality of the supply process (e.g. supplier inability to deliver, supplies of unacceptable supplies quality etc). The usual assumption here is that supply deliveries either materialize or fail according to a Bernoulli process (e.g. Okyay et al. 2014). Clearly, with this type of disruptions, the length of "wet" and "dry" periods directly relate to the supply Bernoulli process. So, endogenous disruptions effectively constitute a limiting case of proportional supply yield (with 100% or zero yield losses). Güllü et al. (1997) investigated a base-stock policy under deterministic dynamic demand with the supply following an independent non-stationary Bernoulli process and proposed a Newsvendor-like solution for the optimal order-up-to level. Argon et al. (2001) studied a deterministic demand system where demand is affected by the previous period backorders under an independent supply Bernoulli process and numerically obtained the order-up-to level that maximizes profit. Warsing et al. (2013) studied a (S, T) base stock policy under stochastic demand and a correlated Bernoulli supply process. Using a discrete time Markov chain analysis to determine steady-state probabilities, they analytically evaluated optimal order-up-to levels, which for specific demand processes, are obtained in closed-form (so they can be directly evaluated). Recent reviews of models with disruptions, mainly focusing on exogenous disruptions are given in Schmitt et al. (2015), Paul et al. (2016), Snyder et al. (2016) and Schmitt et al. (2017).

A common feature of the above studies is that they ignored the review interval (or equivalently the order quantity) as a decision variable, only optimizing the order-up-to level. Under the EOQ (with backorders) paradigm and an independent Bernoulli supply process, Skouri et al. (2014) solved a two-dimensional constrained optimization problem, to obtain closed-form expressions for both the optimal order quantity and the re-order level. The analysis is exact for relatively low supply delivery failure probability, but becomes approximate as this probability increases. Subsequent studies have applied and extended this study. Ritha and FrancinaNishandhi (2015) used the model in a single-vendor and multi-buyer context to explore the effects of imperfect supply on system cost. Salehi et al. (2016) adapted the model for partial backorders and lost sales. Taleizadeh (2017) extended the original model allowing for partial backorders and lot prepayments. It is also worth noting that using analogous assumptions to Skouri et al. (2014), Rezai (2016) and Taleizadeh and Dehkordi (2017) have incorporated sampling inspection plans within the EOQ framework, while Taleizadeh et al. (2016) considered reparation of imperfect products.

A final note is necessary. Invariably, in all previous papers that have considered the base stock periodic review (S,T) policy, inventory costing follows an extreme version of discrete accounting, the *end of cycle* scheme. Under this scheme, inventory costing is exclusively based on the end-of-cycle inventory level, totally ignoring inventory profile variations within the cycle. In contrast, the latter is always considered in EOQ-based or continuous review (r,Q) inventory modes, which employ the traditional continuous accounting scheme. Rao (2003) showed how to analyse the base-stock (S,T) policy under continuous cost accounting, while Lagodimos et al. (2012) extended this to other periodic review policies. For the inaccuracies that may result by the deployment of end-of-cycle costing, the studies of Rudi et al. (2009) and Avinadav and Henig (2015) are most interesting. We further explore this issue as part of the present work.

In this paper we study a single-echelon inventory system controlled by the periodic review (S,T) base stock policy. Demand is deterministic with a fixed rate, while the supply process is unreliable characterized by endogenous disruptions following an independent Bernoulli process. The objective is to jointly determine the review interval and the order-up-to level that minimize long-run average total cost (under both continuous and end-of cycle accounting). Other than providing an extended exact formulation and analysis for the approximate model in Skouri et al. (2014), the main contributions of this paper are the following: (i) we present exact closed-form solutions for the optimal order-up-to level S and the review interval T (for both costing schemers) and discuss their association with those of the classical EOQ (with backorders) model; (ii) we present analytical properties relating the optimal decision variables and cost with the supply process quality; (iii) we provide results showing the consequences of using the inaccurate end-of-cycle costing scheme or ignoring the supply process quality on the system cost performance.

The remainder of this paper is organized as follows. Section 2 gives the model notation and assumptions used while Section 3 presents a unified exact long-run average cost model, valid for both costing schemes. The analysis leading to the optimal solution, for continuous and discrete accounting schemes, is presented in Section 4 and 5 respectively, while Section 6 provides numerical results. Finally, Section 7 summarizes the findings and gives directions for future research. An Appendix presents how the unified cost model can be adapted to be used for the analysis of any non-zero lead time.

2. Preliminaries

In this section we present the basic notation used along with the assumptions underlying the operation of the inventory system considered in this paper.

Notation

D demand rate (units per unit time)

S order up-to-level (decision variable)

T review (reorder) interval (decision variable)

K fixed cost (per delivery)

h unit holding cost (per unit per unit time)

b unit backordered cost (per unit per unit time)

$\overline{I}_i$ average inventory for a cycle with starting inventory state i ($i = 1, 2, ...$)

π_i probability of a cycle with starting inventory state i ($i = 1, 2, ...$)

α the ratio $\alpha = b/(h+b)$

x^+, x^- the operators $x^+ = \max(0, x)$ and $x^- = \max(0, -x)$

x^* globally optimal value of variable x

Operating and modelling assumptions

We study a single-echelon inventory installation controlled by the (S,T) policy. So, replenishment orders are released periodically (in time intervals *T*), aimed at restoring inventory up to a level *S*. While demand is deterministic, the supply process is unreliable and characterized by endogenous disruptions, following a stationary Bernoulli process. Therefore, for each replenishment order, there is a probability *p* that the respective delivery will fail to materialize (leading to a delivery failure). By the control policy used, the supply quantity not delivered at the current period routinely increases the next period replenishment order. We also use the following assumptions:

1. The planning horizon is infinite.
2. Demand rate is known and has a fixed rate.
3. Shortages are allowed and fully backordered.
4. Lead time is zero and there are no emergency deliveries.
5. Supply delivery failures (disruptions) are independent of each other.

We seek the policy variables S and T that minimize long run average cost. Except for one assumption, the above problem is equivalent to the one studied in Skouri et al. (2014) that assumed an (r,Q) policy coupled with a fixed delivery schedule (without emergency deliveries) in their analysis. In fact, by the standard transformations $S = r + Q$ and $T = Q/D$, the problem can directly be recast in terms of an (S,T) policy as assumed here. To facilitate modelling, however, Skouri et al. (2014) further assumed that $r \leq 0$, or equivalently that $S \leq DT$, rendering their analysis (at least in part) approximate. No such restrictive assumption is made here, leading to an exact analysis for two different accounting schemes.

A final note is necessary. While apparently restrictive, these assumptions totally comply with the realities of the so-called *fixed* or *cyclic delivery contracts*. Under such contracts, popular in several industries (see Parija and Sarker, 1999, Moinzadeh and Nahmias, 2000, Bahroun et al., 2007), a supplier undertakes to deliver to a buyer according to an agreed fixed equidistance delivery plan. While the timing of any delivery cannot be altered, its exact size is finalized on short notice (representing the lead time, assumed here to be zero). Provided that the demand is not too erratic for EOQ-type models to apply (see Silver et al., 1998), the system analysed here fits well this industrial setting, so the results could prove valuable (to both supplier and buyer) when negotiating the details of the associated delivery plan.

3. Model Formulation

Based on the operating assumptions presented in Section 2, we now provide a general unified formulation of the optimization problem, valid for both cost accounting schemes (i.e. continuous and end-of-cycle) we study here. In this unified formulation, we make use of the non-decreasing function $D(t)$ with $D(T)=DT$, which we denote as the *apparent cumulative demand*. As discussed later, the exact specification of $D(t)$ depends on both the actual demand process and the costing scheme used. In the following we treat $D(t)$ as being the actual cumulative demand imposed on the system over an inventory cycle $[0,T]$.

