

# STOCHASTIC FRONTIER MODELS WITH CORRELATED EFFECTS

Giannis Karagiannis<sup>1</sup> and Magnus Kellermann<sup>2</sup>

<sup>1</sup> Professor, Department of Economics, University of Macedonia, Thessaloniki, Greece;  
[karagian@uom.edu.gr](mailto:karagian@uom.edu.gr)

<sup>2</sup> Researcher, The Bavarian State Research Center for Agriculture, Munich, Germany;  
[magnus.kellermann@yahoo.de](mailto:magnus.kellermann@yahoo.de)

**ABSTRACT:** In this paper we provide several new specifications within the true random effects model as well as stochastic frontiers models estimated with GLS and MLE that enrich modeling choices when distinguishing between heterogeneity and efficiency. The main feature of the proposed specifications is that they enlarge the set of heterogeneity covariates beyond that of Mundlak's adjustment terms to include environmental factors that are not under the control of producers but affect the operation conditions of the production units. These environmental factors may be time varying or time invariant and not all may be correlated with heterogeneity.

**KEYWORDS:** Stochastic frontier models; heterogeneity; adjustment terms; environmental factors

**AKNOWLEDGEMENT:** An earlier version of this paper was presented at the 2017 International Panel Data Conference in Thessaloniki, Greece. We would like to thank session participants and especially Peter Schmidt for a constructive discussion. We would also like to thank two anonymous referees for helpful comments and suggestions.

November 2018

# STOCHASTIC FRONTIER MODELS WITH CORRELATED EFFECTS

## 1. Introduction

The true effects models developed by Greene (2005a, b) are undoubtedly one of the most important recent contributions in stochastic frontier analysis. Their main advantage is that they allow individual effects to exist alongside inefficiency in such a way that we can distinguish between heterogeneity and technical efficiency, thereby providing more accurate performance evaluations. Standard stochastic frontier models have failed one way or the other to deal with this issue. In particular, stochastic frontier models making no distributional assumptions about the one-sided error term capturing technical inefficiency (i.e., Schmidt and Sickles, 1984) confound heterogeneity and inefficiency because whatever is not accounted for by factor inputs is attributed to technical inefficiency (Greene, 2004) while stochastic frontier models making distributional assumptions (e.g., Pitt and Lee, 1981) simply ignored (assumed away) heterogeneity. The direct consequence of these modeling limitations may be inaccurate estimates of efficiency. That is, in the former case, not distinguishing between heterogeneity and efficiency may lead to an overestimation (underestimation) of technical inefficiency for production units that are subject to unfavorable (favorable) individual effects while in the latter case, neglecting heterogeneity that is asymmetrically distributed among production units may result in biased estimates of technology parameters (due to omitted variables), significantly inflated estimates of technical inefficiency (Sherlund, Barrett and Adesina, 2002) and increased dispersion of efficiency scores (Tybout, 2000).<sup>1</sup>

However, the true effects models that distinguish between heterogeneity and inefficiency are unfortunately not without problems. On the one hand, the incidental parameter problem in the true fixed effects model could affect the inefficiency estimates as indicated by Greene's (2005a) simulation.<sup>2</sup> On the other hand, the true random effects model may result in biased estimates of technology parameters when the unobserved factors are correlated with the explanatory variables, i.e., input quantities (Farsi, Filippini and Greene, 2005). The inconsistent estimates of the technology (slope) parameters will then bias the estimated variance of the composed error term used in the Jondrow *et al.* (1982) procedure to

estimate technical efficiency. In addition, as all time-invariant effects are incorporated into the individual effects, we cannot account for any persistent inefficiency and thus, technical inefficiency tends to be underestimated (Last and Wetzel, 2010).<sup>3</sup>

Under these circumstances, the true random effects model with Mundlak's (1978) adjustment terms, proposed by Farsi, Filippini and Kuenzle (2005) and Farsi, Filippini and Greene (2005), appears as a promising alternative because it can at the same time reduce the heterogeneity bias in both the technology (slope) parameters and the inefficiency estimates.<sup>4</sup> The *Mundlak true random effects (M-TRE) model* is based on the assumption that unobserved heterogeneity is correlated with the group means of the explanatory variables, which in the case of production frontiers refers to input quantities. By controlling for (some of the) unobserved heterogeneity and separating the correlation effects by means of the auxiliary equations, the *M-TRE* model decreases the bias in the inefficiency estimates without affecting the consistency of the estimated technology parameters. In fact, when the error term in the estimated equation is a composite asymmetric term, as with stochastic frontier models, heterogeneity bias will be reduced, given that the correlation between the individual effects and the explanatory variables is partly captured in the model (Farsi, Filippini and Kuenzle, 2005).

This paper attempts to contribute to this strand of the literature by proposing two alternative models in the spirit of the *M-TRE* model.<sup>5</sup> In both of them we try to further improve the ability of the true random effects model to account for heterogeneity by enlarging further the set of potential correlates in order to increase the portion of measured heterogeneity and squeeze the impact of heterogeneity bias on the estimated technology parameters and technical efficiency. The first model allows heterogeneity to be correlated with the group means of input quantities as well as a set of relevant environmental variables that are beyond producers' control but account for the operating conditions with which the production units have to cope with and most likely tend to differ across firms. The second model takes even a broader view by assuming that heterogeneity may not be correlated with all of the technology related explanatory variables. For example, heterogeneity is most likely uncorrelated with the neutral component of disembodied technical change as long as production units are not involved in R&D activities, as is the case in agriculture and several service industries. The same may also be true for a subset of environmental variables, such as those affecting the operating conditions in a uniform way but change over time (e.g., policy variables). Notice that both proposed models employ a more general specification of correlated individual effects than the *M-TRE* model.<sup>6</sup>

Besides the true effects type of models, accounting for endogenous individual effects gives rise to several new variants of standard stochastic frontier models. We present two such variants without distributional assumptions, which can be viewed as extensions of the fixed- and the random-effects stochastic frontier models introduced by Schmidt and Sickles (1984) and developed further by Good *et al.* (1993) to account for environmental factors and by Farsi, Filippini and Kuenzle (2005) to incorporate Mundlak's adjustment terms. Moreover, we propose other three models making distributional assumptions that complement previous attempts by Coelli, Perelman and Romano (1999) and Sherlund, Barrett and Adesina (2002) to account for environmental factors in maximum-likelihood type of stochastic frontier models.

The main reason for incorporating endogenous individual effects in applied efficiency analysis is that they may improve econometricians' ability to account for heterogeneity that is unobserved for them but not for producers, who adjust their input decisions conditional on their underlying environmental factors. We expect that at least part of what is considered to be individual heterogeneity may be accounted for by introducing some control variables.<sup>7</sup> These variables include environmental factors affecting firms' operating conditions and Mundlak's adjustment terms, which in the case of production frontiers correspond to individual means of input quantities. The only difference is that in the proposed conventional stochastic frontier models it is assumed that all heterogeneity is captured by the control variables whereas in the true effects models, the included control variables account only for part of the individual effects and the rest is treated as unobserved heterogeneity by means of random effects.

We provide an empirical evaluation of the proposed models using data for a sample of specialized German dairy farms during the period 2003–2008. This is especially appealing due to the fact that in agriculture farms are prone to heterogeneous production conditions. For this reason, models that fail to account for different production and environmental conditions have been criticized in the literature; see for example Sherlund, Barrett and Adesina (2002) and Abdulai and Tietje (2007). We explicitly consider this and we provide some empirical evidence on the extent of the differences in the estimated technical efficiency scores by using alternative specifications to account for endogenous individual effects and/or different model assumptions.

The remaining of this paper is organized as follows: in the next section, we present the proposed model specifications and outline their estimation procedures. The data and the

empirical model are described in the third section. The comparative empirical results are discussed in the fourth section. Concluding remarks follow in the last section.

