Noname manuscript No.
(will be inserted by the editor)

Diffusion of Innovations in Middle Eastern vs Western Markets: A Mathematical Computation Cellular Automata Simulation Model

Konstantinos Petridis · **Nikolaos E. Petridis**

the date of receipt and acceptance should be inserted later

K. Petridis
Department of Applied Informatics
University of Macedonia
156 Egnatia Street
Tel.: +30 2310 891 627
E-mail: k.petridis@uom.edu.gr

Abstract Simulation modelling has gained ground over the years since it can provide various scenarios applied to any scientific area. In this study, a stochastic cellular automata model is proposed, in which agents fall into three distinct categories (adopters, non - adopters and denials). Based on Hofstede's cultural dimension *individualism*, we characterize three major international markets, as perfectly clustered (collective) to perfectly random (individualistic). We investigate innovation diffusion speed, in each network topology. At each time step, the decision of non - adopters to purchase innovative products, is affected by their immediate neighborhood (von Neumman). The speed of diffusion is evaluated using time at which sales reach 50 % of market. Effects of simulation parameters on speed of diffusion, is assessed using a log - normal Accelerated Failure Time (AFT) model. Results demonstrate that diffusion of innovative products accelerates when innovators of a virtual economic system are placed according to a random network and when amount of innovators and imitators in the economic system increases. Slower innovative products' diffusion process is a result of a large amount of denials and of how imitators are placed in the in the virtual economic system. Diffusion in Small - World virtual economic systems lead to small time inflexion points very close to those of a random networked market.

Keywords Agent Based Model · Diffusion of Innovative Products · Small - World Network · Innovators and Denials · Survival analysis · Computational Mathematics

1 Introduction

Sayama [22] defines an automaton as *a technical term used in computer science and mathematics for a theoretical machine that changes its internal state based on inputs and its previous state*.

Cellular automata (CA) are a set of such structures, usually placed along a regular spatial grid, whose cell transit states and this procedure is conducted simultaneously for all cells, according to a function which takes into consideration their neighborhood ([26],[25],[28]). This simulation technique, which belongs to computational mathematics scientific field, has been applied in various research areas such as Biology, Chemistry, Civil Engineering and Marketing among many others.

They are widely used, because of their simple application and due to the fact that they can provide useful insights when real data are not easy to collect. In the most of the cases in relative literature, CA models evolve at discrete time steps, by using a predetermined set of rules, imposed by researchers. As they are computational systems, they are abstract and therefore can provide solutions to algorithmic problems.

In the context of the present study, a CA model is employed, in which each cell is characterized by three states: a) non - adopters (state 0), b) innovators - first adopters (state 1) and c) denials (state -1). At every time step, the mean effect of a Von Neumann neighborhood is assessed, only for those cells, which are assigned in the state 0 (non - adopters). If the mean effect from their direct neighborhood is positive, cells are probable to transit from state 0 to state 1, otherwise they remain in their initial state (non - adopters). Cells of state 1, are placed according to a Small - World topology, with varying coefficient of randomness (β).

Our ultimate aim of this study, is to investigate the speed of innovations diffusion throughout differentiated markets, in terms of coefficient randomness' values. An attempt is made, so as to establish similarities of virtual economic systems with real ones, by using socio - economic indices, proposed by Hofstede [14],[13]. We find evidence that Middle - Eastern markets are clustered, European markets are less clustered but more random and finally

that Northern - American markets are approximated by a completely random Small - World topology, in terms of Hofstede's *individualism* index. Finally, we reach to the conclusion that innovation diffusion faster for Northern - American (completely random) markets and slower for Middle - Eastern ones and that imitation effect in completely random markets (i.e Northern - American), can lead in a prolonged period of innovations acceptance.

2 Literature

In the context of marketing, CA and agent based simulation procedures have gained a lot of attention and have been used as a means of investigating interactions in micro structures, due to their implementation's simplicity (e.g [15],[29]).

Each cell of CA represent a member of a virtual economic system (e.g. a small - market), and his/her surrounded cells represent their immediate neighborhood. This economic system, usually consists of members who are eager to purchase an innovative product and are called *innovators* and those who are eager, yet hesitate, and are called *imitators* [21].

The decision of adopting an innovative product by imitators, depends on the "signal" that they receive from their neighbors, through channels of communications. Innovators send positive messages for products that they purchase and if there is adequate positive information, imitators adopt innovative products as well. These positive "signals", which in relative literature are called *Word - of - Mouth* (WoM) [4], aim at motivating non - adopters' awareness.

Therefore, CA serve as models to investigate interactions between members of a virtual economic system, in a micro - environment. Goldenberg et al. (2001) [9] employed a CA model, assuming that members of this virtual economic system, fall into two categories ("potential adopters" and "adopters"). "Potential adopters" transit to state "adopters", using a probabilistic function, and adoption of innovative products occurs either naturally or utilizing WoM channels.

The above model has been extended by Goldenberg at al., (2007) [10], so as to incorporate the negative WoM. They model adoption, assuming that marker is described by a specific network topology, (i.e. Small - World [27]), and that members of this virtual economic system have the possibility to reject innovative product. Adoption or rejection of an innovative product, is based on positive WoM, advertising and negative WoM, using different probability functions.

Deffuant et al (2005) [7], employ a slightly different method for modelling diffusion of innovation adoptions. They propose a three scale score for adoption interest, which consists of *no*, *maybe* and *yes* and a 5 scale score for information score (*not - concerned*, *information request*, *no adoption*, *pre - adoption*, *adoption*). Their main finding is that high social value and low individual benefits are factors that convey the diffusion of innovations.

Thiriot and Kant (2008) [24] using data collected from the published means-end chains analysis of *iPod*^{TM} and from statistical analysis of reviews provided by consumers on specialized websites, investigated the adoptions decision. They employed a simulation model in which agents' decision to adopt innovative products is based, among other factors, on awareness information seeking and WoM spreading. Members of the virtual economic system, fall in Roger's [21] states of adoption, namely, *early adopter*, *early majority*, *late majority*, and *laggards*. Not all studies, investigate the innovation diffusion, with the use of a 2 dimensional simulation model.

In specific, Gondenberg et al. (2000) [11], proposed a multidimensional CA model, assuming a different subjective adoption threshold for each member of the virtual economic

system. This threshold ranges, randomly, between 0 and 1. Diffusion of innovative products is successful, once members of virtual economic system reach a ”critical mass”, namely if there is an adequate number of neighboring cells that have adopted it. They find that this personal threshold, which varies from cell to cell, facilitates innovation diffusion. The majority studies reviewed, which had a major impact on the investigation of innovation diffusion, assume that interactions take place in a fixed system and agents are connected with each other in a Small - World network topology. Most of them also assume that this topology does not change over time.