Under the periodic review (S,T) policy, with a reliable supply process, the start-of-cycle inventory level is always restored to the nominal state S. So, the long-run system operation comprises of an infinite series of identical inventory cycles. However, under random endogenous disruptions, the start-of-cycle inventory level becomes a random variable, $S-XDT$; depending on the size of the failed delivery XDT, it can thus attain several distinct states i, each occurring with some probability π_i. It is well established (e.g. Skouri et al., 2014) that, when delivery failures follow an independent Bernoulli process, X is a geometric random variable $X \sim Geo(p)$; so the state probabilities π_i coincide with the geometric mass function:

$$\pi_i = P(X = i-1) = p^{i-1}(1-p),\ \ i = 1,2,\ldots.$$

Consequently, under delivery failures, the long run system operation now comprises of an infinite number of different inventory cycles, each occurring with some probability π_i and with starting inventory levels (states) differing by multiples of *DT*. Figure 1 shows a possible sample path for the specific case where the apparent cumulative demand function is $D(t)=Dt$. Observe that, in the event of a delivery failure (disruption), the start-of-cycle inventory does not increase, while it is always restored at the nominal level S following any successful delivery.

Based on the above and using the function $D(t)$ as the cumulative demand, the total system long-run average cost (per unit time) can be expressed as:

$$C(S,T) = \frac{K}{T} + G(S,T) = \frac{K}{T} + \sum_{\tau=1}^{\infty}(h\pi_\tau \overline{I}_\tau^{+} + b\pi_\tau \overline{I}_\tau^{-}) \tag{1}$$

where $G(S,T)$ denotes the average holding/backorders cost over all inventory cycles, while

$$\bar{I}_\tau^+ = \frac{1}{T}\int_0^T \left(S-(\tau-1)DT-D(t)\right)^+ dt \text{ and } \bar{I}_\tau^- = \frac{1}{T}\int_0^T \left(S-(\tau-1)DT-D(t)\right)^- dt \tag{2}$$

give the average non-negative inventory and backorders for any inventory cycle with starting state τ, corresponding to starting inventory level $S-(\tau-1)DT$. We wish to evaluate: $\min_{S,T} C(S,T)$.

Although exact, due to the presence of the x^+, x^- operators, the above formulation does not lead to an easily solvable optimization problem. For this reason, let us now define the new variable $m = \lfloor S/DT \rfloor$. Observing (1) and recalling that the total demand in any cycle is DT, it is not hard to see that $\bar{I}_\tau^+ = \bar{I}_\tau$ for $\tau \le m$ and $\bar{I}_\tau^- = -\bar{I}_\tau$ for $\tau \ge m+2$, while at $\tau = m+1$ it may happen (depending on the specification of $D(t)$) for both $\bar{I}_\tau^+ \ge 0$ and $\bar{I}_\tau^- \ge 0$ to hold simultaneously. Furthermore, observe that from the definition of $\bar{I}_\tau^+$, it also holds that $\bar{I}_\tau - \bar{I}_m = (m-\tau)DT$ for $\tau < m$ while $\bar{I}_\tau - \bar{I}_{m-2} = (\tau-m-2)DT$ for $\tau > m+2$. So replacing these observations in $C(S,T)$ we obtain:

$$\begin{aligned} C(S,T,m) = \frac{K}{T} + \sum_{\tau=1}^{m-1} h\pi_\tau\,(m-\tau)DT + \sum_{\tau=m+3}^{\infty} b\pi_\tau\,(\tau-m-2)DT + \pi_{m+1}(h\,\bar{I}_{m+1}^+ + b\bar{I}_{m+1}^-) \\ + h\,\bar{I}_m^+ \sum_{\tau=1}^{m} \pi_\tau + b\,\bar{I}_{m+2}^- \sum_{\tau=m+2}^{\infty} \pi_\tau \end{aligned} \tag{3}$$

where π_i are the state probabilities defined above. So, under this general formulation, the following constrained optimization problem prevails: $\min_{S,T,m} C(S,T,m)$ s. t. $m = \lfloor S/DT \rfloor$. To solve this problem, we first need to customize it for each costing scheme, so to obtain workable models for the integrals $h\bar{I}_{m+1}^+ + b\bar{I}_{m+1}^-$, $\bar{I}_m^+$ and $\bar{I}_{m+2}^-$. For this purpose, the detailed specification of the apparent cumulative demand function $D(t)$ as applied to each scheme is necessary.

As discussed in Rao (2003) and Lagodimos et al. (2012), the key factor differentiating accounting schemes for inventory control is demand information. Continuous costing assumes that demand information is fully available. So, the inventory profile used for cost evaluation directly reflects the imposed demand process (deterministic or stochastic), so $D(t)$ coincides with the actual cumulative demand. In discrete costing, however, inventory information is only available in discrete time periods, reflecting the total demand in each period. So, exact inventory evaluation is only possible at the end of each period. To obtain the inventory profile within each period, the end-of-period inventory is then assumed to represent inventory level throughout the period. Hence, the apparent cumulative demand now becomes $D(t) = d_\tau$, where d_τ is the total demand in period τ and t is any instant within this period. Note that the end-of-cycle costing scheme is an extreme version of discrete costing, with the costing period taken to be equal with the inventory cycle.

4. Continuous Cost Model Analysis

This costing scheme has been traditionally used in the analysis of deterministic EOQ type models as well as that of stochastic models employing the continuous review (r,Q) policy. To our knowledge, Rao (2003) was first to analyze the periodic review (S,T) policy under continuous costing, effectively allowing the results for this policy to be directly associated with those of the former models. Now, since under continuous costing, $D(t)$ coincides with the actual cumulative demand (see Section 2), and we assume that demand is deterministic with a fixed rate D, clearly $D(t)=Dt$, for any $t\in[0,T]$. So, the integrals in (2) become:

$$h\overline{I}_{m+1}^{+}+b\overline{I}_{m+1}^{-}=\frac{h(S-mDT)^2}{2TD}+\frac{b\left[(m+1)DT-S\right]^2}{2TD},\quad \overline{I}_{m}^{+}=\left[S-mDT+\frac{DT}{2}\right],\quad \overline{I}_{m+2}^{-}=\left[mDT-S+\frac{3DT}{2}\right]$$

The above follow directly by noticing that $h\overline{I}_{m+1}^{+}+b\overline{I}_{m+1}^{-}$ effectively represents an inventory profile virtually identical to that of the classical EOQ (with backorders) model, $\overline{I}_{m}^{+}$ and $\overline{I}_{m+2}^{-}$ represent trapezoidal profiles whose exact size follows from the definitions in (2). Therefore, replacing in (3) and using the definitions of π_i, after some algebra the long-run average cost (per unit time) under continuous costing prevails:

$$C(S,T,m)=\frac{K}{T}+\frac{hDT\left\lfloor m(1-p)-1+p^m\right\rfloor}{1-p}+\frac{bTDp^{m+2}}{1-p}+\frac{(1-p)p^m h(S-mDT)^2}{2TD}+\frac{(1-p)p^m b\left[(m+1)DT-S\right]^2}{2TD}+$$
$$+h(1-p^m)\left[S-mDT+\frac{DT}{2}\right]+bp^{m+1}\left[mDT-S+\frac{3DT}{2}\right]\qquad(4)$$

So we need to minimize the above s. t. $m=\lfloor S/DT\rfloor$. We are now in the position to formally analyze this problem to evaluate the optimal values of the decision variables (S,T,m).

Lemma 4.1: (i) For any given m, the function $C(S,T,m)$ is jointly convex in (S,T); (ii) The optimal m is given by $m^*=\lfloor\log(1-\alpha)/\log p\rfloor$, where $\alpha=b/(h+b)$.