## 2. Models Specification and Estimation

Following Greene (2005a, b) let the true effects stochastic production frontier model be given as:

$$y_{it} = \beta_0 + f(x_{it}) + \alpha_i - u_{it} + v_{it} \quad (1)$$

where  $i$  is used to index production or decision making units and  $t$  time periods,  $y$  refers to (the log of) output quantity and  $x$  to (the log of) input quantities,  $f(\cdot)$  is the functional form of the production function not including the intercept term  $\beta_0$ ,  $\alpha_i$  represents firm-specific (individual) effects or unobserved heterogeneity,  $-u_{it}$  is a one-sided non-negative error term measuring (the log of) technical efficiency, and  $v_{it}$  is a symmetric and normally distributed error term, which plays the role of statistical noise accounting for (i) unanticipated production shocks that firms do not observe when making their input decisions and (ii) econometricians' weaknesses related to omitted explanatory variables, measurement errors in the dependent variable, and functional form discrepancies. In addition, the following distributional assumptions are made for these error terms: both are independent and identically distributed (*iid*) as  $v_{it} \sim N(0, \sigma_v^2)$  and  $u_{it} \sim N^+(0, \sigma_u^2)$  (half-normal) and are also uncorrelated with (or distributed independently of) input quantities  $x$  and each other. The former presupposes that producers are unaware of their technical efficiency before they make their input decisions.<sup>8</sup> In such a case and assuming that they maximize expected profits, the quantities of (variable) inputs are largely predetermined and hence uncorrelated with technical efficiency (Zellner, Kmenta and Dreze, 1966).<sup>9</sup> Moreover, technical efficiency is assumed to be stochastically (randomly) time varying as no particular form is specified for its time pattern. In other words, inefficiency is not persistent and "each period brings about new idiosyncratic elements thus new sources of inefficiency. This is a reasonable assumption, particularly in industries that are constantly facing new technologies" (Farsi, Filippini and Greene, 2005, p. 77). The implication of this assumption is that the observations of the same production unit are considered as independent sample points.

Following Greene (2005a, b) there are alternative ways of modelling the firm-specific effects in (1). One way is to treat  $\alpha_i$  as fixed effects that are correlated with input quantities even though  $\alpha_i$  and  $x_{it}$  are assumed to be uncorrelated with both  $u_{it}$  and  $v_{it}$ . This specification corresponds to the *true fixed effects (TFE) model*. Another way is to think of

heterogeneity as being an independent and identically distributed (*iid*) variable with  $\alpha_i \sim N(0, \sigma_\alpha^2)$ . This results in the *true random effects (TRE) model*, where  $u_{it}$ ,  $v_{it}$  and  $\alpha_i$  are assumed to be uncorrelated with input quantities and each other.

On the other hand, the *M-TRE* model, as given by Farsi, Filippini and Kuenzle (2005) and Farsi, Filippini and Greene (2005), is based on the assumption that the individual effects are a linear function of the group means of all explanatory variables (i.e., input quantities) across time, namely:

$$\alpha_i = \pi' \bar{x}_i + \delta_i \quad (2)$$

where  $\pi$  are parameters to be estimated and a bar over a variable denotes its group mean, i.e.,  $\bar{x}_i = (1/T_i) \sum x_{it}$  and  $\delta_i$  refers to individual heterogeneity. By substituting (2) into (1) we obtain an estimable form of the *M-TRE* model:

$$y_{it} = \beta_0 + f(x_{it}) + \pi' \bar{x}_{it} + \delta_i + e_{it} \quad (3)$$

where  $e_{it} = -u_{it} + v_{it}$  is a composite asymmetric error term equal to the sum of two orthogonal error terms, one reflecting inefficiency and the other statistical noise. In estimating (3), Farsi, Filippini and Kuenzle (2005) and Farsi, Filippini and Greene (2005) have treated  $\delta_i$  as pure unobserved heterogeneity, reflecting that part of firm-specific (individual) effects which cannot be explained by observed factors, namely, the group means of input quantities. To deal with this unobservable component they assumed that it is independent and identically distributed (*iid*) as  $\delta_i \sim N(0, \sigma_\delta^2)$ . As a random-effects model, (3) assumes that  $e_{it}$  (and thus  $v_{it}$  and technical inefficiency) is uncorrelated with pure unobserved heterogeneity  $\delta_i$  and input quantities. However, with some fixed-effects elements being inherent in Mundlak's adjustment terms,  $\alpha_i$  is correlated with the group means of input quantities as in (2), which is expected to deteriorate the heterogeneity bias. Under this setup, the *M-TRE* model is estimated with simulated maximum likelihood (see Greene, 2005a, b) and technical efficiency estimates are obtained as  $E(u_{it}|e_{it})$  by using the Jondrow *et al.* (1978) estimator (Greene, 2004).<sup>10</sup>

A potential limitation of the *M-TRE* model is that it accounts only for heterogeneity reflected in input quantities. The impact of other variables affecting heterogeneity, such as those related to “environmental factors”, is not accommodated in the *M-TRE* model.<sup>11</sup> We consider as environmental “those factors which (...) are taken as not within the management's field of choice” (Hall and Winsten, 1959, p. 72) and thus are not under its control but account

for the operating conditions with which the production units have to cope. Most likely they tend to differ across firms and perhaps some of them also across time. Although there are industries in which firms have considerable control over their operating conditions, one can find examples for which this is not the case. For example, population density and per capita income might be considered as two environmental factors in evaluating the performance of retail distribution firms (Hall and Winsten, 1959) while several geographic and demographic features of the regions or countries served are important to accurately estimate efficiency in the airline industry (Coelli, Perelman and Romano, 1999). Agro-ecological conditions, topography, pest infestation and (plant or animal) diseases are important environmental factors in estimating farm efficiency (Mundlak, 1961; Sherlund, Barrett and Adesina, 2002).

To accommodate these concerns into (2) we extend the *M-TRE* model in two directions: *first*, we adopt a variant of Mundlak's model, first appearing in Maddala (1987), that includes time-invariant environmental factors  $z_i$  into the auxiliary equation explaining individual effects and *second*, we incorporate time-varying environmental factors  $z_{it}$  into the production function  $f(\cdot)$ . With these modifications, the auxiliary equation (2) is written as:

$$\alpha_i = \pi' \bar{x}_i^* + \gamma' z_i + \delta_i \quad (4)$$

where  $\gamma$  are parameters to be estimated and  $x_{it}^* = (x_{it}, z_{it})$ . Thus with (4) we attempt to improve the ability of the *TRE* model to account for heterogeneity by including a set of relevant environmental factors along with firm-specific means of input quantities. By substituting (4) into (1) we obtain the following equation:

$$y_{it} = \beta_0 + f(x_{it}^*) + \pi' \bar{x}_i^* + \gamma' z_i + \delta_i + e_{it} \quad (5)$$

which we refer to as the *Mundlak-Maddala true random effects (MM-TRE) model*. As the *M-TRE* model, the *MM-TRE* model is estimated with simulated maximum likelihood and technical efficiency estimates are obtained as  $E(u_{it}|e_{it})$  by using the Jondrow *et al.* (1978) estimator. From (5) and (3) one can verify that the *M-TRE* is nested in the *MM-TRE* model. Hence, the decision as to which model should be used for the data at hand can be based on the likelihood ratio test.

Yet another modeling alternative emerges by noticing that the firm-specific effects are not necessarily correlated with all technology and environmental variables; in a completely different setup this idea was introduced by Hausman and Taylor (1981). For example, individual effects are most likely uncorrelated with quasi-fixed inputs and neutral

disembodied technical change. According to Griliches and Mairesse (1998, p. 385) “if one accepts the notion that the quasi-fixed inputs are predetermined for the duration of the relevant observation period” then their quantity is uncorrelated with the individual effects. Similarly, for disembodied technical change (modeled via a time trend) that has a common impact (i.e., shift) on the production technology of all producers is uncorrelated with heterogeneity. On the other hand, all symmetrically distributed time varying environmental variables may have an impact on the position of the production function but that is going to be uniform for all producers. Such variables could be dummies reflecting policy changes and even rainfall measures for tightly defined geographical areas. However, under certain circumstances the uncorrelatedness assumption can also apply to input quantities; e.g., in their study for the US airline industry, Cornwell, Schmidt and Sickles (1990) argued that labor and material inputs were uncorrelated with individual effects.

We accommodate these aspects by partitioning the  $x_{it}^*$  vector into a group of technology and environmental variables  $x_{1i}^* = (x_{1i}, z_{1it})$  that are correlated with heterogeneity (firm-specific effects) and another group of variables  $x_{2it}^* = (x_{2it}, z_{2i})$  that are not. Then we can write the auxiliary equation as:

$$\alpha_i = \pi_1' \bar{x}_{1i}^* + \gamma' z_i + \delta_i \quad (6)$$

where  $\pi_1$  are parameters to be estimated. By substituting (6) into (1) we obtain the following equation:

$$y_{it} = \beta_0 + f(x_{it}^*) + \pi_1' \bar{x}_{1i}^* + \gamma' z_i + \delta_i + e_{it} \quad (7)$$

which we refer to as the *Hausman-Taylor true random effects (HT-TRE) model*. As for the other two models, the *HT-TRE* model is estimated with simulated maximum likelihood and technical efficiency estimates are obtained as  $E(u_{it}|e_{it})$  by using the Jondrow *et al.* (1978) estimator. The empirical validity of the *HT-TRE* model can be tested against both the *M-TRE* and the *MM-TRE* models using the likelihood ratio test.