Hofstede's model of national culture ([14],[13]) consists of six dimensions, some of which have been used in the investigation of innovation's diffusion (e.g [6],[5], [12] and many others). These dimensions, refer to a quantification of culture and exhibit independent preferences for one attitude over another. In addition, cultural dimensions are measured in a country rather than individual level. He defines six cultural dimensions as follows:

power distance index (PDI): this dimension measures the degree to which weak members of a society are in accordance with the fact that authorities distribute power in an unequal way. When power distance index score is large, members tend to perceive a ranked society, without asking the source of this authority. Furthermore, countries with high PDI, are characterized by a great amount of societal disturbances and inequality. On the other hand, societies in which PDI score is low, members are trying to achieve a more equal system of power distribution.

Individualism vs collectivism (IDV): this dimension measures the degree to which society's members tend to care and show affection only for themselves and their close relatives, whereas in collective societies individuals consider relatives and/or members of a particular ingroup, to protect and take care of them. In the case when society's IDV score is large, it members' decisions are based on ”I”, whereas in collectivism societal systems, individuals' choices are based on ”We”.

Masculinity versus feminity (MAS): this dimension measures the extent to which societies are characterized by achievement, heroism, assertiveness, and material rewards for success (masculine) or cooperation, modesty, caring for the weak and quality of life (feminine). In the context of business, rarely these cultures are reported as ”tough” versus ”tender” accordingly.

Uncertainty avoidance index (UAI): this dimension expresses the degree to which members of a society are in fond of taking risks, living in an uncertain environment and ambiguity. Countries, whose societies' UAI score is large, present low to zero tolerance to any behaviour or ideas, which diverge from the ones that they have set. In societies, in which UAI score is low, practice is more important aspect than principle.

Long term versus short term normative orientation (LTO): this dimension expresses the degree to which members of a society, tend to apply solutions from the past, for problems of the present or the future. Tradition, weighs more in societies with low LTO scores, whereas education is more important for societies with high LTO scores.

Indulgence versus restraint (IVR): this dimension expresses the degree to which members of a society are free to enjoy life and have fun (indulgence), while in societies whose IVR score is low, there are strict rules and members are constrained.

In this paper we conduct a comparison between three major markets; Northern American, European and Middle Eastern, in terms of speed of diffusion innovations. We propose a CA simulation model, of a fixed size, which does not vary over time, however, we assume 3 types of cells; a) non - adopters (state 0) who are eager and willing to adopt, if they are convinced, b) adopters (state 1) and c) denials (state -1). Adoption of innovative products, for agents of state 0 (non - adopters) depends on the mean effect of agents in their direct

neighborhood. Innovators (state 1) are placed on a grid, which varies according to a Small - World topology, from completely regular to completely random. At a second stage, data collected from simulation procedure, are evaluated using survival analysis, and in specific accelerated failure time models as well as Kaplan - Meier estimators. The rest of our paper includes methodology of simulation process, results and conclusions.

3 Methodology

3.1 Simulation Model

Members of the economic system, are initially placed on a 2 dimension grid virtual economic system, and are characterized by three states; a) *non - adopters*, b) *adopters* and c) *denials*.

Non - adopters (state 0) are actually individuals, who are willing, yet either cannot afford an innovative product especially when it is launched in a market, when their prices are extremely high. In order to adopt innovative products, they need motivation from neighboring environment, namely from adopters.

Adopters (state 1) are informed, wealthy and educated individuals, who fear no change or challenge. Adopters of the initial state, at $t = 1$, are called *innovators*.

Denials (state -1) are individuals who belong to an economic system and are afraid to adopt innovative products and are not willing at all to do so. They are responsible for negative Word - of - Mouth (WoM) and hinder the diffusion of innovative products within a market.

Let the state of a random cell be c_i at dicrete time $t = 0,1,2,...,T$, $s_t(c_i)$, then its possible values are $s_t(c_i) = [-1,0,+1]$, and the states of neighboring cells to cell c, at time t, be $s_t(N(c_i))$, we define the following quantities:

$$\begin{aligned} n^- &= \sum_{i=1}^{N(c_i)} I[s_t(N(c_i)) = -1] \\ n^+ &= \sum_{i=1}^{N(c_i)} I[s_t(N(c_i)) = +1] \end{aligned} \tag{1}$$

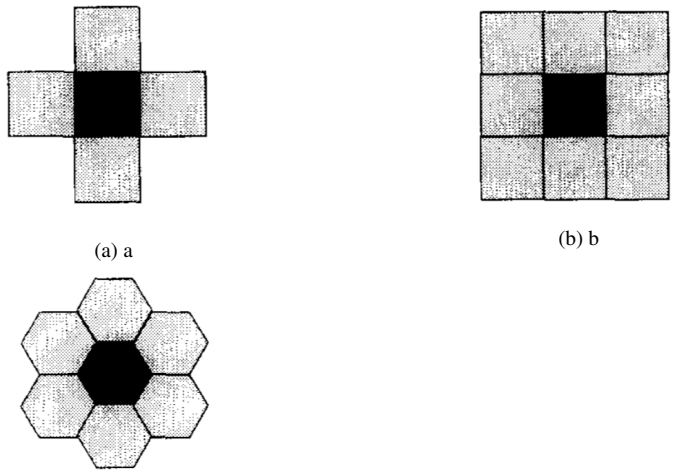
where n^-, the amount of neighboring cells which are in the state -1 (denials), n^+, the amount of neighboring cells which are in the state +1 (innovators), $N(c_i)$ cell's c_i neighborhood size and $I(\cdot)$ is a logical function. In relative literature, 3 types of neighborhoods have been proposed, *von Neumann*, *Moore* and *Uniform* (Fig. 1).

In the context of the present study, we use von Neuman neighborhood (Fig. 1, a), in which the target cell (black) $(c_{i,j})$, receives effects from the upper $(c_{i,j+1})$, right $(c_{i+1,j})$, down $(c_{i,j-1})$ and left neighboring cell $(c_{i-1,j})$. Therefore, the number of neighboring cells which send positive effect at time t, $(s_t(N(c_i)) = +1)$ is $n^+ \in [0,4]$, while number of neighboring cells which send positive effect at time t $(s_t(N(c_i)) = -1)$ is $n^- \in [4-n^+,4]$.

At $t+1$ time step, we evaluate each cell, c_i, which is in state 0, namely $s_t(c_i) = 0$. His/her decision to adopt the product, subjects to the following rules:

Rule 1: The amount of adopters, (n^+), outnumbers the amount of denials (n^-), in $N(c_i)$ direct neighborhood, namely $n^+ > n^-$, and

Rule 2: the probability that person i adopts an innovative product exceeds a subjective personal threshold, namely:



(a) a

(b) b

(c) (c)

Fig. 1: Different types of CA neighborhood a) Von Neumann, b) Moore and c) Uniform neighborhood.

$$\begin{aligned} &P(s_{t+1}(c_i)=1) = 1-(1-p)^{n^+}(1-q)^{n^+-n^-}(1-d)^{n^-} > a \\ &a \sim U(0,1) \end{aligned} \tag{2}$$

In Eq. 2, p stands for the propensity that a person is innovator and spread positive Word - of - Mouth (WoM), q is the propensity that a cell, c_i, imitates adopters, and d is the propensity that a person rejects innovations, regardless of neighborhood's positive effect. Eventually, the cell, c_i, transits to $s_{t+1}(c_i) = 1$ given that $s_t(c_i) = 0$, according to the following update function:

$$P[s_{t+1}(c_i)=1|s_t(c_i)=0] = \begin{cases} 1, \ (n^+ > n^-) \cap [P(s_{t+1}(c_i)=1) > U(0,1)] \neq \emptyset \\ 0, \ (n^+ > n^-) \cap [P(s_{t+1}(c_i)=1) > U(0,1)] = \emptyset \end{cases}$$

Simulation terminates when there are no cells to transit from state 0 to state 1 or at $t = 80$ time steps. At the end of each simulation we record the time, at which 50% of market penetration is achieved. This time correspond to inflexion point, $T_{50\%}$.