Proof: (i) For any m, the first and second order partial derivatives of $C(S,T,m)$ in S and T are:

$$\frac{\partial C(S,T,m)}{\partial T}=-\frac{K}{T^2}-hD\left(\frac{p}{1-p}+\frac{1}{2}\right)+(h+b)p^m(1-p)\left(\frac{m^2D}{2}-\frac{S^2}{2DT^2}\right)+\frac{(h+b)D}{2(1-p)}p^m(2m-2mp+p+1),$$

$$\frac{\partial C(S,T,m)}{\partial S} = \frac{p^m(1-p)(h+b)S}{TD} + h - p^m(h+b) - mp^m(1-p)(h+b),$$

$$\frac{\partial^2 C(S,T,m)}{\partial T^2} = \frac{2K}{T^3} + (h+b)p^m(1-p)\frac{S^2}{DT^3}, \quad \frac{\partial^2 C(S,T,m)}{\partial S^2} = \frac{p^m(1-p)(h+b)}{DT} \text{ and}$$

$$\frac{\partial^2 C(S,T,m)}{\partial S \partial T} = -(h+b)p^m(1-p)\frac{S}{DT^2}.$$

Since $\frac{\partial^2 C(S,T,m)}{\partial S^2} = \frac{p^m(1-p)(h+b)}{DT} > 0$ and

$$H = \frac{\partial^2 C(S,T,m)}{\partial S^2} \cdot \frac{\partial^2 C(S,T,m)}{\partial T^2} - \left[\frac{\partial^2 C(S,T,m)}{\partial T \partial S}\right]^2 = \frac{2K(h+b)(1-p)p^m}{DT^4} > 0,$$

So, $C(S,T,m)$ is jointly convex in (S,T).

(ii) Setting $\partial C(S,T,m)/\partial S = 0$ we obtain the unique, by (i) above, optimal S for given (T,m):

$$S = \frac{DT}{1-p}\left(m(1-p) + 1 - \frac{h}{(h+b)p^m}\right). \tag{5}$$

Since (by the constraint) $m = \lfloor S/DT \rfloor$, we clearly need that $m \le (S/DT) < m+1$ is satisfied. Replacing S/DT from (5), these inequalities become $p^m \ge 1-\alpha$ and $p^{m+1} < 1-\alpha$, while by taking logarithms lead to the unique $m^* \in N$ that satisfies the required relation. □

Interestingly, expression (5) giving the locally optimal S for given (T,m) corresponds exactly to a standard Newsvendor formulation. As discussed in deriving in (4), there are m inventory cycles with only positive inventory and one with both positive inventory and backorders. So, the Newsvendor condition becomes $\sum_{\tau=1}^{m}\pi_\tau + \pi_{m+1}(S - mDT)/DT = b/(h+b)$, where the last term gives the fraction of time the inventory level of cycle $m+1$ is non-negative. Replacing the probabilities π_τ, (5) follows. This observation permits us to physically interpret m^* as the floor below which any order-up-to level S cannot satisfy the Newsvendor probability. So, we expect m^* to increase with an increase in either the system uncertainty (i.e. delivery failure probability p) or the service level target $\alpha = b/(h+b)$ imposed (as actually indicated by Proposition 4.1.ii).

Proposition 4.2: The globally optimal policy variables (S^*, T^*) are:

$$T^* = \sqrt{\frac{2K(1-p)p^{m^*}(h+b)}{D[-h^2+(h+b)^2 p^{2m^*+1} + h(h+b)(1+2m^*)(1-p)p^{m^*}]}} \text{ and } S^* = \frac{DT^*}{1-p}(m^*(1-p)+1-\frac{h}{(h+b)p^{m^*}}),$$ while

the optimal total cost is $C^* = 2K/T^*$.

Proof: Replacing S (at the optimal $m = m^*$) from (5) in (4), we obtain the partially optimal (in S) average total cost $C(T,m^*)$:

$$C(T,m^*) = \frac{K}{T^*} + \frac{DT}{2(1-p)(h+b)p^{m^*}}\left[-h^2+(h+b)^2 p^{2m^*+1} + h(h+b)(2m^*+1)(1-p)p^{m^*}\right].$$

By Lemma 4.1.(i), $C(T,m^*)$ is clearly convex in T; so, the optimal T^* prevails as the unique solution of $\partial C(T,m^*)/\partial T = 0$ as given above, while S^* follows directly. It can be shown that since $p^{m^*} \geq h/h+b$, the denominator of T^* is always positive. Finally setting $T = T^*$ in $C(T,m^*)$, the expression for C^* prevails. □

There is a remarkable similarity of the problem globally optimal solution as given above with that of the classical EOQ (with backorders) problem. First notice that, for perfect supply process quality, $m^* = \lim_{p \to 0^+} \lfloor \log(1-\alpha)/\log p \rfloor = 0$; replacing in the expressions of Proposition 4.2, the corresponding solutions of the EOQ (with backorders) model prevail:

$$S_0 = \frac{bDT_0}{h+b},\ T_0 = \sqrt{\frac{2K(h+b)}{hbD}} \text{ and } C_0 = 2K/T_0.$$

Equally important, however, is that the structure of the problem solution in Proposition 4.2 is identical to that of the EOQ solution, only differing by constants (representing the supply process uncertainty). It can be safely stated, therefore, that the continuous costing model we have presented here is a direct extension of the EOQ (with backorders) model so to incorporate independent Bernoulli delivery failures.

Lemma 4.3. The following properties hold: (i) The optimal m^* is step-wise increasing in p; (ii) The optimal T^* is decreasing in p; (iii) The ratio S^*/DT^* is increasing in p; (iv) The optimal cost C^* is increasing in p.

Proof. (i) Let $f(p) = \log(1-\alpha)/\log p$, so $m^* = \lfloor f(p) \rfloor$. But $f(p)$ is increasing in p, since $df(p)/dp = -\log(1-\alpha)/p(\log p)^2 \geq 0$ and the required result follows.

(ii) Since $T^* = T^*(p, m^*(p))$ and $m^*(p)$ varies as (i) above, it is sufficient to show that T^* is continuous in p for every $p \in (0,1)$ and that

$$\frac{\partial T^*}{\partial p} = \frac{p^{m^*-1}(h+b)K\left[(h+b)^2(-1+m^*(p-1))p^{1+2m^*} + h^2(p+m^*(p-1))\right]}{DT^*\left[h^2 + h(h+b)(1+2m^*)(p-1)p^{m^*} - (h+b)^2 p^{1+2m^*}\right]^2} < 0 .$$

Let partition $(0,1)$ into $(0, \mathrm{p}_1), [p_1, \mathrm{p}_2), ..., [p_n, \mathrm{p}_{n+1}), ..., [p_k, 1)$, where p_n and p_{n+1} are such that $m^*(p) = n \in \mathbb{N}$ for $p \in [p_n, p_{n+1})$ and $m^*(p) = n+1$ for $p \in [p_{n+1}, p_{n+2})$. At the points p_n it holds that $n = \log(1-\alpha)/\log p \in \mathbb{N}$ or equivalently that $p_n^n = h/(h+b)$. In any partition $[p_{n-1}, p_n)$, T^* is clearly continuous in p so we need to show continuity at p_n. Replacing $p_n^n = h/(h+b)$ in T^* we can form the functions $T^*(p,\ m^*(p) = n-1)$ and $T^*(p,\ m^*(p) = n)$. Since it can be shown that $\lim_{p \to p_n^-} T^*(p,\ m^*(p) = n-1) = \lim_{p \to p_n^+} T^*(p,\ m^*(p) = n) = T^*(p_n) = \sqrt{K/nhD}$, T^* is continuous at points p_n and the proof is complete.

Considering now $\partial T^* / \partial p$ as given above, we need to show that its numerator is negative. Substituting again $p^{m^*} \geq h/h+b$, after some algebra the numerator becomes $-p^{m^*-1}(h+b)Kh^2(1-p^2) \leq 0$, so the required relation holds.