In the previous models we have followed Greene (2005a, b) by interpreting the  $\delta_i$  terms in (3), (5) and (7) as pure unobserved heterogeneity. Therefore, we have assumed that the group means of input quantities and of environmental factors only partially explain individual effects. Alternatively, the auxiliary equations in (2), (4) and (6) can be thought of as pure regression equations with  $\delta_i$  representing statistical noise, where individual effects are related to a vector of observed characteristics, including environmental factors and group

means of input quantities, subject to a statistical error. This is equivalent to saying that if we include all the relevant variables controlling for heterogeneity in (2), (4) and (6), then measured heterogeneity can account for individual effects up to a statistical error. In this case,  $v_{it} + \delta_i$  is an independent and identically distributed (*iid*) error term with zero mean and constant variance i.e.:  $(v_{it} + \delta_i) \sim iid N(0, \sigma_v^2 + \sigma_\delta^2)$ . Equations (3), (5) and (7) may be adjusted accordingly as:

$$y_{it} = \beta_0 + f(x_{it}) + \pi' \bar{x}_i + \varepsilon_{it} \quad (8)$$

$$y_{it} = \beta_0 + f(x_{it}^*) + \pi' \bar{x}_i^* + \gamma' z_i + \varepsilon_{it} \quad (9)$$

$$y_{it} = \beta_0 + f(x_{it}^*) + \pi_1' \bar{x}_{1i}^* + \gamma' z_i + \varepsilon_{it} \quad (10)$$

where  $\varepsilon_{it} = (v_{it} + \delta_{it}) - u_{it}$  is a composite asymmetric error term equal to the sum of two orthogonal error terms one reflecting statistical noise (the sum of the two terms in the parenthesis) and the other technical inefficiency. As it is necessary to make distributional assumption to estimate models (8)-(10) and we follow the same line of reasoning as above in referring to them as the *M-MLE*, *MM-MLE* and *HT-MLE* models, respectively. These models complement and enrich previous work by Coelli, Perelman and Romano (1999) and Sherlund, Barrett and Adesina (2002) in accounting for environmental factors in MLE stochastic frontier models.

In contrast to the true effects models in (3), (5) and (7) and to keep track with the tradition of MLE stochastic frontier models, we assume that technical inefficiency is deterministically time-varying. In particular, we adopt Battese and Coelli (1992) formulation  $u_{it} = \exp(-\eta(t - T))u_i$ , where  $\eta$  is a parameter to be estimated and  $u_i$  is time invariant technical efficiency which is assumed to be independent and identically distributed (*iid*) with  $N^+(0, \sigma_u^2)$ . If the estimated value of  $\eta$  is positive (negative), technical efficiency tends to improve (deteriorate) over time, while if  $\eta = 0$  technical efficiency is time-invariant. This formulation allows inefficiency to evolve smoothly through time, with its movement being monotonic and the same for all production units. Moreover, the most efficient firm does not change over time. However, these two aspects of the Battese and Coelli (1992) formulation are consistent with the stylized facts, as documented in Bartelsman and Doms (2002) survey, on the uniformity of changes in efficiency across production units as well as the persistence of efficiency differentials over time.

In the previous models, we treat the  $\delta_i$ 's as either pure unobserved heterogeneity or statistical noise. These are not however the only possible interpretations. A third alternative, offered by Hay and Liu (1997), views  $\delta_i$  and  $u_{it}$  as two distinct components of technical

inefficiency, namely long- and short-run. Long-run inefficiency reflects persistent differences in the quality of management due to innate abilities and business experience as well as differences in the ability or lack of expertise to utilize the available technology. On the other hand, short-run inefficiency is allowed to vary over time because we expect management to raise its effort in response to internal and external (i.e., competitive) pressure. The aforementioned interpretation of long-run inefficiency can be accommodated by treating  $\delta_i$  as a one-sided error term that is independent and identically distributed (*iid*) with constant mean (which is positive) and variance.<sup>12</sup>

However, if the assumption of time-invariant efficiency is tenable, then we can assume away short-run (time-varying) technical inefficiency; that is,  $u_{it} = 0$ .<sup>13</sup> The idea of time invariant inefficiency is inherent in Jovanovic's (1982) "passive learning" model, where firms are "born" with a fixed level of efficiency. Then, firms endowed with a relatively low efficiency level eventually have to exit the market while the surviving firms exhibit efficiency persistence. Under these circumstances and following the initiative of Farsi, Filippini and Kuenzle (2005), who proposed (11) below, we could extend the class of random effect models suggested by Schmidt and Sickles (1984) with the following three models by adjusting accordingly (3), (5) and (7) as:

$$y_{it} = \beta_0^* + f(x_{it}) + \pi' \bar{x}_i + \omega_{it} \quad (11)$$

$$y_{it} = \beta_0^* + f(x_{it}^*) + \pi' \bar{x}_i^* + \gamma' z_i + \omega_{it} \quad (12)$$

$$y_{it} = \beta_0^* + f(x_{it}^*) + \pi_1' \bar{x}_{1i}^* + \gamma' z_i + \omega_{it} \quad (13)$$

where  $\beta_0^* = \beta_0 - E(\delta_i)$  and  $\omega_{it} = v_{it} - [\delta_i - E(\delta_i)]$  is an independent and identically distributed (*iid*) error term with zero mean and constant variance, and  $E$  refers to the expectation operator.<sup>14</sup> The above three models, which we refer to as *M-GLS*, *MM-GLS*, and *HT-GLS*, can be estimated with GLS along the lines suggested by Schmidt and Sickles (1984) without making distributional assumptions about the  $\delta_i$  term as in the true effects models. Moreover, the choice among the three aforementioned models can be made by means of a Wald test on the joint significance of the additional variables.

Firm-specific estimates of technical efficiency are obtained by first computing an estimate of  $\beta_0^* + \omega_{it}$  as  $y_{it} - \hat{f}(x_{it}) - \hat{\pi}' \bar{x}_i$ ,  $y_{it} - \hat{f}(x_{it}^*) - \hat{\pi}' \bar{x}_i^* - \hat{\gamma}' z_i$ , and  $y_{it} - \hat{f}(x_{it}^*) - \hat{\pi}_1' \bar{x}_{1i}^* - \hat{\gamma}' z_i$  using respectively the estimated parameters (denoted by a hat) of the models (11), (12) and (13). From those residuals we can recover estimates of the individual-firm intercepts in (11), (12) and (13) as  $\hat{\beta}_i = (1/T) \sum \xi_{it}$  (Schmidt and Sickles, 1984). Then using

the normalization  $\hat{\beta}_0 = \max_i(\hat{\beta}_i)$ , we may derive estimates of the firms' inefficiency from  $\hat{\delta}_i = \hat{\beta}_0 - \hat{\beta}_i$ .

### 3. Empirical Model and Data Description

We apply the three types of models discussed in the previous section to a panel dataset of 466 Bavarian farms specialized in dairy production. The data covers the period 2003–2008 and includes a total of 2409 observations.<sup>15</sup> We estimate the three types of models in their conventional specification, augmented with environmental factors as in Good *et al.* (1993) for the GLS models and Coelli, Perelman and Romano (1999) for the MLE models as well as with the three discussed extensions. Hence in total we estimate 15 different stochastic frontier models and examine their respective results. In order to keep the empirical exercise simple we employ a single-output, multiple input Cobb-Douglas production function to represent the production technology. Needless to say, all discussed models are in no way limited *a priori* to this functional form.

The output variable includes farm's total revenues from dairy production. As input variables we consider the following: *labor*, measured in full-time equivalents (including family labor as well as hired workers); *land*, measured in ha; *material*, including expenses for forage production, veterinary services, purchased feed and other related expenses and *capital*, measured as the end-of-year value of all farm related machinery, equipment and buildings as well as the livestock. All monetary values were deflated using appropriate price indices obtained from the German Federal Statistical Office (Destatis, 2012). We show the descriptive statistics of the input and output variables in the upper part of Table 1.