In the present study, we assume that innovators are placed on an evolving grid according to parameter β, with dimensions $N \times N$, which remain fixed throughout simulation procedure. Watts & Strogatz,(1998) [27], proposed a one parameter model, which interpolates between a ragular lattice and a completely random network. This parameter, β, is defined as *coefficient of randomness*.

In specific, a graph is denoted as $G(V,E)$, where V the amount of innovators in the economic system, while E denotes the amount of edges or connections that these innovators have established.

When virtual economic system is described by a ring - lattice (Fig. 2, 1st column), innovators are in a circle and are connected with each other. In Fig.2, a picture of the virtual

economic system is depicted, which corresponds to a ring lattice network. Red cells are innovators, while yellow ones are either non adopters or denials. Innovators are concentrated into a diagonal and it is very probable that cells near this diagonal, which are in state 0 (non-adopters), to transit first into state 1, due to large number of cells in state 1, in the von Neumann neighborhood.

Ring lattice's degree, k, namely, the number of connections each cell at state 1 has, is at $t = 1$ fixed and equal to $p*N$, expressed as integer. As innovation diffuses throughout virtual economic system, new connections are added and therefore network becomes more dense. Obviously, innovators are clustered within a partition of virtual economic system. Therefore, for the case of a virtual market, which is described by a ring lattice, the clustering coefficient CC, namely the degree to which innovators tend to form groups, is equal to $CC(g) = \frac{3k-3}{4k-2}$. The degree distribution, $P(k)$ of a ring lattice topology, which expresses the proportion of nodes that have k connections is $P(k) = k/N = K$, where K is the mean degree.

By varying β, ring lattice economic virtual system, changes its structure and innovators are placed in a more random way (Fig. 2, 2nd column). Rewiring existing connections between innovators, a new structure is created. In specific, the amount of rewired connections is equal to $\frac{\beta*N*K}{2}$, namely $K = k/N$ and $0 < \beta < 1$. Some innovators are still concentrated in a partition of economic system, while others have now been dispersed throughout the market. The new structure, exhibits lower clustering of innovators. Degree distribution of such topology, is a combination of Poisson and Binomial distributions [3]:

$$P(k) = \sum_{n=0}^{f(k,K)} \binom{K/2}{n} (1-\beta)^n * \beta^{(K/2)-n} * \frac{(\beta * K/2)^{k-(K/2)-n}}{(k-(K/2)-n)!} * e^{-\beta*K/2} \tag{3}$$

When $\beta = 1$, innovator's topology is completely random [8] (Fig. 2, third column). Innovators are completely dispersed to every partition of virtual economic system. Clustering of such topology is $CC = \frac{k}{N}$, while the degree distribution is Binomial, if N is small, $P(k) = \binom{k}{N-1} * p^k * (1-p)^{N-1-k}$, where p is the probability of connection between any two innovators.

Comparing three types of innovators' network topologies, it is obvious that regular lattice ($\beta = 0$) creates a structure which is completely clustered, while random network topologies ($\beta = 1$) a dispersed virtual economic system. The Small - World case lies somewhere between these two extreme cases ($\beta \simeq 0.5$).

The varying parameters of simulation process are β, p, q and d, the combination result of which produces a total of $20*12*12*2 = 5,760$ observations, which correspond to a full factorial (Table 1).

Table 1: Simulation parameters

Parameter	Symbol	Value
Size of economic system	N	100
Time horizon	t	80
Coefficient of randomization	β	0 - 1 (20 values)
Propensity to innovate	p	0.001 - 0.03 (12 values)
Propensity to imitate	q	0.1 - 0.3 (12 values)
Percent of denials	d	0.01 - 0.028 (2 values)

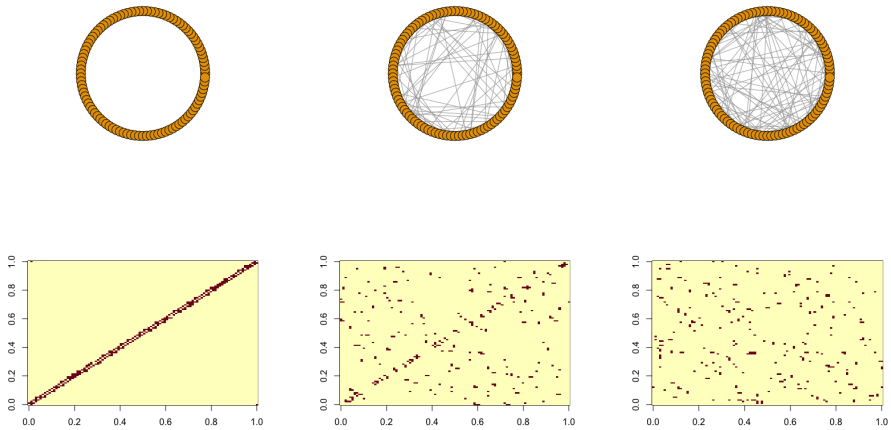


Fig. 2: Adopters' placement in a) regular, b) small - world and c) random virtual economic system.

3.2 Market characteristics

We employ the above simulation model, within a dynamic network varying environment. According to Hofstede, IDV exhibits the extent to which societies are collective (low IDV score) or collective (high IDV score). In Table 2, IDV score's summary statistics are presented, for three major regions; Northern America, Europe and Middle East.

Northern America region, which consists of USA and Canada, is characterized by a large mean value of IDV (M = 85.5, SD = 7.8). This implies that members of such societies, make decisions on their own rather than based on beliefs of relatives or friends. This means, that individuals are less clustered and act solely. We assume that markets of countries with high degree of IDV, are better described from a random network, in which innovators are not clustered to a partition.

Mean IDV score for Europe region, which consists of UK, Germany, France, Italy, Norway, Finland, Denmark, Belgium, Netherlands, Spain, Portugal, Greece, Austria, Slovakia, Slovenia, Poland, Estonia, Lithuania, Malta, Hungary, Ireland and Croatia, is 61 with standard deviation equal to 17.2. This suggests, that Europe's society is considered to be semi - individualistic, semi - collectivistic. Therefore, we expect that European market, would be best described by a Small - World network topology, as according to this network structure, innovators are more clustered compared to a completely random network, however, there will be some innovators dispersed throughout this virtual social system.