(iii) The proof of continuity is directly analogous to (ii) above. Since

$$\frac{\partial}{\partial p}\left(m^* + \frac{1}{1-p}\left[1 - \frac{h}{(h+b)p^{m^*}}\right]\right) = \frac{(b+h)p^{m^*+1} + h(m-(1+m)p)}{(b+h)(1-p^2)p^{m^*+1}} \geq \frac{hm(1-p)}{(b+h)(1-p^2)p^{m^*+1}} > 0 ,$$

the required relation holds.

(iv) It follows directly since T^* is decreasing in p. □

The above analytical properties relate to the behaviour of the optimal cost and the respective optimizers as functions of the supply process quality p. Observe that, as p increases, then C^* and m^* increase while T^* decreases. This behaviour is totally in line with that of stochastic systems under increased levels of (demand or supply) uncertainty, which tend to reduce the ordering frequency and increase the necessary safety stock. Also note that, since S^* is linear in DT^* (by Proposition 4.3), we have only analyzed the ratio S^*/DT^* and found it increasing in p (as expected). For the data shown, Figures 2-4 provide examples of the behaviour of these functions in p for different service level targets. All graphs clearly verify the theoretical predictions.

We are now in the position to compare the above exact analysis with that of Skouri et al. (2014) which instigated this work. Viewed as a continuous costing model, Skouri et al. (2014) effectively modelled and solved the problem: $\min_{S,T} C(S,T,m=0)$ s.t. $m=\lfloor S/DT \rfloor = 0$. In general, this is an approximate formulation easier to solve (since, by arbitrarily fixing variable m, the exact three-dimensional problem became two-dimensional). It happens, however, when by the exact model $m^*=0$ (actually obtained for relatively small p), both the Skouri et al. (2014) model and its respective solution to be actually exact.

Figure 5 presents a comparison between the exact and the approximate optimal cost as function of p, for the example data shown. To plot the graph, for each p, we first found the optimal (S^*,T^*) for each model separately. We then evaluated the resulting cost by using the true cost expression (4). Observe that (as expected) both models perform identically for $p \le 0.25$ (when optimal $m^*=0$). However, for larger p, the Skouri et al. (2014) model becomes approximate, leading to solutions that can considerably increase total cost.

5. End-of-cycle Cost Model Analysis

The end-of-cycle costing schemes has been traditionally used for cost evaluation in periodic review inventory systems. Under this scheme, inventory is evaluated at the end of each cycle and is assumed to represent inventory for the entire cycle (see Section 3). Therefore, for a fixed-rate deterministic demand, $D(t)=DT$ for any $t\in[0,T]$. Using this, the inventory profile (in all inventory cycles) has a rectangular shape. Therefore, it always holds that $\bar{I}^+_{m+1}=0$ and $\bar{I}^-_{m+2}=\bar{I}^-_{m+1}+DT$, and replacing in (3) we obtain:

$$C(S,T,m)=\frac{K}{T}+\sum_{\tau=1}^{m-1} h\pi_\tau\,(m-\tau)DT+\sum_{\tau=m+2}^{\infty} b\pi_\tau\,(\tau-m-1)DT+h\,\bar{I}^+_m\sum_{\tau=1}^{m}\pi_\tau+b\,\bar{I}^-_{m+1}\sum_{\tau=m+1}^{\infty}\pi_\tau \qquad (6)$$

Now, also taking into account that $\bar{I}^+_m=S-mDT$ and $\bar{I}^-_{m+1}=(m+1)DT-S$, the following result restricts the state space of decision variable S.

Lemma 5.1. For any T, the optimal policy satisfies: $S/DT=m\in\mathbb{Z}^+$ and $\bar{I}^+_m=0$

Proof. Consider any $S \in \mathrm{R}^+$. Since $\bar{I}_m^+ + \bar{I}_{m+1}^- = DT$, we can write $\bar{I}_m^+ = xDT$ and $\bar{I}_{m+1}^- = (1-x)DT$ where $x \in [0,1]$. Replacing this in (6) together with the definition of the probabilities π_i and observing that the first two terms in (6) represent weighted sums of arithmetic series, we can finally rewrite $C(S,T,m)$ as:

$$C(S,T,m) = \frac{K}{T} + \frac{hDT\left[m(1-p)-1+p^m\right]}{1-p} + \frac{bDTp^m}{1-p} + DTx\left[h - p^m(h+b)\right].$$

The last term above is linear in x. So, depending on the sign of $h - p^m(h+b)$, C is minimized at either $x = 0$ or $x = 1$; hence, at optimum, $S/DT = m \in \mathbb{Z}^+$ and $\bar{I}_m^+ = 0$. □

Therefore, in order to minimize $C(S,T,m)$, we need to only consider values $S \in \mathrm{R}^+$ that satisfy Lemma 5.1; so the problem reduces to evaluating $\min_{S,T,m} C(S,T,m)$ subject to $S/DT = m \in \mathbb{Z}^+$. Using (6) with $S = mDT$ and $m \in \mathbb{Z}^+$, the long-run average cost (per unit time) under end-of-cycle costing becomes:

$$C(T,m) = \frac{K}{T} + \frac{DT\left[hm(1-p) - h + (h+b)p^m\right]}{1-p}. \qquad (7)$$

Note that, since the above only holds for $m \in \mathbb{Z}^+$, the constraint in the general problem formulation on (3) has now been effectively embedded in objective function. We can now evaluate the optimal decision variables (S,T).

Lemma 5.2. (i) The function $C(T,m)$ is discretely convex in m. (ii) The optimal m is given by $m^* = \lceil \log(1-\alpha)/\log p \rceil$, where $\alpha = b/(h+b)$.

Proof. (i) Since $m \in \mathbb{Z}^+$, we need to examine the first forward difference of $C(T,m)$, being $\Delta C(T,m) = C(T,m+1) - C(T,m)$, where Δ is the forward difference operator. Since now $\Delta C(T,m+1) - \Delta C(T,m) = DT(h+b)(1-p)p^m \geq 0$, the first forward difference of $C(T,m)$ is non-decreasing in m and so is discretely convex in m (see Yüceer, 2002).

(ii) The first forward difference of $C(T,m)$ is

$$\Delta C(T,m) = DT\left[h - (h+b)p^m\right].$$

Clearly $\Delta C(T,m)=0$ when $p^m = h/h+b$. Since, by (i) above, $C(T,m)$ is discretely convex in m, there always exists a unique m^* such that $\Delta C(T,m)<0$ for all $m<m^*$ and $\Delta C(T,m)\geq 0$ for all $m\geq m^*$. For this m^* the inequalities $\Delta C(T,m^*-1)<0$ and $\Delta C(T,m^*)\geq 0$ need be satisfied simultaneously, which imply that $p^{m^*} \leq h/(h+b) < p^{m^*-1}$. Solving for m^* the required expression follows. □

This result corresponds directly to the standard Newsvendor condition, as applied to discrete demand systems (e.g. Schmitt et al., 2010). Despite the analogies with its continuous costing counterpart in Lemma 4.1, there is a difference that needs to be pointed out. For the end-of-cycle model, by Lemma 5.1 $S=mDT$ so the determination of m^* leads directly to the locally optimal S (for given T); but, since $m\in\mathbb{Z}^+$, one expects abrupt increases in S whenever $m^*\to m^*+1$. This does not happen for the continuous model where m^* there only represents the floor of the respective S. It should be noted that an identical expression to m^* for the end-of-cycle model above was previously derived by Warsing et al. (2013), as the limiting (deterministic) case of a stochastic system (under conditions equivalent to end-of-cycle costing).