The basic production frontier specification of all three types of models includes just the four inputs as explanatory variables. For the models including Mundlak's adjustment terms (i.e., (3), (8) and (11)) we construct an additional set of variables  $\bar{x}_{ik} = \frac{1}{T_i} \sum x_{itk}$ , where  $k = 1, \dots, 4$ . For the models including the Mundlak-Maddala adjustment terms (i.e., (5), (9) and (12)), we also include a set of environmental variables, which contains time-invariant dummy variables for part-time farming and dummy variables indicating the location of the farm in different regions in Bavaria.<sup>16</sup> As a time-varying environmental variable we consider is the share of owned land. The descriptive statistics for these variables are given in the lower panel of Table 1.

The intuition that only some and not all environmental and technology factors are correlated with unobserved heterogeneity leads to the Hausman-Taylor adjustment as in (7),

(10) and (13). In principle, any technology or environmental variable could be assumed to be independent of unobserved heterogeneity. However, for many variables, especially inputs, this is a strong assumption that has to be based on conclusive arguments. On the other hand, as previously discussed, there are also natural candidates such as quasi-fixed inputs or disembodied technical change modeled via a time trend. Since we cannot fully justify the uncorrelatedness assumption for any of the aforementioned input or environmental variables we proceed with the Hausman-Taylor adjustment by simply adding a linear time trend, which by construction is uncorrelated with time invariant effects.

#### 4. Discussion of Empirical Results

As a first check for correlated individual effects, we test the null hypotheses that  $H_0: \alpha_i \perp x_{itk}$  using a Hausman test on the conventional random effects model. The test clearly rejects the hypotheses of no correlation between the effects and the input variables, with a test statistic of 317.1 against a critical value of  $\chi^2_{(4;0.01)} = 13.3$ . Even if this test only applies to models estimated by GLS, the result indicates that most likely all models assuming exogenous random effects will provide biased estimates for the technology parameters.

We present the estimated coefficients of all model specifications in Tables 2 to 4. The estimated parameters for the input variables obtained from the *M-GLS* model are, as was shown by Mundlak (1978), identical to those of a fixed effects model and thus are unaffected by heterogeneity bias.<sup>17</sup> From the third column in Table 2 we can see that all four parameters related to Mundlak's adjustment terms are significantly different than zero. Under the assumption that the remaining time-invariant differences between farms are due to inefficiency, we obtain the respective efficiency scores from the group means of the residuals by applying the normalization described in Schmidt and Sickles (1984). The estimated standard deviation  $\sigma_\delta$  is therefore equivalent to the standard deviation of the inefficiency term. If we compare  $\sigma_\delta$  from columns 1 and 3 in Table 2 we can see a slight decrease in the estimated standard deviation of the inefficiency term from 0.1417 to 0.130. This is expected as the *M-GLS* model is intended to capture all unobserved time-invariant heterogeneity that is correlated with the input variables. This in turn helps to reduce heterogeneity bias in the inefficiency estimates. Hence, as in the Schmidt and Sickles (1984) fixed effects SFA model, the *M-GLS* model provides in principle unbiased estimates of the slope parameters, without capturing all time-invariant differences in the fixed-effects inefficiency term.

The results in column 2 of Table 2 refer to the specification proposed by Good *et al.* (1993). We find that the technology parameters related to the input variables lie in between

those of the conventional random effects model estimated by GLS and the *M-GLS* model. In addition, six out of seven environmental variables are statistically significant. Comparing the standard deviation of the inefficiency term  $\sigma_\delta$  from columns 1 and 2 we can see a marginal reduction from 0.1417 to 0.1368. These findings imply that the inclusion of the environmental variables helps to reduce heterogeneity bias in the technology parameters and to a lesser extent in the inefficiency estimates.

In the *MM-GLS* specification, we combine Mundlak's adjustment terms with the set of environmental variables. In this specification, the term  $\alpha_i = \pi_{\bar{x}}\bar{x}_i + \pi_{\bar{z}}\bar{z}_i + \gamma z_i$  takes into account not only unobserved heterogeneity correlated with input into but also heterogeneity related to time-varying and time-invariant environmental factors. As a result, the estimated standard deviation of the time-invariant error component  $\sigma_\delta$  is further reduced to 0.1261. In our case, we don't find an effect on  $\sigma_\delta$  but rather on  $\sigma_v$ , namely the estimated standard deviation of the idiosyncratic error component, because the trend variable that is considered as a time-varying environmental factor has no in-between group's variance. On the other hand, we see almost no effect on  $\sigma_v$  between the conventional random effects model estimated by GLS model and the *M-GLS* and the *MM-GLS* models. This is so as the only additional time-varying variable is the share of owned land.<sup>18</sup>

The estimated parameters of the random effects models estimated by ML and GLS are similar (see Table 3). The input parameters in the conventional model have similar values and the same is true for Mundlak's adjustment terms and the environmental variables. The estimated standard deviation of the inefficiency component  $\sigma_u$  is reduced from 0.2709 in the conventional specification to 0.2101 in the *MM-MLE* model. Interestingly,  $\sigma_u$  increases again to 0.2389 in the *HT-MLE* model. We assign this increase to the change in the sign of the parameter  $\eta$ , which although positive in all other specifications (indicating an increase in efficiency over time) it turns negative in the *HT-MLE* model, due most likely to the inclusion of the trend variable in the production function. This leads to an increased dispersion in the inefficiency values over time and therefore to the higher value in  $\sigma_u$ .

Similarly to the random effects models estimated by GLS and ML, inclusion of Mundlak's adjustment terms into the TRE model reduces the unexplained variation from  $\sigma_{\beta_0}=0.1789$  to 0.1378 (see Table 4). On the other hand, the inclusion of environmental variables only slightly reduces the standard deviation of the random component from  $\sigma_{\beta_0}=0.1789$  to 0.1700. However, these two sets of variables do not have a distinct effect on inefficiency estimates. Comparing the columns 1-5 in Table 4 we find almost identical estimates for  $\sigma_u$  and  $\sigma_v$  as all time-invariant differences between farms are captured by the

random parameter, inefficiency estimates do not contain any time-invariant component and are not contaminated with heterogeneity. As a result, only the variability of the random component is reduced. Nevertheless, as in the *M-GLS* and the *M-MLE* models, Mundlak's adjustment terms mitigate heterogeneity bias in the estimated technology parameters. We find those estimated parameters in the TRE models to evolve across the different specifications in a similar way as in the random effects models estimated by GLS and ML. Including environmental variables leads, as in the *MM-TRE* model, to a further decrease in  $\sigma_{\beta_0}$ . Once again the inefficiency estimates are almost unaffected. In the *HT-TRE* model the estimated time trend parameter is significant; however, the changes in the time-invariant and time-varying error components are only minor.

The different specifications within the three types of models are nested. Hence, we can use the Likelihood-Ratio and the Wald tests to identify the most suitable specification for each type of models. The results of these tests are summarized in Table 5. Consistently across the three types of models, we find that all specifications that attempt to model heterogeneity are preferred over the conventional specifications. In addition, in all three types of models, the specification with Hausman-Taylor adjustment terms is preferred. However, deciding as to whether the GLS, ML or simulated ML is the appropriate estimator choice is not as straightforward. It is rather based on the assumptions one is willing to make about the  $\delta_i$  terms in the auxiliary equations.

We next present and compare the efficiency scores derived from the different models and specifications.<sup>19</sup> In the upper panel of Figure 1 we present the distributions of the estimated efficiency scores from models estimated by GLS. From there it becomes clear that the additional variables included account for heterogeneity and therefore help to reduce the respective contamination of the efficiency scores. In our case, mean efficiency scores vary from 0.687 in the conventional model estimated by GLS and 0.719 in the *MM-GLS* model. As we can see from the middle panel of Figure 2 the same applies to models estimated by ML. In both the *M-MLE* and *MM-MLE* models, the distribution of efficiency scores shifts rightwards and raises mean efficiency from 0.786 in the conventional specification to 0.821 and 0.828, respectively. The efficiency results for the TRE models, presented in the lower panel Figure 1, also fit into this set of results. On the other hand, adding time-invariant variables, mostly environmental factors, to the estimation equation doesn't have a substantial effect on the efficiency scores, obtained from the TRE models.<sup>20</sup>

In order to examine how the efficiency scores obtained from the different models and specifications relate to each other we calculate their Pearson correlation coefficients (see

Table 6). For the models with time-varying efficiency we use their group mean efficiency score over the observed period for calculating the correlation coefficient. The coefficients vary between 0.443 and 0.998. We find a consistently high correlation between the various models estimated by GLS and ML, in particular for the M-, MM- and HT-specifications that take heterogeneity into account. The correlation between the efficiency scores of those models and the efficiency scores from the TRE models is notably lower. Thus it seems that the correlation between the efficiency scores from models that treat the residual term  $\delta_i$  in the auxiliary equations as pure unobserved heterogeneity (i.e., TRE models) and models that treat it as either statistical noise (i.e., models estimated by ML) or persistent efficiency (i.e., models estimated by GLS) is low.