On the other hand, IDV mean score for countries which belong to Middle Eastern region, is equal to 32.7 with standard deviation equal to 9.1. IDV was computed for Iran, Iraq, Turkey, Israel, Kuwait, Saudi Arabia, UAE, Jordan, Lebanon, Egypt, Syria, Qatar, and Israel. Middle Eastern societies, on average, are characterized by a large degree of collectivism compared to European and Northern America's societies. Members of such societies, are based on their families, and this suggests a large degree of clustering. This assumption is proved by Ali et al., (2010) [2], who is characterizing Arabs in general as a collectivist

society, due to their Islamic way of thought, link to the past and honouring Arabic traditions and respect for family. Therefore, we expect that Middle Eastern markets, to be described by a regular lattice network topology in which strong bonds are established between members of society.

Table 2: IDV scores for several regions

Region	Mean	Standard Deviation	Minimum	Minimum
Northern America	85.5	7.8	80	91
Europe	61.0	17.2	27	89
Middle East	32.7	9.1	25	54

3.3 Survival analysis

In order to investigate the speed of diffusion, as simulation parameter values vary, time at which 50% of innovative products' cumulative sales (inflexion point) is assessed. In addition, we employ accelerated failure time parametric survival models, in order to evaluate the impact of simulation parameters, p, q, β, and d, on the time at which inlexion point takes place.

In specific, parametric survival models are assumed to follow some distributions, whose density functions, $f(t)$, contain unknown parameters. In the general form, the cumulative probability function $F(t)$, probability density function , $f(t)$, survival function $S(t)$, and hazard function, ht are given below ([16]):

$$\begin{aligned} F(t) &= Pr(T \leqslant t) \\ f(t) &= \frac{dF(t)}{dt} \\ S(t) &= \int_t^{\infty} f(u)du \\ h(t) &= \frac{-d[S(t)]/dt}{S(t)} \end{aligned} \tag{4}$$

In specific, let T_i be the random variable which corresponds to the time until inflexion point occurs and δ_i a binary random variable which receives value 0, when inflexion time has not taken place and 1 otherwise, then a accelerated time failure parametric model, assuming p, q,β, and d as covariates, can modelled as follows:

$$log(T_i) = \beta_0 + \beta_1 \cdot p_i + \beta_2 \cdot q_i + \beta_3 \cdot d_i + \sigma \cdot \varepsilon_i \tag{5}$$

In Eq. 5, σ is a scaling parameter, ε is the vector of residuals, and $\beta = [\beta_0 \beta_1 \beta_2 \beta_3]^T$ denotes vector of unknown parameters to be estimated.

Several distributions were applied, so as to find the appropriate accelerated failure time model. The evaluation of models' appropriateness was based on three criteria; a) Log - Likelihood value, b) Akaike Information Criterion (AIC) ([1]) and c) Schwarz Bayesian Information Criterion (BIC) ([23]).

Based on all three criteria, normal distribution seems to be the most appropriate, and therefore a log - normal parametric survival model was applied, due to the fact that log - likelihood's value is the least compared to the corresponding values of Weibull and Log - logistic distribution, and so are the values for AIC and BIC criteria (Table 3).

Table 3: Distributions applied

Distribution	Log - Likelihood	AIC	BIC
Weibull	-3767.829	7547.657	7604.058
Log - normal	-3223.125	6458.25	6514.65
Log - logistic	-3570.171	7152.342	7208.742

Log - normal accelerated failure time parametric model assumes that $\varepsilon \sim N(0,1)$. In addition, hazard function of T, $\lambda_0(t)$, which is computed if in Eq. 5 we set each β equal to 0, is given as follows ([30]):

$$\lambda_0(t) = \frac{\phi(\frac{log(t)}{\sigma}}{[1-\Phi(\frac{log(t)}{\sigma}]\cdot\sigma\cdot t} \tag{6}$$

where $\phi(x) = \frac{1}{\sqrt{2\cdot\pi}}\cdot e^{-x^2/2}$ is the probability density function and $\Phi(x) = \int_{-\infty}^{x}\frac{1}{\sqrt{2\cdot\pi}}\cdot e^{-u^2/2}du$ the cumulative distribution of the standard normal distribution ($N(0,1)$. Given these, the hazard function of T in logarithms can be given as $log(\lambda_0(t|x)) = log[\lambda_0(t\cdot e^{-x^T\beta})] - x^T\beta$ while the survival function of the log - normal model is expressed as $S(t|x) = \Phi[\sum_{i=1}^{p}\beta_i^* x_i - \alpha log(t)]$, where $\beta_i^* = \beta_i/\sigma$ and $\alpha = 1/\sigma$. Accelerated failure time log - normal model's coefficients (β), are estimated using maximum likelihood method:

$$\begin{aligned} L(\theta|D_n^t) &= \prod_{i=1}^{n}[f_i(t_i|\theta)]^{\delta_i}[S_i(t_i|\theta)]^{1-\delta_i} \\ \theta &= (\beta,\sigma) \\ D_n^t &= [T_t,\delta_i,x_t] \end{aligned} \tag{7}$$

while taking logarithms and using appropriate transformations, likelihood functions can be expressed as follows:

$$l(\theta|D_n^t) = \sum_{i}^{N}[\delta_i ln(h_i(t_i|\theta) + ln(S_i(t_i|\theta))] \tag{8}$$

Kaplan – Meier estimator is a non – parametric statistical procedure, which is used to estimate the survival function from censored data. In the context of the present study, it is used so as to investigate the probability that a fast diffusion of innovative products, among different types of network environments [17].

4 Results

When virtual economic system is described by a regular network (Middle - Eastern markets), where coefficient of randomness (β) is zero, adopters are located in the same neighborhood, and virtual economic system is characterized by low propensity to innovate ($p = 0.001$), imitate ($q = 0.1$) and small amount of denials ($d = 0.01$) (Fig.3).

As time evolves, the WoM effect takes place and the diffusion process launches. At time step, $t = 20$, a small amount of non - adopters (white colour) decided to eventually adopt the innovative product (red colour), while at time step $t = 40$, more than half of agents who are in state 0 (non - adopters), remain uninformed and ultimately do not proceed in adopting the innovative product. At the end of the simulation horizon, namely at time step $t = 80$, a considerable amount of non - adopters, remained in the same state and located in the same neighborhood, but there are several non - adopters dispersed throughout this regular networked economic system.

In the second column of Fig.3, the evolution of innovative products' adoption is presented, when diffusion of such products takes place in a small - world network environment ($\beta = 52.6\%$). Unlike regular economic system, in small - world network, innovators are dispersed, and therefore diffusion of innovative products begins from different points. As a result, innovators (state 1) tend to motivate non adopters (state 0), without "forming queues", and therefore, diffusion is faster and eventually achieves the maximum market penetration level. At $t = 40$ steps, a considerable amount of consumers, have eventually adopted the innovative product, while at the end of simulation procedure ($t = 80$ time steps), almost all agents who are in the state 0 (non - adopter), were convinced and purchased the innovative product.