Proposition 5.3: The globally optimal policy variables (S^*,T^*) are:

$T^* = \sqrt{\dfrac{K(1-p)}{D\left[hm^*(1-p)-h+(h+b)p^{m^*}\right]}}$ and $S^*=m^*DT^*$, while the optimal total cost is $C^*=2K/T^*$.

Proof. By simply setting $m=m^*$ in (7), we obtain the partially optimal (in m) total cost $C(T,m^*)$. But this is convex in T, since

$$\frac{\partial^2 C(T,m^*)}{\partial T^2} = \frac{2K}{T^3} > 0$$

for every T; so, the optimal T^* prevails as the unique solution of $\partial C(T,m^*)/\partial T = 0$ as given above, while S^* follows directly. As in Proposition 4.2 the denominator of T^* is always positive. Finally setting $T=T^*$ in $C(T,m^*)$, the expression for C^* prevails. □

It is worth noting that, despite using a different costing scheme, the optimal cost and the optimizers of the end-of-cycle costing model are still structurally identical to those of the EOQ (with backorders) model.

Lemma 5.4. The following properties hold: (i) The optimal m^* is step-wise increasing in p; (ii) The optimal T^* is decreasing in p; (iii) The optimal cost C^* is increasing in p.

Proof. (i) Identical to proof of Lemma 4.3.(i).

(ii) As in the proof of Lemma 4.3(ii), we need to show that T^* is both continuous in p for every $p \in (0,1)$ and that

$$\frac{\partial T^*}{\partial p} = \frac{K(hp-(b+h)(p+m^*(1-p))p^{m^*})}{2DpT(hm^*(1-p)-h+(h+b)p^{m^*})^2} \leq 0 .$$

Continuity follows by using identical steps as in the proof of Lemma 4.3(ii). Now by a similar substitution in this Lemma the numerator of $\partial T^* / \partial p$ after some algebra becomes $-K(b+h)p^{m^*-1}(1-p)(m^*-1) \leq 0$ which completes the proof.

(iii) Identical to proof of Lemma 4.3. (iv). □

Therefore, the behaviour of the optimal cost and its optimizers is directly analogous to that for the continuous costing scheme (see Section 4). Using identical data to those used for the examples of continuous costing, Figures 6 and 7 demonstrate the effects of decreased supply process quality on the optimal cost and optimizers.

6. Numerical Comparisons

Computations were performed in order to obtain insight into the following basic managerial questions, directly related with deciding the parameters of an inventory control policy in real applications: What is the effect of deploying heuristics that either ignore the existence of supply disruptions or use an approximate model on system performance?

To address these issues, we have considered the continuous costing model optimal solution to represent the exact optimal system cost (as happens for deterministic fixed-rate demand). We then used the EOQ (with backorders) solution (for the heuristic that ignores the supply process quality) and the end-of-cycle model solution (for the approximate cost heuristic). Using either of these models, we first evaluated their respective optimal policy variables and then replaced them to the continuous costing cost function in (4) to determine the respective costs. We thus calculated the following performance indicators:

$$\Delta C_0 = \frac{C(S_0, T_0, m_0 = 0) - C^*}{C^*} \text{ and } \Delta C_e = \frac{C(S_e^*, T_e^*, m_e^*) - C^*}{C^*}$$

where C^* represents the optimal cost under the continuous costing model, while the subscripts “0” and “e” refer to an optimal policy variable evaluated using to the EOQ and the end-of-cycle model respectively. Table 1 provides some indicative computational results, all obtained for $K = 100$, $D = 4000$ and $h = 2$, while the other input parameters (b and p) is indicated on the table.

First considering the impact of ignoring the effects of supply disruptions, the results indicate a uniform behaviour across all settings: as process supply quality deteriorates (p increases), the cost difference ΔC_0 always increases. The same holds for the respective optimizers that gradually diverge from their optimal counterparts. This behaviour is totally expected, since the EOQ optimizers carry no implicit safety stock to alleviate the effects of uncertainty. Accordingly, cost differences ΔC_0 increase and become greater with increased of the unit backorders cost. The conclusion is, therefore, clear: the use of models which effectively ignore the process supply quality may have very serious cost implications and should clearly be avoided.

More interesting are the results of using the approximate end-of-cycle cost model for determining the ordering policy variables, primarily since this scheme is dominant in the analysis of periodic review policies (see Section 3). As indicated in the table, the use of the end-of-cycle model generally increases the cost differences ΔC_e. The increase, however, is not uniform, showing a generally smooth trend in p, intercepted by sudden ΔC_e jumps (observe cases b=4 and p=0.4 as well as b=16 and p=0.15). Since this behaviour is always linked with an increase in m_e^*, its cause lies with the structure of the end-of-cycle costing model optimizers, with S_e^* abruptly increasing when optimal $m_e^* \to m_e^* + 1$ (see discussion for Lemma 5.2). At these points, the difference $S_e^* - S^*$ increases, hence the observed deviations in ΔC_e. As expected, the cost impact of this behaviour diminishes for greater m_e^* values (occurring for increased b and p).

On the managerial question, therefore, regarding the suitability of the EOQ model or the end-of-cycle approximate model for the inventory policy parameters evaluation, our results allow the following conclusion: The end-of-cycle costing model, whilst outperforming EOQ, is still suboptimal, leading to serious (even very serious) cost increases; hence, its deployment can only be recommended when the real demand process justifies its basic costing assumption (i.e. $D(t) = DT$ for any $t \in [0,T]$ in Section 5).

7. Concluding Remarks

Deterministic-demand inventory analysis recently surpassed one hundred years of continuing research (Choi, 2014, Cárdenas-Barrón et al., 2014). Following this tradition, we studied a single-echelon inventory installation controlled by the (S,T) inventory policy with endogenous supply disruptions following an independent Bernoulli process. In order to enhance understanding of how costing practices may impact inventory decisions, two different accounting schemes were considered. After modeling long-run average cost, we determined the optimal policy variables in closed-form. While differing in their details, for both costing schemes, the optimal order-up-to level is the solution of a Newsvendor formulation, while the optimal review interval prevails by optimizing the partially optimal (in S) cost function. It is worth noting that the form and the solution of the continuous costing model, constitutes a direct extension of the classical EOQ (with backorders) model, if we allow for no delivery failures (and reduces to the later when $p \to 0$).

We provided analytical properties, demonstrating the impact of the supply process quality on the optimal total cost and the respective optimizers. The behaviour observed for both costing schemes totally agrees with that generally conjectured for stochastic systems under increased (demand or supply) uncertainty. Considering the continuous costing model as an exact representation of the actual cost, we explored the effect of heuristic decision variables determination, using the solution of either the (inaccurate) end-of-cycle model or the classical EOQ model (that ignores supply disruptions). The results demonstrated that, whilst the end-of-cycle model clearly outperformed EOQ, the cost increase resulting from either heuristic may be considerable.

There are several possibilities for further research. One promising direction is to deploy the analytical results for the deterministic-demand system for optimizing stochastic-demand systems. Consider, for example, the model by Warsing et al. (2013), which assumes stochastic demand, while all other assumptions coincide to those of the end-of-cycle cost analysis in Section 5. Recall that Warsing et al. (2013) (as all work with stochastic demand in the area) only optimized the order-up-to-level S. By appropriately applying Jensen's inequality (see Zheng, 1992 and Lagodimos et al., 2017), it appears possible to show that, for any

T, the locally optimal (in S) cost of the stochastic demand system is bounded below by the respective deterministic demand system cost $C(T, m^*)$. But, since the latter is convex in T (by Proposition 5.3), a finite search on T guarantees the determination of the stochastic system globally optimal cost. In addition, it would be interesting to explore the performance of a heuristic where the review interval of the stochastic system is determined by the EOQ-type closed-form solution in Proposition 5.3. Given the good performance of such heuristics observed for inventory systems without disruptions (e.g. Zheng, 1992; Axsater, 1996; Rao, 2003), we expect the results of such an investigation to be positive.