## 5. Concluding Remarks

In this paper we examine several extensions of stochastic frontier models that attempt to take heterogeneity, observable or unobserved, into account and to reduce heterogeneity induced biases in the estimated efficiency scores and technology parameters. For these purposes, we consider not only group means of input quantities in modeling unobserved heterogeneity in terms of TRE stochastic production frontier models but also environmental variables that affect the conditions under which production is taking place. In particular, we consider three variants: *first*, the conventional correlated effects model including only group means of input quantities; *second*, an extended version of the correlated effects model including both group means of input quantities and time-invariant environmental factors; and *third*, a version of correlated effects model where the effects are not necessarily correlated with all explanatory variables (inputs and environmental factors). The second extension is that we adapt this modeling of heterogeneity not only to the TRE models but also to random effects models that can be estimated by either GLS or ML. This way, we extend the range of models and broaden the toolbox the empirical analysis in a straightforward, usable way.

Our empirical results indicate that the third variant of the correlated effects model, namely the one where the effects are not necessarily correlated with all explanatory variables (inputs and environmental factors) is preferred compared to other two regardless of the estimation method. Nevertheless, the choice among the estimation methods is not as straightforward as among variants, which are nested to each other. The choice depends primarily on the assumption one is willing to make or the interpretation is going to give to the residual terms  $\delta_i$  in the auxiliary equations. These terms are treated as unobserved

heterogeneity in the TRE models, as statistical noise in the models estimated by ML, and as persistent efficiency in the models estimated by GLS.

## References

- Abdulai, A. and H. Tietje. Estimating Technical Efficiency under Unobserved Heterogeneity with Stochastic Frontier Models: Application to Northern German Dairy Farms, *European Review of Agricultural Economics*, 2007, 34, 393-416.
- Baltagi, B.H. *Econometric Analysis of Panel Data*, J. Wiley & Sons, 1995.
- Bartelsman, E. and M. Doms. Understanding Productivity: Lessons from Longitudinal Microdata, *Journal of Economic Literature*, 2000, 38, 569-94.
- Battese G.E. and T.J. Coelli. Frontier Production Functions, Technical Efficiency and Panel Data: With Application to Paddy Farmers in India, *Journal of Productivity Analysis*, 1992, 3, 153-69.
- Battese G.E. and T.J. Coelli. A Model for Technical Inefficiency Effects in a Stochastic Frontier Production for Panel Data, *Empirical Economics*, 1995, 20, 325-32.
- Chamberlain, G. Multivariate Regression Models for Panel Data, *Journal of Econometrics*, 1982, 18, 5-46.
- Coelli, T.J., Perelman, S. and E. Romano. Accounting for Environmental Influences in Stochastic frontier Models: with Application to International Airlines, *Journal of Productivity Analysis*, 1999, 11, 251-73.
- Colombi, R., Kumbhakar, S.C., Martini, G. and G. Vittadini. Closed-skew Normality in Stochastic Frontiers with Individual Effects and Long/short-run Efficiency, *Journal of Productivity Analysis*, 2014, 42, 123-36.
- Destatis (Statistisches Bundesamt Deutschland). 2012. Sachgebiete und Statistiken, Preise. (<http://www-genesis.destatis.de/genesis/online/online>)
- Farsi, M., Filippini, M. and W. Greene. Efficiency Measurement in Network Industries: Application to the Swiss Railway Companies, *Journal of Regulatory Economics*, 2005, 28, 69-90.
- Farsi, M., Filippini, M. and M. Kuenzle. Unobserved Heterogeneity in Stochastic Cost Frontier Models: An Application to Swiss Nursing Homes, *Applied Economics*, 2005, 37, 2127-41.
- Filippini, M. and W. Greene. Persistent and Transient Productive Inefficiency: A Maximum Simulated Likelihood Approach, *Journal of Productivity Analysis*, 2016, 45, 187-96.
- Good, D., Nadiri, M., Roller, L.H. and R.C. Sickles. Efficiency and Productivity Growth Comparisons of European and U.S. Air Carriers: A First Look at the Data, *Journal of Productivity Analysis*, 1993, 4, 115-25.
- Greene, W. Reconsidering Heterogeneity in Panel Data Estimators of the Stochastic Frontier Model, *Journal of Econometrics*, 2005a, 126, 269-303.

- Greene, W. Fixed and Random Effects in Stochastic Frontier Models, *Journal of Productivity Analysis*, 2005b, 23, 7-32.
- Greene, W. Distinguishing between Heterogeneity and Inefficiency: Stochastic Frontier Analysis of the World Health Organization's Panel Data on National Health Care Systems, *Health Economics*, 2004, 13, 959-80.
- Griliches, Z. and J. Mairesse. Production Functions: The Search for Identification, in Griliches, Z. *Practicing Econometrics: Essays in Method and Application*, Edward Elgar, 1998, 383-411.
- Hausman, J.A. and E.E. Taylor. Panel Data and Unobservable Individual Effects, *Econometrica*, 1981, 49, 1377-98.
- Hall, M. and C. Winsten. The Ambiguous Notion of Efficiency. *Economic Journal*, 1959, 69, 71-86.
- Hay, D.A. and G.S. Lin. The Efficiency of Firms: What Difference does Competition Make?, *Economic Journal*, 1997, 107, 597-617.
- Jondrow, J., Lovell, C.A.K., Materov, I.S. and P. Schmidt. On the Estimation of Technical Inefficiency in the Stochastic Frontier Production Function Model, *Journal of Econometrics*, 1982, 19, 233-38.
- Jovanovic, B. Selection and Evolution of Industry, *Econometrica*, 1982, 50, 649-70.
- Kumbhakar, S.C. and A. Heshmati. Efficiency Measurement in Swedish Dairy Farms: An Application of rotating Panel Data, 1976-88, *American Journal of Agricultural Economics*, 1995, 77, 660-74.
- Last, A.K. and H. Wetzel. The Efficiency of German Public Theaters: A Stochastic Frontier Analysis Approach, *Journal of Cultural Economics*, 2010, 34, 89-110.
- Maddala, G.S. Recent Developments in the Econometrics of Panel Data Analysis, *Transportation Research*, 1987, 21, 303-26.
- Mundlak, Y. Empirical Production Functions Free of Management Bias, *Journal of Farm Economics*, 1961, 44-56.
- Mundlak, Y. On the Pooling of Time Series and Cross Section Data, *Econometrica*, 1978, 46, 69-85.
- Schmidt, P. and R.C. Sickles. Production Frontiers and Panel Data, *Journal of Business and Economic Statistics*, 1984, 2, 367-74.
- Sherlund, S.M., Barrett, C.B. and A.A. Adesima. Smallholder Technical Efficiency Controlling for Environmental Production Conditions, *Journal of Development Economics*, 2002, 69, 85-101.
- Train, K. *Discrete Choice Models with Simulation*, 2nd Edition, Cambridge University Press

- Tybout, J.R. Manufacturing Firms in Developing Countries: How Well they Do and Why?, *Journal of Economic Literature*, 2000, 38, 11-44.
- Wang, H.J. and C.W. Ho. Estimating Fixed-effect Panel Stochastic Frontier Models by Model Transformation, *Journal of Econometrics*, 2010, 157, 286-96.
- Zellner, A., Kmenta, J. and J. Dreze. Specification and Estimation of Cobb-Douglas Production Function Models, *Econometrica*, 1966, 34, 784-95.