This finding suggests that diffusion of innovative products is slow and those products do not reach the maximum penetration level, when virtual economic system (market) is described by a regular small - world network topology.

The previous finding is enhanced when diffusion curves, generated by a combination of several values of p, q, d, and β, are presented together (Fig.4). During the first scenario, where values of innovation, imitation propensity as well as amount of denials, are small, diffusion is slow when small - world network is completely regular ($\beta = 0\%$) (blue line) and becomes faster when coefficient of randomness (β) becomes approximately equal to 1 (black line), leading to a fully randomized network topology.

Interesting is the fact that in the case when network topology is fully regular, maximum penetration level of an innovative product reaches approximately 80% at the end of time horizon ($t = 80$), while when it is fully randomized, innovative products not only are adopted by nearly all members who belong in the virtual economic system (except for denials), but also point of inflexion does not exceed $t = 20$ time step. In addition, results do not differ significantly among the cases when β varies from 43 to 71%.

This finding suggests that diffusion of innovative products, under the same conditions, in a small - world network ($\beta \simeq 50\%$), is as successful as it would be in a fully randomized small - world structure ($\beta \simeq 100\%$). When market is characterized by a large value of innovation propensity (p), and small values of q and d, diffusion of innovative products, in small - world network, in which coefficient of randomness is strictly larger than 0, is faster compared to the previous case. A large amount of innovators in a market, placed in a small - world topology, drives a successful diffusion of innovative products, even when network topology is completely structured ($\beta = 0\%$), as maximum penetration exceeds 80% of the market and inflexion point is located at $t = 60$, as opposed to the previous case where inflexion point was located at approximately $t = 70$ time step.

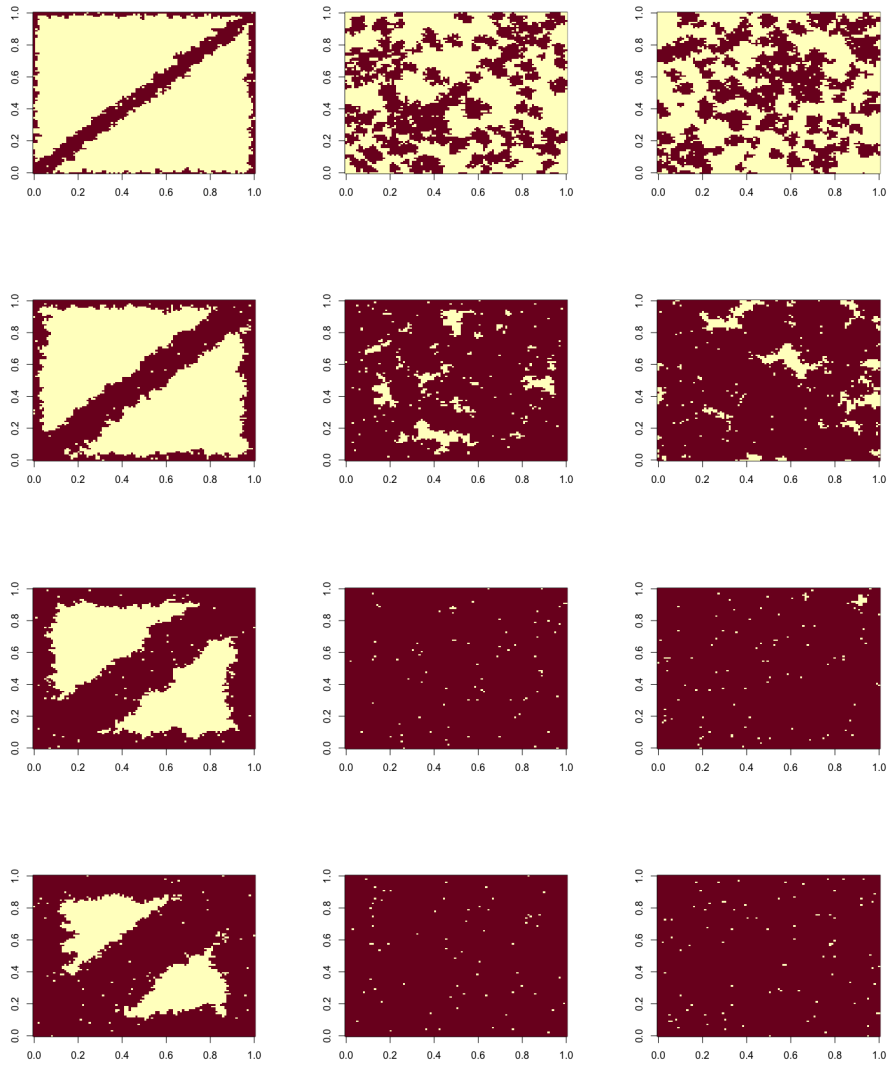
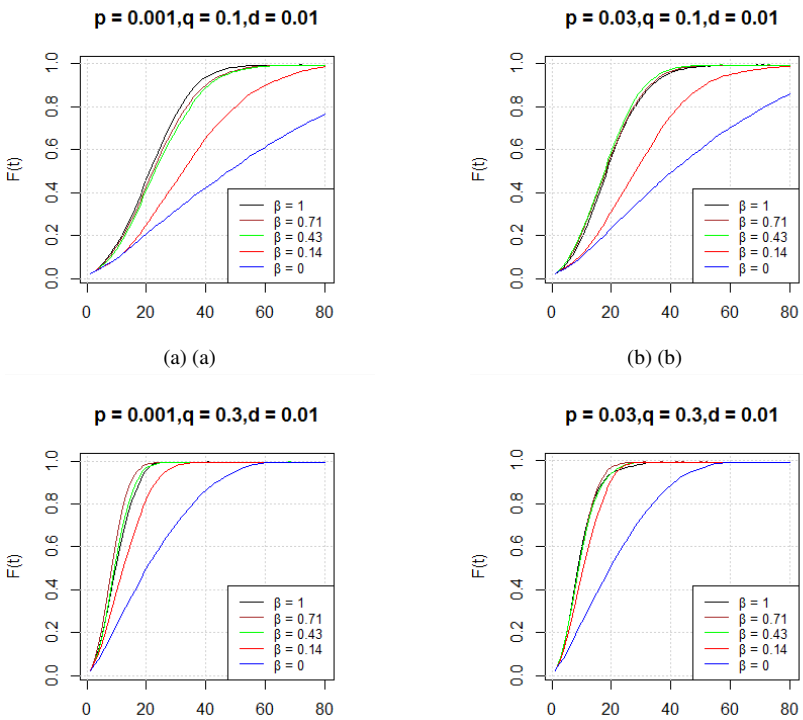


Fig. 3: 1st column: Diffusion in regular economic system ($\beta = 0\%$) for t = 20, 40, 60 and 80 time steps, 2nd column: Diffusion in small world economic system ($\beta = 52.6\%$) for t = 20, 40, 60 and 80 time steps, and 3rd column: Diffusion in random economic system ($\beta = 100\%$) for t = 20, 40, 60 and 80 time steps.