Another direction of research relates with adapting the unified cost model in Section 3 to analyze problem variants obtained by relaxing the initial assumptions. One such assumption is the zero lead time assumption which, although realistic for certain settings, does not universally apply. To facilitate such research, the Appendix shows how the state probabilities π_i and the order-up-to level S of the original cost model need be modified to accommodate a deterministic lead time $L \geq 0$. Notice that, even under such a lead time, the state probabilities (although more complicated) are still computable, since they are given as a mixture of known proximity distributions. Therefore, while it does not appear probable that the analysis of this relaxed model could lead to a closed-form optimal solution, the determination of the respective optimal policy is still possible numerically (using the results of the zero lead time system as bounds).

We feel that more important, however, is to relax the assumption of independent supply disruptions, allowing for the probability of a disruption to decrease if a disruption has already occurred. As in the case of a positive lead time, provided that the resulting state probabilities π_i could be evaluated, the unified cost model in Section 3 (simply by replacing the original π_i by the appropriate new state probabilities) could be directly used for the analysis. Again, even if no closed-form solutions were found to exist, computational results could still demonstrate the impact of dependent disruptions for managerial decision making.

Appendix: General lead time

This appendix presents how the unified cost model in Section 3 needs be modified in order to be used when the zero lead time assumption is relaxed.

Let us suppose a deterministic lead time $L \geq 0$. Since the possible states of the system (i.e. the start-of-cycle inventory level) remain unaffected by the lead time, we need to reevaluate the associated state-probabilities π_i. For this purpose we need to carefully examine the system dynamics.

With a lead time $L \geq 0$, the base stock policy releases replenishment orders in regular time intervals T, so as to raise the system inventory position (i.e. quantity on-hand plus on order) at some target order-up-to level $\tilde{S}$. Therefore, immediately after an ordering decision at any t, there will be exactly $n = \max\{\lceil L/T \rceil, 1\}$ orders in the pipeline, forming the set $\{B_{t-kT} : k = 0,...,n-1\}$. Assuming now that all information on supply losses (due to possible disruptions) is only available at the instant an order is received, then (for any t):

$$B_t = \theta_t B_{t-nT} + DT$$

where θ_t is a Bernoulli random variable with $P(\theta_t = 0) = p$. Therefore, since each order only affects the size of the order to be placed nT time units after its release, the pipeline orders above are mutually independent and identically distributed; building on the exposition in Section 3, $B_{t-kT} = X_{t-kT} DT$ (for $k = 0,...,n-1$) where the random variables X_{t-kT} are independent and follow a geometric distribution $Geo(p)$. Hence, given an ordering decision at t, we can exactly model the system inventory level, J_{t+L} say, at $t+L$:

$$J_{t+L} = \tilde{S} - DL - \sum_{k=0}^{n-1}(1-\theta_{t+(n-k)T}) \cdot B_{t-kT} = \tilde{S} - DL - DT\sum_{k=0}^{n-1}(1-\theta_{t+(n-k)T}) \cdot X_{t-kT}, \tag{A.1}$$

where the second equality follows by replacing the size of each pipeline order. Now, the last term above is a random variable and represents the supply losses associated with the potential failure of pipeline orders to materialize into supplies (due to disruption at the time of their receipt). For example, if all n pipeline orders materialize (that is $\theta_{t+(n-k)T} = 1$ for $k = 0,...,n-1$), there will be zero pipeline losses. Otherwise the losses will directly depend on the exact combination of orders that will not materialize.

To model the probability associated with the pipeline losses, it is convenient to define the random variable

$$V_n = \sum_{k=1}^{n}(1-\theta_k) \cdot X_k$$

where, since all terms in (A.1) are stationary, the random variable indices have been simplified; we seek the probability mass function $\rho_i = P(V_n = i)$, where $i = 0,1,...$ are the pipeline losses. By conditioning on the number of pipeline orders that may fail (a new random variable N) and carefully considering the cases that may occur, we can eventually write:

$$\rho_0 = P(V_n = 0) = (1-p)^n, \quad i = 0$$
$$\rho_i = P(V_n = i) = \sum_{\nu=1}^{n} P(N=\nu)P(V_n = i \,|\, N=\nu) = \sum_{\nu=i}^{n} P(N=\nu)P\left(\sum_{k=1}^{\nu} X_k = i\right) \quad i = 1,2,...$$

Notice the second summation, which ensures that any losses i presuppose the existence at least $N \geq i$ failed pipeline orders (since the size of a pipeline order is at least $1 \cdot DT$). Despite its complicated form, the above mass function is (at least numerically) computable. To see this, observe that the random variable N is binomial $N \sim B(n, p)$ while each of the variables $\sum_{k=1}^{\nu} X_k$ (being sums of ν independent identically distributed geometric variables) follows a negative binomial distribution $\sum_{k=1}^{\nu} X_k \sim NB(\nu, p)$.

On the basis of the above, in order for the unified cost model in (3) to hold for analyzing systems with any positive lead time, we simply need to replace

$$S \to \tilde{S} - DL \text{ and } \pi_{i+1} \to \rho_i, \quad i = 0,1,2,... .$$

Finally, an interesting special case arises when $L < T$ always. Since this implies that $n = 1$, it is straightforward that the above probabilities for ρ_i reduce to the geometric $Geo(p)$ distribution mass function, as used in the analysis presented in the paper (focusing on the limiting case where $L = 0$).

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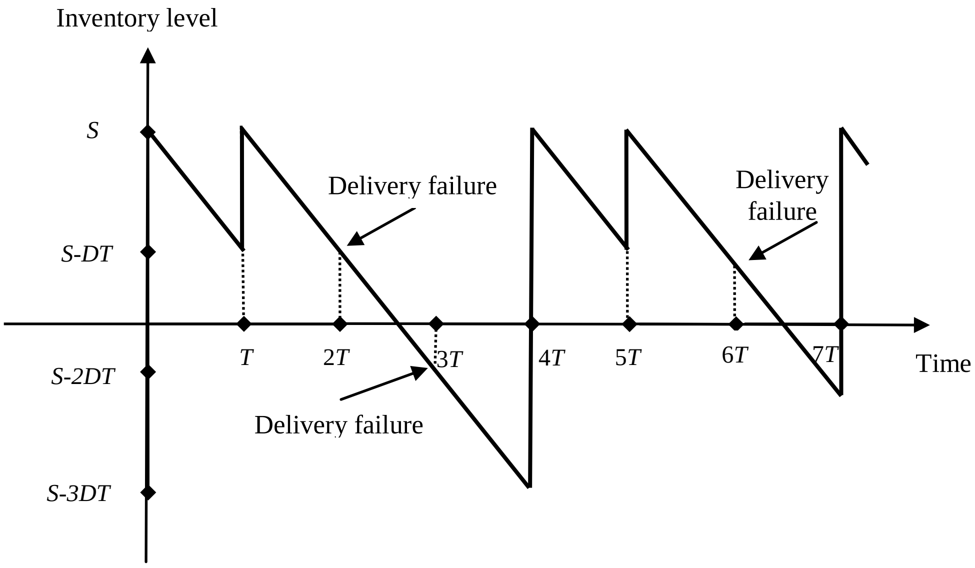

Figure 1. Sample path of the inventory level over time (assuming $D(t) = Dt$)