**Table 1: Descriptive Statistics for input- and output- and environmental variables**

		Mean	S.D.	Min	Max
Output	Revenues (1000 €)	110.3	53.6	13.4	402.6
Inputs	Labor (mwu)	1.61	0.49	0.30	3.86
	Land (ha)	52.6	28.8	5.6	318.3
	Material (1000 €)	57.7	30.7	5.7	247.1
	Capital (1000 €)	223.6	132.6	12.9	1063.5
Environmental variables	share of owned land (%)	48.59	24.10	0.00	100.00
	ag. Region 1 <sup>1</sup>	8.3			
	ag. Region 2 <sup>1</sup>	2.7			
	ag. Region 3 <sup>1</sup>	33.7			
	ag. Region 4 <sup>1</sup>	43.0			
	ag. Region 5 <sup>1</sup>	6.9			
	ag. Region 6 <sup>1</sup>	5.4			
	part time farming <sup>1</sup>	6.5			

<sup>1</sup>Dummy variable

**Table 2: Estimated GLS stochastic frontier models**

	Conventional		Conventional with environmental variables		M-GLS (11) <sup>d</sup>		MM-GLS (12) <sup>d</sup>		HT-GLS (13) <sup>d</sup>	
	Coeff.	Std.Err.	Coeff.	Std.Err.	Coeff.	Std.Err.	Coeff.	Std.Err.	Coeff.	Std.Err.
$\beta_0$ (Constant)	0.0046	(0.0072)	-0.0724	(0.0290) <sup>b</sup>	0.0134	(0.0067) <sup>b</sup>	0.0043	(0.0281)	-0.0510	(0.0285) <sup>c</sup>
$\beta_{x1}$ (Labor)	0.0832	(0.0151) <sup>a</sup>	0.0772	(0.0152) <sup>a</sup>	0.0344	(0.0172) <sup>b</sup>	0.0422	(0.0172) <sup>b</sup>	0.0534	(0.0168) <sup>a</sup>
$\beta_{x2}$ (Land)	0.2539	(0.0151) <sup>a</sup>	0.3132	(0.0178) <sup>a</sup>	0.3765	(0.0218) <sup>a</sup>	0.4456	(0.0270) <sup>a</sup>	0.3110	(0.0294) <sup>a</sup>
$\beta_{x3}$ (Material)	0.5106	(0.0139) <sup>a</sup>	0.4980	(0.0140) <sup>a</sup>	0.3913	(0.0171) <sup>a</sup>	0.3906	(0.0171) <sup>a</sup>	0.3752	(0.0167) <sup>a</sup>
$\beta_{x4}$ (Capital)	0.0687	(0.0091) <sup>a</sup>	0.0675	(0.0092) <sup>a</sup>	0.0150	(0.0109)	0.0168	(0.0109)	0.0574	(0.0113) <sup>a</sup>
$\pi_{\bar{x}1}$					0.1827	(0.0347) <sup>a</sup>	0.1478	(0.0358) <sup>a</sup>	0.1412	(0.0356) <sup>a</sup>
$\pi_{\bar{x}2}$					-0.4233	(0.0312) <sup>a</sup>	-0.4376	(0.0372) <sup>a</sup>	-0.3090	(0.0388) <sup>a</sup>
$\pi_{\bar{x}3}$					0.3066	(0.0291) <sup>a</sup>	0.2798	(0.0295) <sup>a</sup>	0.2937	(0.0292) <sup>a</sup>
$\pi_{\bar{x}4}$					0.1502	(0.0196) <sup>a</sup>	0.1528	(0.0201) <sup>a</sup>	0.1149	(0.0203) <sup>a</sup>
$\beta_{z1}$ (owned land)			0.0013	(0.0004) <sup>a</sup>			0.0018	(0.0004) <sup>a</sup>	0.0012	(0.0004) <sup>a</sup>
$\pi_{\bar{z}1}$							-0.0012	(0.0005) <sup>b</sup>	-0.0005	(0.0005)
$\gamma_1$ (Region 1)			0.1297	(0.0352) <sup>a</sup>			0.0531	(0.0331)	0.0550	(0.0331) <sup>c</sup>
$\gamma_2$ (Region 2)			0.1450	(0.0467) <sup>a</sup>			0.0309	(0.0440)	0.0302	(0.0440)
$\gamma_3$ (Region 3)			0.0621	(0.0283) <sup>b</sup>			-0.0079	(0.0271)	-0.0092	(0.0270)
$\gamma_4$ (Region 4)			0.0034	(0.0274)			-0.0327	(0.0258)	-0.0331	(0.0258)
$\gamma_5$ (Region 6)			-0.1282	(0.0385) <sup>a</sup>			-0.0972	(0.0360) <sup>a</sup>	-0.0932	(0.0359) <sup>a</sup>
$\gamma_6$ (Part time)			-0.0673	(0.0291) <sup>b</sup>			-0.0389	(0.0289)	-0.0357	(0.0289)
$\beta_t$ (Trend)									0.0154	(0.0015) <sup>a</sup>
$\sigma_\delta$	0.1417	-	0.1368	-	0.1300	-	0.1261	-	0.1261	-
$\sigma_v$	0.1010	-	0.0102	-	0.1014	-	0.1010	-	0.0985	-

Notes: a,b,c indicate parameter significant different from zero on the 1%, 5% and 10% level respectively

d the number in parenthesis refers to the corresponding equation number in the text

**Table 3: Estimated MLE stochastic frontier models**

	Conventional		Conventional with environmental variables		M-MLE (8) <sup>d</sup>		MM-MLE (9) <sup>d</sup>		HT-MLE (10) <sup>d</sup>	
	Coeff.	Std.Err.			Coeff.	Std.Err.	Coeff.	Std.Err.	Coeff.	Std.Err.
$\beta_0$ (Constant)	0.2545	(0.0059) <sup>a</sup>	0.1495	(0.0223) <sup>a</sup>	0.2137	(0.0055) <sup>a</sup>	0.1693	(0.0289) <sup>a</sup>	0.0870	(0.0303) <sup>a</sup>
$\beta_{x1}$ (Labor)	0.0827	(0.0128) <sup>a</sup>	0.0746	(0.0132) <sup>a</sup>	0.0345	(0.0142) <sup>b</sup>	0.0403	(0.0145) <sup>a</sup>	0.0560	(0.0142) <sup>a</sup>
$\beta_{x2}$ (Land)	0.2211	(0.0103) <sup>a</sup>	0.2754	(0.0132) <sup>a</sup>	0.3100	(0.0135) <sup>a</sup>	0.3718	(0.0180) <sup>a</sup>	0.3096	(0.0176) <sup>a</sup>
$\beta_{x3}$ (Material)	0.4997	(0.0091) <sup>a</sup>	0.4988	(0.0092) <sup>a</sup>	0.3844	(0.0096) <sup>a</sup>	0.3838	(0.0095) <sup>a</sup>	0.3741	(0.0094) <sup>a</sup>
$\beta_{x4}$ (Capital)	0.0970	(0.0079) <sup>a</sup>	0.0983	(0.0087) <sup>a</sup>	0.0435	(0.0104) <sup>a</sup>	0.0438	(0.0104) <sup>a</sup>	0.0559	(0.0102) <sup>a</sup>
$\pi_{\bar{x}1}$					0.1809	(0.0252) <sup>a</sup>	0.1277	(0.0283) <sup>a</sup>	0.1155	(0.0290) <sup>a</sup>
$\pi_{\bar{x}2}$					-0.3681	(0.0214) <sup>a</sup>	-0.3550	(0.0309) <sup>a</sup>	-0.2757	(0.0312) <sup>a</sup>
$\pi_{\bar{x}3}$					0.2836	(0.0206) <sup>a</sup>	0.2543	(0.0227) <sup>a</sup>	0.2453	(0.0234) <sup>a</sup>
$\pi_{\bar{x}4}$					0.1370	(0.0146) <sup>a</sup>	0.1365	(0.0173) <sup>a</sup>	0.1177	(0.0172) <sup>a</sup>
$\beta_{z1}$ (owned land)			0.0013	(0.0003) <sup>a</sup>			0.0015	(0.0003) <sup>a</sup>	0.0012	(0.0003) <sup>a</sup>
$\pi_{\bar{z}1}$							-0.0007	(0.0005)	-0.0003	(0.0005)
$\gamma_1$ (Region 1)			0.1301	(0.0305) <sup>a</sup>			0.0787	(0.0281) <sup>a</sup>	0.0906	(0.0303) <sup>a</sup>
$\gamma_2$ (Region 2)			0.0951	(0.0583)			0.0112	(0.0487)	0.0289	(0.0527)
$\gamma_3$ (Region 3)			0.0730	(0.0220) <sup>a</sup>			0.0234	(0.0246)	0.0267	(0.0264)
$\gamma_4$ (Region 4)			0.0068	(0.0198)			-0.0111	(0.0236)	-0.0049	(0.0252)
$\gamma_5$ (Region 6)			-0.0824	(0.0278) <sup>a</sup>			-0.0943	(0.0307) <sup>a</sup>	-0.0730	(0.0369) <sup>b</sup>
$\gamma_6$ (Part time)			-0.0537	(0.0211) <sup>b</sup>			-0.0403	(0.0205) <sup>b</sup>	-0.0332	(0.0220)
$\beta_t$ (Trend)									0.0184	(0.0020) <sup>a</sup>
$\lambda$	2.6086	(0.0167) <sup>a</sup>	2.4105	(0.0182) <sup>a</sup>	2.1631	(0.0188) <sup>a</sup>	2.0989	(0.0194) <sup>a</sup>	2.4162	(0.0179) <sup>a</sup>
$\sigma_u$	0.2709	(0.0011) <sup>a</sup>	0.2483	(0.0009) <sup>a</sup>	0.2184	(0.0006) <sup>a</sup>	0.2101	(0.0005) <sup>a</sup>	0.2389	(0.0008) <sup>a</sup>
$\sigma_v$	0.1038	-	0.1030	-	0.1010	-	0.1001	-	0.0989	-
$\eta$	0.0436	(0.0040) <sup>a</sup>	0.0458	(0.0045) <sup>a</sup>	0.0483	(0.0045) <sup>a</sup>	0.0480	(0.0049) <sup>a</sup>	-0.0166	(0.0086) <sup>c</sup>