(a) (a) (b) (b)

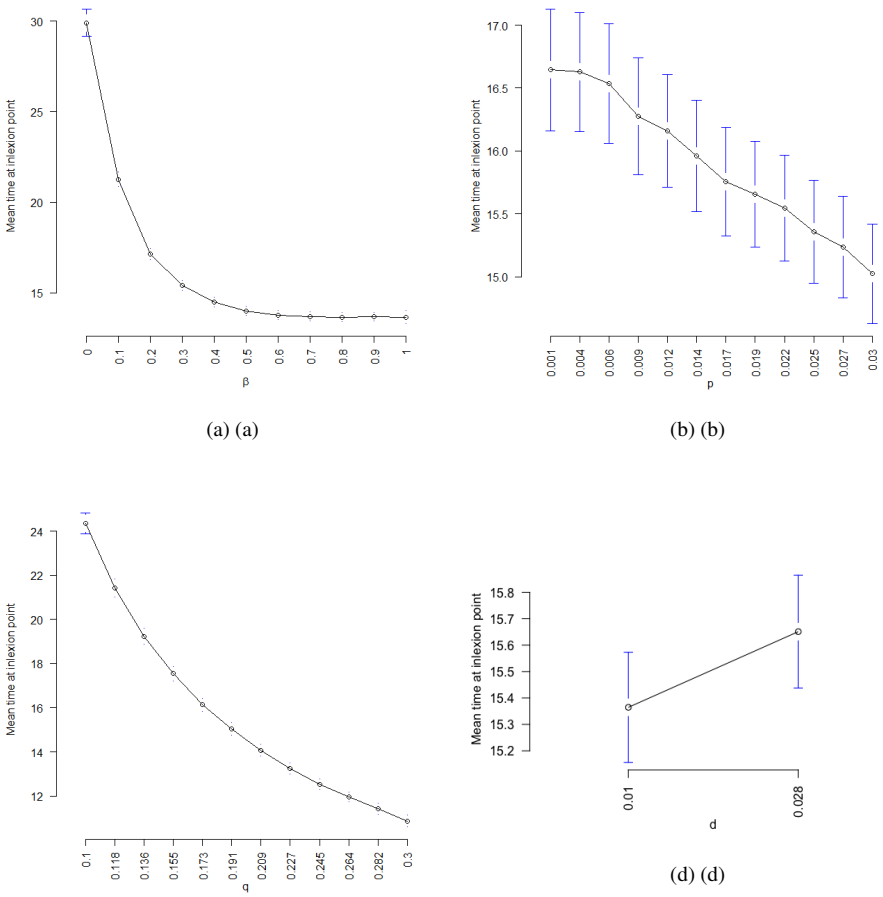
(c) (c) (d) (d)

Fig. 4: Diffusion process of innovative products in small - world networks with varying β for markets with (a) small values of p, q and d, (b) large value of p and small values of q and d, (c) small value of p and d and large of q and (d) large values of p, q and small value of d.

Results are extremely different when imitation parameter receives a large value. Innovators may be an important role in a successful innovative products' diffusion process, however, imitators are responsible to spread the Word - of - Mouth and eventually aware more uniformed consumers. Diffusion process, which correspond to the maximum value of imitation propensity ($q = 0.3$), is faster than in the previous cases and innovative products achieve almost the maximum penetration level in each value of β.

In Fig.5, the mean time at which inflexion point is achieved, is presented against varying values of simulation parameters. Increase in coefficient of randomness (β), causes an exponential descrease in mean inlfexion point time. Specifically, in regular small - world network topologies, across combinations of all other simulation parameters, mean inflexion time corresponds to approximately 30 time steps, while when coefficient of randomness' value is 50%, the mean time at inflexion point corresponds to less than 15 time steps. For values of β larger than 50%, mean time at which inflexion point is achieved, does not decrease significantly.

The number of innovators in the virtual economic system, seems to affect positively the speed of diffusion. In specific, a low intensity innovation propensity ($p = 0.001$), re-



(a) (a)

(b) (b)

(c) (c)

(d) (d)

Fig. 5: Mean time at which inflexion point is achieved for various values of (a) β, (b) p (c) q and (d) d.

sults in mean inflexion point time at approximately 16.5 time steps. A 22 times innovation propensity increment $p = 0.022$, would lead in a less than 3 times steps decrease in mean inflexion point time. When innovation propensity receives its maximum value ($p = 0.03$), mean inflexion point time is nearly 15 time steps.

Results for mean time in inflexion point, are different for various values of imitation propensity (q). When agents are characterized by a small value of imitation propensity ($q = 0.1$), mean time at inflexion point is approximately 24 time steps. At every increment of q, a large decrease in mean time is observed. When q receives its maximum value, a nearly 100% decrease in mean inflexion point time has taken place. This suggests that, along with coefficient of randomness (β), imitation propensity (q) is an important factor that leads

to successful diffusion of innovative products, within virtual economic systems which are characterized by small - world network properties.

Table 4: Estimation of Log - normal AFT models

	Dependent variable:		
	log(Time at inflexion point)		
	(Model 1)	(Model 2)	(Model 3)
p	-3.894^{***}	-3.736^{***}	-3.501^{***}
	(0.186)	(0.306)	(0.498)
q	-4.269^{***}	-4.343^{***}	-4.308^{***}
	(0.026)	(0.042)	(0.068)
β	-0.697^{***}	-0.722^{***}	-0.722^{***}
	(0.005)	(0.019)	(0.019)
d	1.154^{***}	1.148^{***}	1.689^{**}
	(0.188)	(0.309)	(0.694)
$p*\beta$	-	-0.331	-0.331
		(0.517)	(0.517)
$q*\beta$	-	0.157^{**}	0.158^{**}
		(0.071)	(0.071)
$\beta*d$	-	0.018	0.017
		(0.523)	(0.523)
$p*d$	-	-	-12.377
			(20.614)
$q*d$	-	-	-1.811
			(2.842)
Constant	4.077^{***}	4.089^{***}	4.079^{***}
	(0.007)	(0.011)	(0.016)
Log Likelihood	$-3{,}223.125$	$-3{,}220.535$	$-3{,}220.131$
χ^2	$18{,}780.760^{***}$ (df = 4)	$18{,}785.940^{***}$ (df = 7)	$18{,}786.750^{***}$ (df = 9)
R^2	74.47%	74.46%	74.47%
Note:		*p<0.1; **p<0.05; ***p<0.01	

Results from log - normal accelerated failure time model confirm findings from descriptive statistics. In Table 4, the estimation of three different models assuming a normal distribution for residuals, is presented. Model 1, assumes that logarithm of time until inflexion point, is affected by innovation propensity (p), imitation propensity (q), coefficient of randomness (β) and percent of denials in the virtual economic system (d). Model 2 contains the previous explanatory variables, plus interactions of p, q and d with coefficient of randomness (β), while in Model 3 interactions of p and q with d are added.