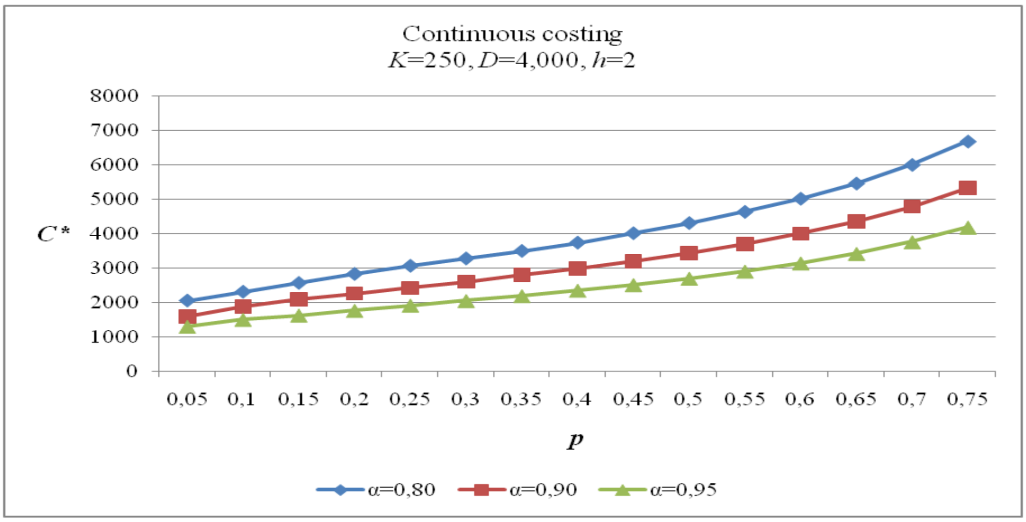

Figure 2. C^* as a function of p (with parameter α).

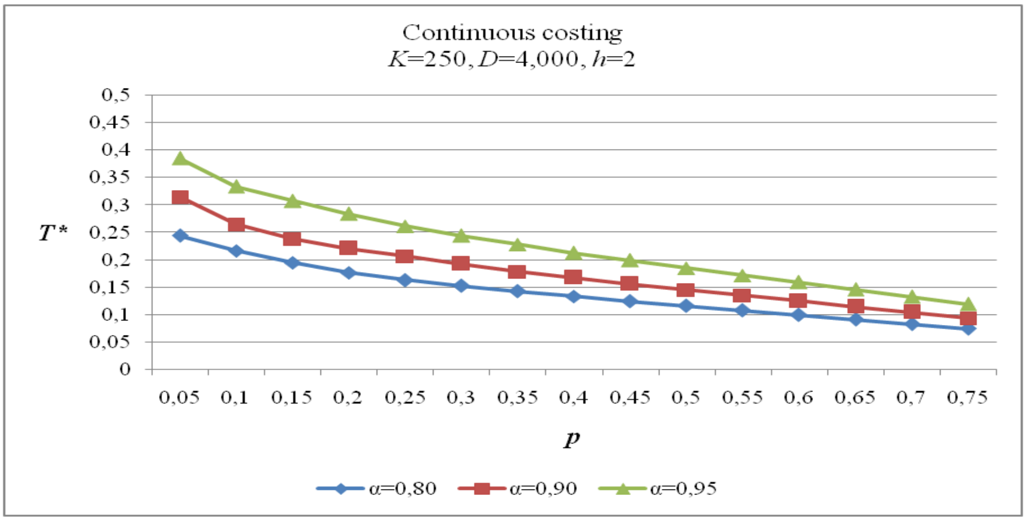

Figure 3. T^* as a function of p (with parameterα).

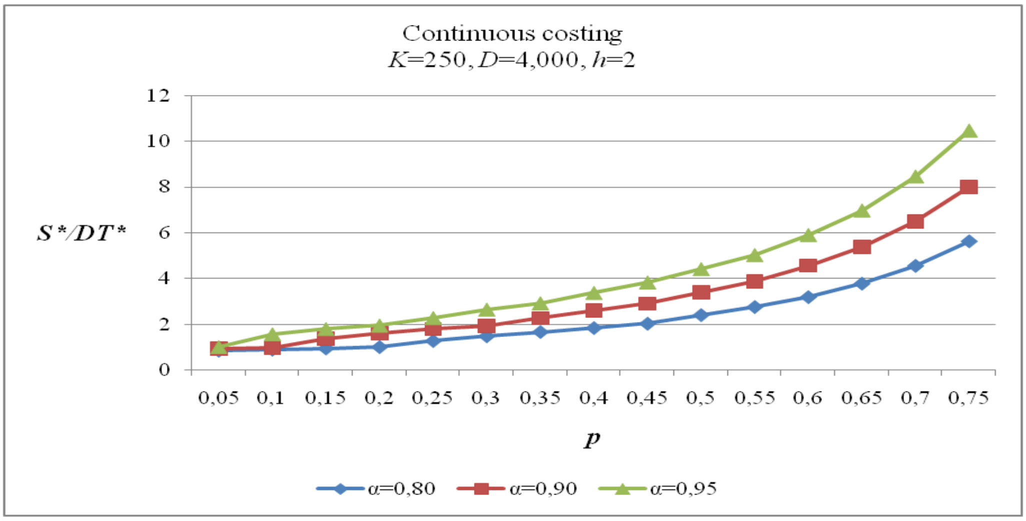

Figure 4. S^*/DT^* as a function of p (with parameter α).

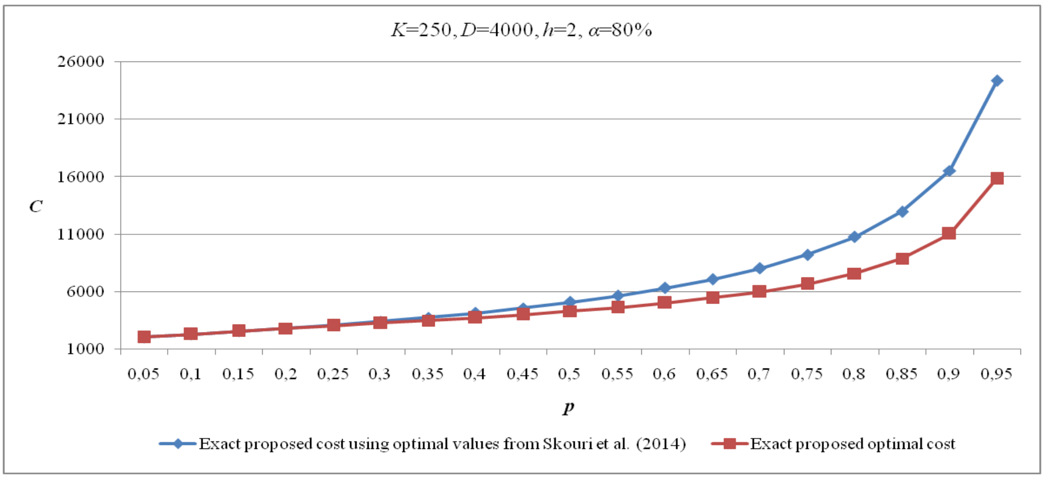

Figure 5. Comparison of optimal and approximate cost by Skouri et al. (2014).

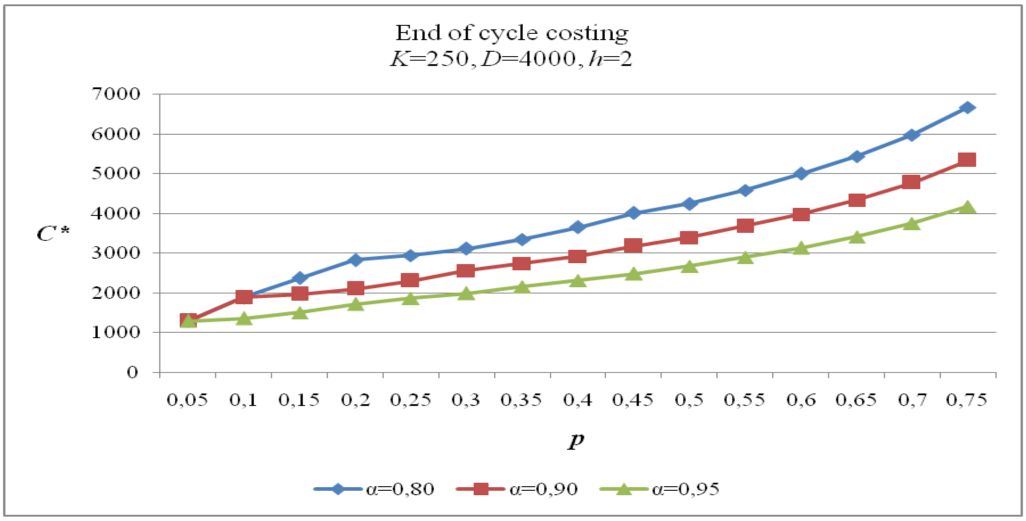

Figure 6. C^* as a function of p (with parameter α).