Notes: a,b,c indicate parameter significant different from zero on the 1%, 5% and 10% level respectively  
d the number in parenthesis refers to the corresponding equation number in the text

**Table 4: Estimated TRE stochastic frontier models**

	Conventional		Conventional with environmental variables		M-TRE (3) <sup>d</sup>		MM-TRE (5) <sup>d</sup>		HT-TRE (7) <sup>d</sup>	
	Coeff.	Std.Err.	Coeff.	Std.Err.	Coeff.	Std.Err.	Coeff.	Std.Err.	Coeff.	Std.Err.
$\beta_0$ (Constant)	0.0865	(0.0043) <sup>a</sup>	0.0090	(0.0101)	0.0986	(0.0046) <sup>a</sup>	0.0763	(0.0103) <sup>a</sup>	0.0383	(0.0104) <sup>a</sup>
$\sigma_{\beta_0}$	0.1789	(0.0024) <sup>a</sup>	0.1700	(0.0024) <sup>a</sup>	0.1378	(0.0019) <sup>a</sup>	0.1315	(0.0020) <sup>a</sup>	0.1303	(0.0020) <sup>a</sup>
$\beta_{x1}$ (Labor)	0.0727	(0.0080) <sup>a</sup>	0.0635	(0.0084) <sup>a</sup>	0.0317	(0.0134) <sup>b</sup>	0.0362	(0.0134) <sup>a</sup>	0.0414	(0.0137) <sup>a</sup>
$\beta_{x2}$ (Land)	0.2733	(0.0066) <sup>a</sup>	0.3265	(0.0077) <sup>a</sup>	0.3810	(0.0125) <sup>a</sup>	0.4243	(0.0129) <sup>a</sup>	0.3034	(0.0174) <sup>a</sup>
$\beta_{x3}$ (Material)	0.5015	(0.0064) <sup>a</sup>	0.4958	(0.0066) <sup>a</sup>	0.3887	(0.0106) <sup>a</sup>	0.3892	(0.0105) <sup>a</sup>	0.3713	(0.0103) <sup>a</sup>
$\beta_{x4}$ (Capital)	0.0658	(0.0048) <sup>a</sup>	0.0658	(0.0050) <sup>a</sup>	0.0167	(0.0093) <sup>c</sup>	0.0173	(0.0093) <sup>c</sup>	0.0601	(0.0095) <sup>a</sup>
$\pi_{\bar{x}1}$					0.2012	(0.0162) <sup>a</sup>	0.1521	(0.0167) <sup>a</sup>	0.1567	(0.0164) <sup>a</sup>
$\pi_{\bar{x}2}$					-0.4324	(0.0143) <sup>a</sup>	-0.4149	(0.0146) <sup>a</sup>	-0.3099	(0.0192) <sup>a</sup>
$\pi_{\bar{x}3}$					0.3235	(0.0130) <sup>a</sup>	0.2918	(0.0131) <sup>a</sup>	0.3050	(0.0127) <sup>a</sup>
$\pi_{\bar{x}4}$					0.1437	(0.0106) <sup>a</sup>	0.1578	(0.0109) <sup>a</sup>	0.1178	(0.0109) <sup>a</sup>
$\beta_{z1}$ (owned land)			0.0013	(0.0003) <sup>a</sup>			0.0011	(0.0001) <sup>a</sup>	0.0011	(0.0003) <sup>a</sup>
$\pi_{\bar{z}1}$							-0.0004	(0.0003)	-0.0004	(0.0003)
$\gamma_1$ (Region 1)			0.1124	(0.0112) <sup>a</sup>			0.0447	(0.0111) <sup>a</sup>	0.0521	(0.0107) <sup>a</sup>
$\gamma_2$ (Region 2)			0.1467	(0.0153) <sup>a</sup>			0.0229	(0.0150)	0.0249	(0.0144) <sup>c</sup>
$\gamma_3$ (Region 3)			0.0418	(0.0090) <sup>a</sup>			-0.0242	(0.0090) <sup>a</sup>	-0.0221	(0.0087) <sup>b</sup>
$\gamma_4$ (Region 4)			-0.0104	(0.0087)			-0.0395	(0.0086) <sup>a</sup>	-0.0402	(0.0083) <sup>a</sup>
$\gamma_5$ (Region 6)			-0.1348	(0.0121) <sup>a</sup>			-0.0947	(0.0119) <sup>a</sup>	-0.0857	(0.0116) <sup>a</sup>
$\gamma_6$ (Part time)			-0.0394	(0.0093) <sup>a</sup>			-0.0722	(0.0083) <sup>a</sup>	-0.0686	(0.0084) <sup>a</sup>
$\beta_t$ (Trend)									0.0163	(0.0011) <sup>a</sup>
$\lambda$	1.4723	(0.0993) <sup>a</sup>	1.4747	(0.1001) <sup>a</sup>	1.4853	(0.1114) <sup>a</sup>	1.4501	(0.1104) <sup>a</sup>	1.5996	(0.1145) <sup>a</sup>
$\sigma_u$	0.1154	-	0.1148	-	0.1127	-	0.1105	-	0.1134	-
$\sigma_v$	0.0784	-	0.0779	-	0.0759	-	0.0762	-	0.0709	-