In Model 1, propensity to innovate seems to affect positively on the time at which inflexion point occurs and this effect is statistically significant at 1% level of significance ($b = -3.894, p < 0.01$). Coefficient values for propensity to innovation in Models 2 and 3 are

very close to the corresponding of Model 1 ($b = -3.736, p < 0.01$ and $b = -3.501, p < 0.01$ respectively).

After "seeds" of economic system adopt innovative products, agents who spread a positive "Word - of - Mouth" take action. Their actions decrease the time at which 50% of the sales are achieved ($b = -4.269, p < 0.01$) statistically significantly, and their effect is larger than the effect of innovators. Coefficient of q is quite the same across all estimated models ($b = -4.343, p < 0.01$ and $b = -4.308, p < 0.01$ respectively.

Coefficient of randomness (β) also affects time at which inflexion time occurs positively and statistically significantly ($b = -0.697, p < 0.01$). When β increases, namely, economic system turns from completely regular ($\beta = 0\%$) to completely ($\beta = 100\%$), adoption of innovative products is not only faster, but also achieve the maximum adoption penetration, which in the case of this study ranges from 95.4% to 99%. Estimated coefficient values of β are very close one another in all three estimated AFT models ($b = -0.722, p < 0.01$).

Obviously, amount of denials in an economic system, will hinder the speed of innovative products' diffusion, regardless other parameters which prove to facilitate adoption of innovative products. In Model 1, number of denials is positive and statistically significant ($b = 1.154, p < 0.05$), which means that a potential increase of denials in an economic system, would suffer a prolonged inflexion time, and therefore a slower diffusion process.

Models 2 and 3 include interaction terms of simulation parameters. Interaction term $p * \beta$ seems to affect positively time until inflexion, however, in both models this effect is not statistically significant. On the other hand, interaction term's $q * \beta$ coefficient, is positive and statistically significant at 5% level of significance ($b = 1.154, p < 0.05$). This means that when intense imitation takes places in a completely random economic system, diffusion of innovative products is slow, while when imitators interact with agents in a completely regular networked economic system, innovative products' adoption is fast.

Interaction terms $p * d$ and $q * d$ (Model 3), are not statistically significant, and this finding suggests that maybe the negative effect, which is caused by an increase of denials in an economic system, is cancelling off by an increase in the innovators and/or imitators. All three models are statistically significant at 1% level of significance, and exhibit an R^2 index which exceeds 70%.

In Fig. 6, the Kaplan - Meier estimators are presented (solid lines) along with fitted values from a log - normal AFT parametric model (dashed lines). Probability that inflexion point occurs at $t = 10$ time steps, is larger when the economic system is described by a completely random network (solid black line).In completely regular networked economic systems, it is more probable that inflexion point takes places in $t = 25$ steps. Economic systems that can be described by a small world network (solid green line), tend to behave similar to a random network, rather than a completely regular one.

5 Conclusions

Diffusion of innovative products is a multi - level product and its investigation is a complex procedure. Successful diffusion does not depend only on product characteristics per se (e.g price, quality utility, complexity and others), but also on market characteristics.

In the present study, we conduct a comparison between three major international markets, namely Northern American, European and Middle - Eastern, using a CA simulation model. We assumed that in a virtual market, there are three types of members; a) adopters, b) non - adopters and c) denials.

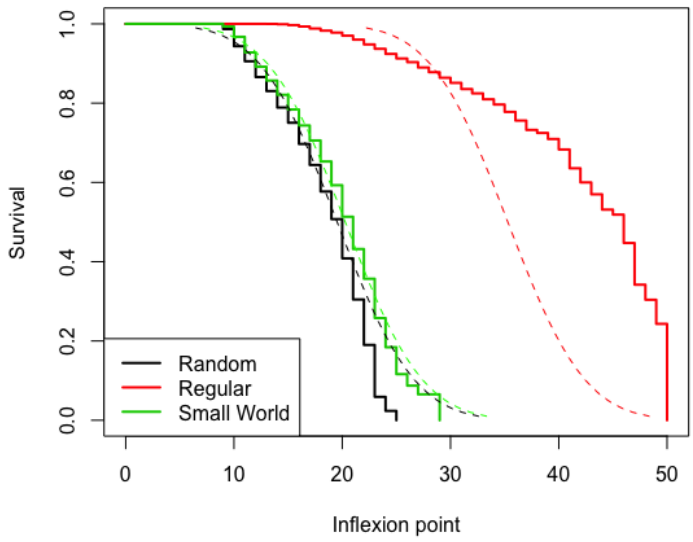


Fig. 6: Kaplan - Meier estimators for different values of randomness coefficients

Interactions between those virtual members, subject to rules, and in specific, adoption choice is a function of the mean neighborhood effect. We used von Neumann neighborhood, so as to investigate how positive and negative WoM affects speed of diffusion, in each market.

We characterized each market (Northern American, European and Middle - Eastern) according to mean IDV dimension score, obtained from Hofstede's website. Northern American market exhibits large values of IDV, therefore they are assumed to be a perfectly random network structure. Middle - Eastern markets, present a low value of mean IDV dimension score, and therefore, they are assumed to be more clustered and best described by a regular lattice. European markets, tend to perform a mean IDV dimension score, which lies between the two extreme cases. For that reason, we characterized the European market as a Small - World network topology.

Diffusion in regular small - world network topologies, which are used as proxy for Middle - Eastern markets, is slow and innovative products' penetration level is smaller than 100%, when the simulation model was assessed for $t = 80$ time steps. On the other hand, diffusion of innovations is rapid for European and Northern American markets, and innovation products' penetration level reach 100%. Mean inflexion time is 30 time steps for Middle - Eastern markets ($\beta = 0\%$), approximately 15 time steps for European markets ($\beta = 50\%$) and below 15 time steps for Northern American markets ($\beta = 100\%$). This result indicates that the clustering of individuals in a virtual economic system, largely affects the speed of diffusion and the innovation's market penetration level. Imitation effect, in Middle - Eastern markets, leads to prolonged diffusion time, whereas, in European and Northern American markets, imitation affects positively innovations diffusion speed.

The simulation model, assumes the existence of three types of consumers, a) non - adopters, b) adopters and c) denials, who interact within a small - world network virtual market. Such categorization was also adopted by Richins, (1983) [20] and Moldovan et al., (2004) [18], however, their models were cellular automata, without assuming that a market is characterized by a network which evolves over time.

Propensity to innovate (p), has a positive effect on speed of diffusion process, however, this effect is smaller compared to the effect of propensity to imitate(q). This is because innovators, who exist in this virtual economic system, are responsible to introduce the innovative product into the market. The topology at which they should be placed along the virtual market, plays an important role in the evolution of innovative products' diffusion process. In the case where they are all located in the same networking neighborhood, (e.g regular networked economic system), diffusion will be slow, as innovators are connected only to innovators, namely consumers who have already adopted the innovative products. In the case where they are dispersed, in a random way, within the virtual economic system, they can affect more non - adopters and in that way, diffusion is fast and successful. Therefore, number of innovators per se, is important but not adequate condition, for a successful diffusion process of innovative product.