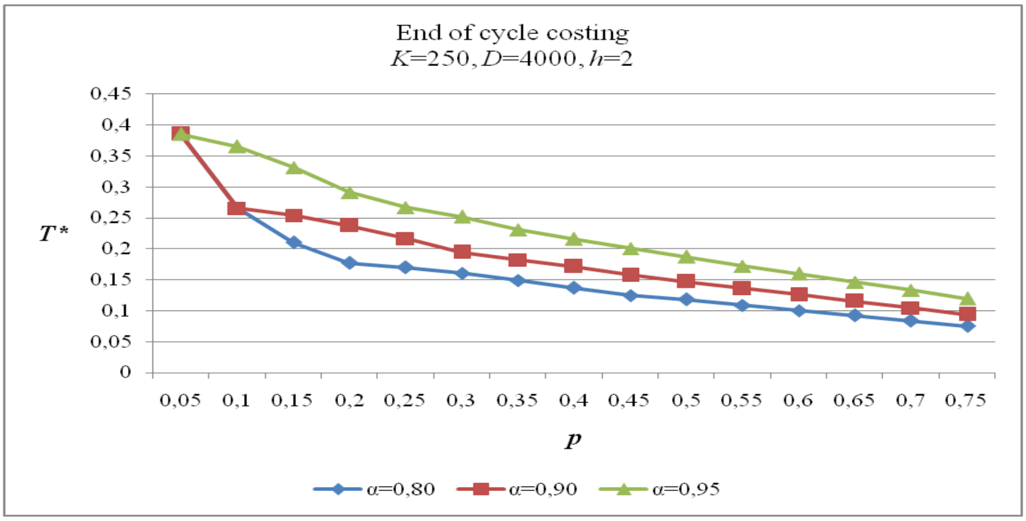

Figure 7. T^* as a function of p (with parameter α).

b	p	Optimal Solution				EOQ Heuristic			End-of-cycle Heuristic			
		m^*	T^*	S^*	C^*	T_0	S_0	$\Delta C_0\%$	m_e^*	T_e^*	S_e^*	$\Delta C_e\%$
4	0.05	0	0.176	494.1	1136.3	0.194	516.4	0.7	1	0.345	1378.4	54.2
	0.10	0	0.161	477.4	1241.3	0.194	516.4	2.6	1	0.237	948.7	21.4
	0.15	0	0.148	465.1	1348.9	0.194	516.4	5.6	1	0.188	752.8	9.5
	0.20	0	0.137	456.4	1460.6	0.194	516.4	9.6	1	0.158	632.5	3.9
	0.25	0	0.127	450.7	1577.6	0.194	516.4	14.6	1	0.137	547.7	1.3
	0.30	0	0.116	447.8	1701.5	0.194	516.4	20.5	1	0.121	483.0	0.2
	0.35	1	0.109	468.4	1833.1	0.194	516.4	27.6	2	0.110	882.6	14.1
	0.40	1	0.102	520.4	1964.1	0.194	516.4	36.7	2	0.105	840.2	8.1
	0.45	1	0.095	560.4	2100.6	0.194	516.4	47.7	2	0.099	788.6	3.9
	0.50	1	0.089	592.3	2250.9	0.194	516.4	60.6	2	0.091	730.3	1.4
	0.55	1	0.083	619.2	2423.2	0.194	516.4	75.5	2	0.083	667.7	0.2
	0.60	2	0.077	666.1	2624.6	0.194	516.4	92.8	3	0.077	921.4	3.4
	0.65	2	0.070	729.0	2856.5	0.194	516.4	114.1	3	0.071	849.1	0.7
	0.70	3	0.064	788.3	3139.8	0.194	516.4	140.1	4	0.064	1021.3	2.0
16	0.05	0	0.136	508.1	1473.4	0.168	596.3	3.1	1	0.172	689.2	4.5
	0.10	0	0.115	456.0	1732.8	0.168	596.3	10.1	1	0.119	474.3	0.1
	0.15	1	0.103	536.5	1946.0	0.168	596.3	21.9	2	0.109	868.0	12.5
	0.20	1	0.095	592.7	2099.7	0.168	596.3	39.2	2	0.102	816.5	5.8
	0.25	1	0.089	619.6	2247.6	0.168	596.3	59.2	2	0.094	751.5	2.0
	0.30	1	0.083	630.7	2409.3	0.168	596.3	81.0	2	0.085	680.3	0.3
	0.35	2	0.077	661.5	2591.7	0.168	596.3	104.6	3	0.0780	935.9	6.3
	0.40	2	0.072	724.2	2772.0	0.168	596.3	132.3	3	0.074	885.9	2.2
	0.45	2	0.067	761.1	2964.7	0.168	596.3	164.0	3	0.068	820.6	0.3
	0.50	3	0.063	808.9	3186.8	0.168	596.3	199.4	4	0.063	1011.9	2.7
	0.55	3	0.058	872.3	3428.4	0.168	596.3	240.9	4	0.059	941.8	0.3
	0.60	4	0.054	939.2	3710.8	0.168	596.3	288.9	5	0.054	1084.7	1.1
	0.65	5	0.0495	1014.0	4040.5	0.168	596.3	346.1	6	0.050	1190.3	1.4
	0.70	6	0.045	1114.4	4440.3	0.168	596.3	415.7	7	0.045	1263.6	0.9
24	0.05	0	0.125	485.3	1601.6	0.165	607.6	5.0	1	0.141	562.7	1.0
	0.10	1	0.104	525.1	1914.1	0.165	607.6	17.1	2	0.110	879.9	15.0
	0.15	1	0.096	601.0	2094.1	0.165	607.6	38.9	2	0.103	827.7	6.3
	0.20	1	0.089	628.5	2251.8	0.165	607.6	64.6	2	0.094	755.9	2.1
	0.25	1	0.083	634.6	2424.2	0.165	607.6	92.2	2	0.085	676.1	0.2
	0.30	2	0.076	674.8	2617.0	0.165	607.6	121.6	3	0.078	931.9	6.1
	0.35	2	0.072	735.6	2797.6	0.165	607.6	156.3	3	0.073	881.0	2.0
	0.40	2	0.067	766.8	2989.4	0.165	607.6	195.4	3	0.068	813.5	0.2
	0.45	3	0.062	818.6	3208.6	0.165	607.6	238.5	4	0.063	1007.7	2.6
	0.50	3	0.058	876.7	3439.6	0.165	607.6	288.7	4	0.059	939.6	0.3
	0.55	4	0.054	939.8	3706.4	0.165	607.6	345.6	5	0.054	1087.0	1.3
	0.60	5	0.050	1003.2	4008.6	0.165	607.6	412.4	6	0.050	1198.0	1.9
	0.65	5	0.046	1092.9	4364.8	0.165	607.6	491.5	6	0.046	1100.6	0.02
	0.70	7	0.042	1204.0	4797.4	0.165	607.6	587.3	8	0.042	1336.6	0.7

Table 1. Results for optimal policy and heuristics under different settings (b and p)