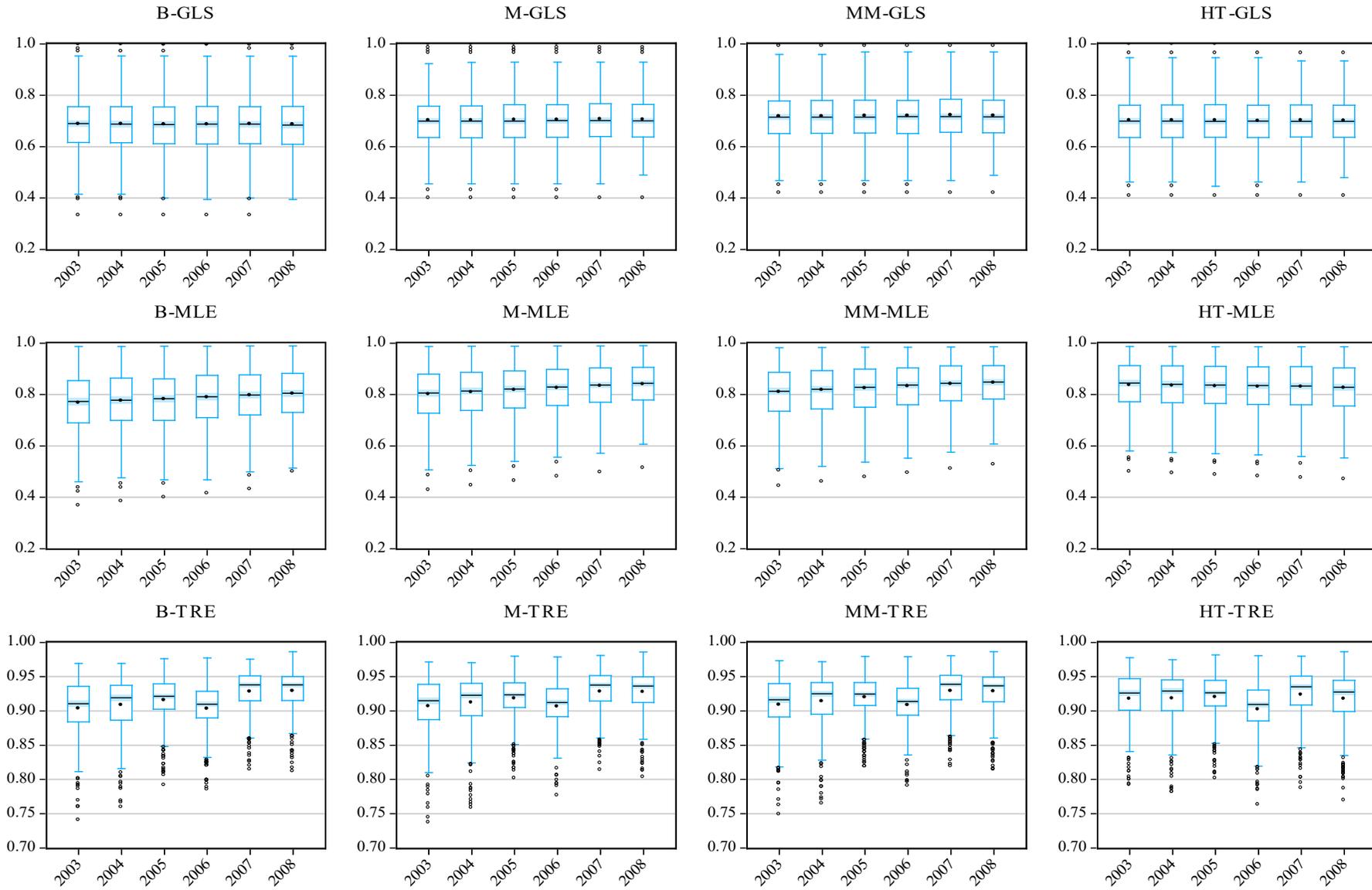
Notes: a,b,c indicate parameter significant different from zero on the 1%, 5% and 10% level respectively  
d the number in parenthesis refers to the corresponding equation number in the text



**Table 5: Specification tests**

Model		F- / LR- statistic	critical value ( $\alpha = 0.05$ )
GLS	conventional vs. M-GLS	87.53	$F_{(4,1935)} = 2.38$
	conventional vs. conventional with environmental variables	12.71	$F_{(7,1932)} = 2.01$
	M-GLS vs. MM-GLS	6.08	$F_{(8,1927)} = 1.94$
	conventional with environmental variables vs. MM-GLS	75.28	$F_{(5,1927)} = 2.22$
	MM-GLS vs. HT-GLS	104.55	$F_{(1,1926)} = 3.85$
MLE	conventional vs. M-MLE	276.39	$\chi_4^2 = 9.49$
	conventional vs. conventional with environmental variables	94.64	$\chi_7^2 = 14.07$
	M-MLE vs. MM-MLE	60.32	$\chi_8^2 = 15.51$
	conventional with environmental variables vs. MM-MLE	242.06	$\chi_5^2 = 11.07$
	MM-MLE vs. HT-MLE	52.59	$\chi_1^2 = 3.84$
TRE	conventional vs. M-TRE	319.82	$\chi_4^2 = 9.49$
	conventional vs. conventional with environmental variables	73.63	$\chi_7^2 = 14.07$
	M-TRE vs. MM-TRE	49.81	$\chi_8^2 = 15.51$
	conventional with environmental variables vs. MM-TRE	296.00	$\chi_5^2 = 11.07$
	MM-TRE vs. HT-TRE	119.32	$\chi_1^2 = 3.84$

**Figure 1: Box plot presentation of efficiency scores from different model specifications per year**



**Table 6: Correlation between efficiency estimates**

	B-GLS	M-GLS	MM-GLS	HT-GLS	B-MLE	M-MLE	MM-MLE	HT-MLE	B-TRE	M-TRE	MM-TRE	HT-TRE
B-GLS	1.0											
M-GLS	0.809	1.0										
MM-GLS	0.785	0.969	1.0									
HT-GLS	0.783	0.967	0.998	1.0								
B-MLE	0.985	0.823	0.801	0.799	1.0							
M-MLE	0.799	0.977	0.950	0.948	0.838	1.0						
MM-MLE	0.780	0.935	0.972	0.970	0.820	0.960	1.0					
HT-MLE	0.811	0.928	0.964	0.966	0.850	0.955	0.993	1.0				
B-TRE	0.519	0.462	0.449	0.443	0.539	0.483	0.475	0.473	1.0			
M-TRE	0.507	0.620	0.598	0.592	0.534	0.632	0.609	0.590	0.821	1.0		
MM-TRE	0.509	0.621	0.629	0.623	0.536	0.634	0.641	0.622	0.821	0.983	1.0	
HT-TRE	0.526	0.621	0.620	0.617	0.553	0.635	0.631	0.625	0.802	0.957	0.968	1.0

## Footnotes

---

<sup>1</sup> Sherlund, Barrett and Adesina (2002) also mentioned that not controlling for unobserved heterogeneity results in an upwards bias in the estimates of technical inefficiency in deterministic frontier models.

<sup>2</sup> For purely estimation related problems of the true fixed effect model see Greene (2005a, b) and Wang and Ho (2010).

<sup>3</sup> Colombi *et al.* (2014) and Filippini and Greene (2016) models help to solve this problem.

<sup>4</sup> The term heterogeneity bias was used by Chamberlain (1982) to refer to the bias induced by the correlation between the individual effects and the explanatory variables in a random effects model.

<sup>5</sup> Both the proposed models as well as the *M-TRE* model itself fall in the boarder category of hierarchical or multilevel models as referred to by Greene (2004).

<sup>6</sup> Baltagi (1995, pp. 116-20) has used the term to “endogenous effects” refer to Mundlak (1978) and Hausman and Taylor (1981) type of panel data models.

<sup>7</sup> This should be distinguished from the inefficiency effect model, e.g., Battese and Coelli (1995), where a number of (control) variables are used to explain efficiency differences across production units.

<sup>8</sup> If a firm knew its level of technical efficiency at the time it makes its production decisions, its input choices would be affected (Schmidt and Sickles, 1984).

<sup>9</sup> For comparison purposes we maintain this assumption even in the GLS models presented later in this section, restricting ourselves to random effects GLS models.

<sup>10</sup> Notice however that the resulting estimated parameters are not the within estimates, as in the original Mundlak (1978) model because  $e_{it}$  is a composite asymmetric rather than a symmetric error term (Farsi, Filippini and Kuenzle, 2005).

<sup>11</sup> The discussion on the role of “environmental factors” in efficiency measurement is dated back to Hall and Winsten (1959) and Mundlak (1961).

<sup>12</sup> Even though not considered here, this interpretation of long-run inefficiency can in principle be accommodated in Greene’s (2005a, b) true effects models.

<sup>13</sup> We may estimate the model even if both  $\delta_i$  and  $u_{it}$  are present. In fact, this is Kumbhakar and Heshmati (1995) four error-components model.

---

<sup>14</sup> Note that (12) may be obtained by combining Good *et al.* (1993) and Farsi, Filippini and Kuenzle (2005a) formulations.

<sup>15</sup> The term “specialized” means that at least 75% of the farms revenues have to come from dairy production.

<sup>16</sup> The agricultural production regions are defined by the Bavarian Agricultural Research Institute. We arbitrarily choose the Franconian region as reference group.

<sup>17</sup> In the presence of correlated effects, the conventional fixed effects estimators of the production function parameters are assumed to be unbiased as thus are used as a benchmark (Farsi, Filippini and Greene, 2005).

<sup>18</sup> Despite being a time varying variable, the in between-groups variance of the *share of owned land* accounts for most (93.2 %) of the variable’s overall variance.

<sup>19</sup> From here on we omit from the discussion the conventional models with the environmental variables but the relevant results are available upon request. In addition, one could calculate the predicted values of the individual effects  $\alpha_i$  and examine how the discussed models account for heterogeneity; the corresponding results are also available upon request.

<sup>20</sup> This might be different in empirical applications where more information about time-varying environmental production conditions is available.