This finding suggests that the process of imitating, or spreading the Word - of - Mouth (WoM), ensures that diffusion of innovative products will be fast. A small increase in propensity of imitation (q), resulted in a multiple times inflexion point mean time decrease. In specific, when non - adopters decided to imitate adopters at 10%, mean time at inflexion point was almost 24 time steps, while when propensity to imitation increased to 30%, mean time at inflexion point fell to less than 12 time steps.

Innovation propensity (p), facilitates the diffusion of innovative products and so does imitation propensity (q), leading in lower inflexion time point, and thus faster diffusion. However, the positive effect of imitation, is larger than that of innovation. As far as the coefficient of randomness (β), it enhances diffusion of innovative products, as it increases, namely, as an economic system turns from completely regular to completely random. Interesting is the fact that virtual economic systems which are characterized by small world topology, tend to be closer to random networks than regular ones, in terms of diffusion speed. Obviously, percent of denials (d), hinders diffusion of innovative products, regardless the network topology of a virtual economic system. However, the negative effect that comes from denials in an economic system, is outperformed by innovation and imitation propensity.

Interesting is the fact that increasing the amount of agents, who are placed into an economic system which is characterized by randomness ($\beta = 100\%$) and tend to imitate rather than take risks and innovate, lead in a slower diffusion. This finding suggests, that not only innovators should be placed according to a random networked economic system, but also imitators. In a different scenario, where innovators are placed on a fully random networked virtual economic system, and imitators are concentrated in a part of it placed according to a regular networked virtual economic system, diffusion of innovative products will not be successful.

Without limiting the value of denials in the virtual economic system, which is a realistic assumption, the positive force of adopters (state 1) and non - adopters who transited from state 0 to state 1, under a fully random virtual economic system, tends to outperform the negative effect of denials and eventually lead to a fast and successful diffusion process, unlike Richins, (1983) [20] and Moldovan et al., (2004) [18].

As future research, the above simulation model could expand in order a comparison between many different network structures to be feasible. As mention in Petridis (2015)

[19], under certain conditions using Operational Research methods, random networks could converge to power - law networks, enabling the investigation of innovative products within evolving network structures, feasible.

The size of every virtual economic system assumed in the present study, is rather limited ($N = 100$ agents), compared to other similar studies, and so is the time horizon ($t = 80$). These limitations are ought to lack of computing capacity, however, they are not a major drawback in generalizing results, due to the fact that the number of observations is adequately large ($T = 5,760$ data points).

References

1. Hirotugu Akaike. A new look at the statistical model identification. *IEEE transactions on automatic control*, 19(6):716–723, 1974.
2. Abbas J Ali, Abdul Aziz Taqi, and Krish Krishnan. Individualism, collectivism, and decision styles of managers in kuwait. *The Journal of Social Psychology*, 137(5):629–637, 1997.
3. Alain Barrat and Martin Weigt. On the properties of small-world network models. *The European Physical Journal B-Condensed Matter and Complex Systems*, 13(3):547–560, 2000.
4. Frank M Bass. A new product growth for model consumer durables. *Management science*, 15(5):215–227, 1969.
5. Deepa Chandrasekaran and Gerard J Tellis. A critical review of marketing research on diffusion of new products. *Review of marketing research*, pages 39–80, 2007.
6. Deepa Chandrasekaran and Gerard J Tellis. Global takeoff of new products: Culture, wealth, or vanishing differences? *Marketing Science*, 27(5):844–860, 2008.
7. Guillaume Deffuant, Sylvie Huet, and Frédéric Amblard. An individual-based model of innovation diffusion mixing social value and individual benefit. *American Journal of Sociology*, 110(4):1041–1069, 2005.
8. Paul Erdős and Alfréd Rényi. On the evolution of random graphs. *Publ. Math. Inst. Hung. Acad. Sci*, 5(1):17–60, 1960.
9. Jacob Goldenberg and Sol Efroni. Using cellular automata modeling of the emergence of innovations. *Technological Forecasting and Social Change*, 68(3):293–308, 2001.
10. Jacob Goldenberg, Barak Libai, Sarit Moldovan, and Eitan Muller. The npv of bad news. *International Journal of Research in Marketing*, 24(3):186–200, 2007.
11. Jacob Goldenberg, Barak Libai, Sorin Solomon, Naeem Jan, and Dietrich Stauffer. Marketing percolation. *Physica A: statistical mechanics and its applications*, 284(1-4):335–347, 2000.
12. Peter N Golder and Gerard J Tellis. Will it ever fly? modeling the takeoff of really new consumer durables. *Marketing Science*, 16(3):256–270, 1997.
13. Geert Hofstede. Cultural dimensions. *www. geert-hofstede. com, consulta*, 13, 2003.
14. Geert Hofstede. Geert hofstede cultural dimensions. 2009.
15. Elmar Kiesling, Markus Günther, Christian Stummer, and Lea M Wakolbinger. Agent-based simulation of innovation diffusion: a review. *Central European Journal of Operations Research*, 20(2):183–230, 2012.
16. David G Kleinbaum and Mitchel Klein. *Survival analysis*, volume 3. Springer, 2010.
17. DG Kleinbaum and M Klein. Survival analysis: A self-learning text, 2005. *New York, NY: Spring Science+ Business Media*, 2011.
18. Sarit Moldovan and Jacob Goldenberg. Cellular automata modeling of resistance to innovations: Effects and solutions. *Technological Forecasting and Social Change*, 71(5):425–442, 2004.
19. Nikolaos E Petridis. Convergence of random to power-law networks using a multi-objective non-linear programming model. *Journal of Multi-Criteria Decision Analysis*, 22(5-6):232–243, 2015.
20. Marsha L Richins. Negative word-of-mouth by dissatisfied consumers: A pilot study. *Journal of marketing*, 47(1):68–78, 1983.
21. Everett M Rogers. *Diffusion of innovations*. Simon and Schuster, 2010.
22. Hiroki Sayama. *Introduction to the modeling and analysis of complex systems*. Open SUNY Textbooks, 2015.
23. Gideon Schwarz et al. Estimating the dimension of a model. *The annals of statistics*, 6(2):461–464, 1978.
24. Samuel Thiriot and Jean-Daniel Kant. Using associative networks to represent adopters'beliefs in a multiagent model of innovation diffusion. *Advances in Complex Systems*, 11(02):261–272, 2008.

25. Tommaso Toffoli and Norman Margolus. *Cellular automata machines: a new environment for modeling*. MIT press, 1987.
26. John Von Neumann et al. The general and logical theory of automata. *1951*, pages 1–41, 1951.
27. Duncan J Watts and Steven H Strogatz. Collective dynamics of 'small-world'networks. *nature*, 393(6684):440, 1998.
28. Stephen Wolfram and A John Mallinckrodt. Cellular automata and complexity. *Computers in Physics*, 9(1):55–55, 1995.
29. Brent Zenobia, Charles Weber, and Tugrul Daim. Artificial markets: A review and assessment of a new venue for innovation research. *Technovation*, 29(5):338–350, 2009.
30. D Zhang. Modeling survival data with parametric regression models. *Analysis of Survival Data*, 2005.